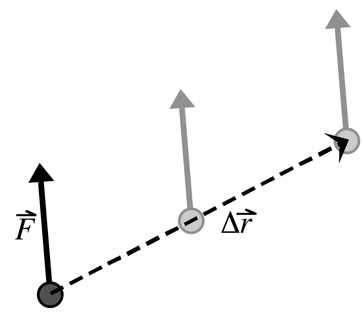
PHYS 2212 Lab Exercise 04: Work, Potential Energy, & Electric Potential

PRELIMINARY MATERIAL TO BE READ BEFORE LAB PERIOD

I. Review of Work: The idea of *work* was introduced in Physics I as being a measure of the *amount of energy transferred* to an object, occurring as the result of the object *experiencing a displacement* (that is, a change of position) *while under the influence of some force*. We calculate the work that is done via a **dot product:** never forget that when you are computing work, you are combining the *vectors* in such a way that a *scalar* quantity results. (This is why the "dot product" is often referred to as the "scalar product".) ***If*** the force that acts is **constant** throughout the displacement (that is, it's magnitude and direction are the same at *every point* passed through by the particle) then we can write:

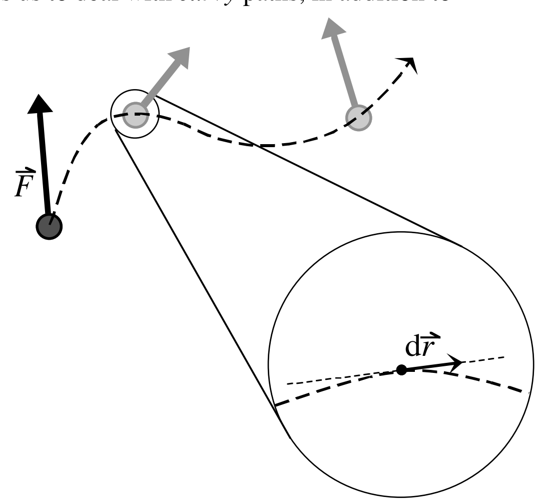


$$W = \vec{F} \cdot \Delta\vec{r}$$

On the other hand, if the force *varies from point to point* as the object moves about, the expression above **will not work**. Instead, we must break the displacement up into sub-displacements, and compute the work along each sub-segment separately, and then add the results together. (We can do this because a dot product is still a "linear product", and satisfies the distributive rule: the sum of products is equal to the product of the sums.) We thus write:

$$W = \sum \Delta W = \sum_i \vec{F}_i \cdot \Delta\vec{r}_i$$

Generally speaking, though, we can's stop there; usually, we have to reduce our sub-segments down to infinitesimal size. This is actually a *good* thing, since it allows us to deal with *curvy* paths, in addition to straight-line segments. Such an infinitesimal sub-segment would be labeled as $d\vec{r}$, and would point *tangent* to the curving path at every point along it. The advantage of using infinitesimal sub-segments is that any *particular* sub-segment will be so tiny that the force can be treated as being effectively *constant* over that region, and we can write:



$$dW = \vec{F}(\vec{r}) \cdot d\vec{r}$$

All that remains is to sum up all of these infinitesimal bits of work, which means—you guessed it—an integral:

$$W_{tot} = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}(\vec{r}) \cdot d\vec{r}$$

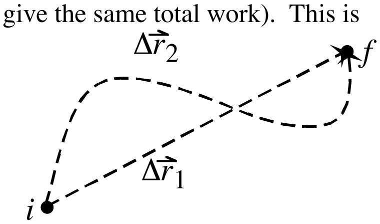
Thus, the most general expression for the work done by a given force is a *line integral* over the *path of displacement* (a.k.a. a "path integral").

The most important thing to keep in mind about this expression (and with it, the less general expression for work by a *constant* force) is the **dot product**—the relative directions of the force and displacement vectors are *crucial* to understanding work. When the vectors are *exactly* parallel, the dot product gives an overall "+1", and *positive* work is done. In fact, as long as the angle between the two vectors is *less than 90°*, you will always obtain a positive value for the work. On the other hand, when the relative angle *exceeds 90°*, the dot product will return a negative value, and we say than *negative* work was done. Pay close heed to the corollary of all this:

- **When the angle between the force and displacement is *exactly 90°*, the work done by the force on the object is *exactly zero*.**

II. Work and Energy: Having established a definition of work, we next recognize that it provides us with a means of measuring *energy transfers* to the object from the source giving rise to the force (or *vice versa*). That is: when *positive* work is done, we say that the force has *given* energy *to* the object; when negative work is done, the force has *removed* energy *from* the object. This transfer of energy can be manifested either in terms of the object speeding up or slowing down (a change in kinetic energy) or in terms of the object *storing* the energy in some manner (a change in *potential* energy).

More specifically, we recognize that for certain, special types of forces (*conservative* forces), the work done as the object moves between two particular points will always be completely independent of the *exact* route taken (that is, *any* two paths between the starting and ending points will always give the same total work). This is advantageous for a number of reasons: first, it makes the "work integral" ***much*** easier to compute, since one can always choose an *optimum* path between the endpoints that avoids the messier aspects of computing a dot product. (In a nutshell: you choose a path consisting of distinct sub-segments, such that on any given sub-segment, you are either displacing *parallel* to the force—dot product = +1—or else *perpendicular* to the force—dot product = 0.) Secondly, when a force is conservative, it becomes possible to account for the "non-kinetic" energy of the particle, simply by *keeping track of* ***where the particle "is".*** This "positional energy" is what we mean when we talk about *potential energy*.



So, when a *conservative* force acts, we say that the "work done by the force" simply measures the exchange of kinetic energy for potential energy (or *vice versa*). The standard convention for this transfer is that **positive work increases the kinetic energy and decreases the potential energy**:

$$\Delta K = +W_{cons}$$
$$\Delta U = -W_{cons}$$

On the other hand, for *non-conservative (NC)* forces, we **cannot** talk about some sort of "potential energy function", but we **can** still interpret the work that is done by these forces as a *kinetic* energy transfer:

$$W_{NC} = \Delta K$$

Thus, if we add together the work done by *all* conservative and non-conservative forces *combined*, we obtain the "Work-Kinetic Energy Theorem":

$$W_{total} = \Delta K$$

Moreover, when multiple forces (conservative and non-conservative) act, we can find that in some situations the object effectively acts as an "intermediary" for allowing the "energy provided by the NC force" to be converted directly into "potential energy of the conservative force". The clearest example of this is the process of lifting an object upwards at steady speed, against the force of gravity. To do so, an "external agent" (you) must exert an upward force (which is "NC"), while at the same time the gravitational force acts downward. Since the speed of the object is *constant*, there is *no net acceleration*, and we can thus conclude that the lifting force and the gravitational force must be exactly *equal in magnitude*. The lifting force will be doing *positive* work (force and displacement are parallel), which would tend to increase the object's kinetic energy. However, at the same time, the gravitational force is doing the exact same amount of *negative* work (force of *same* magnitude, which is *opposite* to the displacement). The negative gravitational work effectively *removes* the kinetic energy added by the lifting force as soon as it "arrives", and converts it into stored potential energy of the gravitational field.

We can account for these sorts of situations by establishing a "General Work-Energy Theorem", which lets us deal with all of the possible "energy rearrangements" due to conservative and NC forces *at the same time*:

$$W_{tot} = W_{NC} + W_{cons} = \Delta K$$
$$W_{NC} + (-\Delta U) = \Delta K$$

$$W_{NC} = \Delta K + \Delta U$$

This tells us that "work by a NC force" can *either* be manifested as an increase in kinetic energy, *or* as an increase in potential energy, or perhaps a "little of both".

III. Electrostatic Work & Electrostatic Potential Energy: It turns out that the electrostatic force described by Coulomb's Law is a "conservative" force—which has some important consequences. For starters, we must recognize that whenever an electric charge is *moved around* within a region containing an electric field, **work is done on the charge by the electric field**.

$$\begin{aligned} W_{elec} &= \int_i^f \vec{F}_e \cdot d\vec{r} \\ &= \int_i^f q\vec{E} \cdot d\vec{r} \\ &= q\int_i^f \vec{E} \cdot d\vec{r} \end{aligned}$$

Moreover, the conservative nature of the field allows us to choose *any* path for this calculation, which can greatly simplify the *details* of computing work in certain circumstances. As before, the "smartest" way to go is to choose a path for the charge consisting of segments either *parallel* or *perpendicular* to the direction of the field. In addition, we recognize that an "Electrostatic Potential Energy Function U_E" exists—and once we've "worked out the details" of this potential energy function for a particular electric field, we can proceed to make energy calculations *without having to resort to the work integral*! That is, we can perform a great deal of calculations using the principle of Conservation of Mechanical Energy:

$$\begin{gathered} \Delta K + \Delta U_E = 0 \\ (K_f - K_i) + (U_{Ef} - U_{Ei}) = 0 \\ K_f + U_{Ef} = K_i + U_{Ei} \end{gathered}$$

The key to figuring out the "appropriate U_E"for a particular field is to note that *positive* electrostatic work implies a *decrease* in electrostatic potential energy (and *vice versa*). So, we **arbitrarily choose** a location "0" that we want to be the "zero PE" location, and we compute the PE at any *other* location by figuring out the work that is done during the move from 0 to the other location:

$$\begin{aligned} U_f &= U_i + \Delta U \\ U(\vec{r}_f) &= U(\vec{r}_i) + [-W_{elec}(i \rightarrow f)] \\ U(\vec{r}) &= U(0) + [-W_{elec}(0 \rightarrow \vec{r})] \\ U(\vec{r}) &= -W_{elec}(0 \rightarrow \vec{r}) \end{aligned}$$

IV. Electric Potential: The idea of electric potential energy is certainly *convenient*, but it does have a major drawback: the amount of potential energy involved in any displacement is always necessarily *dependent upon the* ***amount of test charge*** *q that has been moved*! That is unfortunate, because the *whole point* of electric fields is that their properties exist *even in the absence of test charges*. So, we seek to find a way of characterizing the "energy content" of fields in a manner that is independent of the presence or absence of test charges. The process is *exactly parallel* to what was done in the *Force → Field* transition: we will define the "potential energy *per unit test charge*" as being the "physically relevant quantity". It is to this end that we define the ***Electric Potential, V***:

$$V \equiv \frac{U_E(\text{for test charge } q)}{q}$$

or

$$\Delta V \equiv \frac{\Delta U_E}{q}$$

It is important to note that "Electric Potential" is ***not*** the same thing as "Potential Energy"—these are two *very different* quantities. In particular, the *electric potential* is something that "exists" (i.e. can be measured) at every point in space within an electric field—regardless of whether or not a test charge is there to "feel" it. On the other hand, potential *energy* can only be discussed if an *actual test charge* is placed within the field.

Understanding what "electric potential" actually *means* can be very tricky for many students. The standard mistake that "newbie" physics students make is to confuse the *units* for this quantity with the thing itself. Electric potential is measured in "volts", which motivates students to drop the reference to *potential* entirely, and talk about *voltages* instead. **Danger, Will Robinson!** This misconception encourages students to start thinking about "volts" as being "real things" that can move around the way test charges might. ("Well, gee… there are nineteen volts going through this resistor, which means that…."—**Not hardly likely, Buster!)**

The best way to get a "feel" for what electric potential *represents* is to think of the analogous quantity of "*gravitational* potential"— which is to say "gravitational potential energy per unit test mass". If a mass *m* is placed in the Earth's gravitational field (and presuming that we're near the Earth's surface, wso that the gravitational force is constant), we can express the potential energy of the test mass, when at height *h* above the ground, as:

$$U_g(h) = m\,g\,h$$

We can then to compute the PE *per unit mass*, and call this the Gravitational Potential Γ (Why Γ? Well, I can use any symbol I want, and the greek version of "G" seems well-suited to the task…) We then have:

$$\Gamma(h) = \frac{U_g}{m} = g\,h$$

Thus, the *gravitational potential* is really just a measure of "height in the gravitational field" (albeit with an extra overall scale factor of *g*). That is, the *higher* you are, the *greater* your gravitational potential, and *vice versa*. (Why not just use gravitational potential *energy*, and save ourselves from this mess? Because two *different* masses at the *same* height will have *different* values for *Ug*!) We could then "map out" the Earth's gravitational field by assigning a "potential" to each point in space, ***and this would be exactly equivalent to creating a topographic map of elevations***. Of course, the whole "gravitational potential" step could have been skipped entirely—we could have *started* by simply mapping out the heights "*h*" in the first place.

With *electric* fields, the process is equivalent, *including the final step in which we interpret what "potential"* ***means***: the electric potential is effectively an assignment of "electrostatic height" to each point in space within an electric field, creating an effective "topographic map" of the field. The field will then exert forces on *positive* test charges that cause them to move from high potential to low potential (just like *masses* feel a gravitational force from high elevations to low elevations). The exact direction of that force will be in the *most direct* route to lower potential—which is to say, along the "path of steepest descent" on the topographic map. (In a *gravitational* field, "steepest descent" would be a fancy way of saying "**straight down**".)

Of course, we should also be careful to recognize that *negative* test charges within a field will always experience a force *opposite* to what positive charges would feel. Thus, we conclude that *negative* test charges in a field will feel a force directed from *low* electric potential to *high* electric potential—and that force will be along the shortest and most *direct* route to higher potential ("**straight up**"). In other words: negative charges naturally "fall" from low electric potential to high electric potential, along the "path of steepest *ascent*" on the topographic map. We would have to do *external work,* by forcibly *pushing* such a negative charge, if we wanted to make it move to *lower* potential—and in the process, of course, we would be *increasing its potential energy*.

With this perspective, it is perhaps easier to see why the phrase, "there are nineteen volts going through this resistor" is, well, *just plain silly*: "voltages" are effectively a measure of *height differences* between two points in space (or between two points in a circuit). Height differences don't "go" anywhere. (Have you ever seen "nineteen feet" *go* down through a stairwell? Of course not; the statement is semantically ill-posed—unless you're talking about nine people and a peg-legged pirate.) You would say, instead, "anything that *moves through* the stairwell will *lose* nineteen feet of elevation" (or gain it, or course, depending upon the direction of travel). The same semantics holds true for circuits. The "voltage" that one talks about is a measure of the "electrostatic elevation change" that anything experiences *when it moves between the two points in question*.