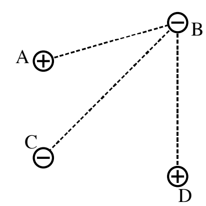
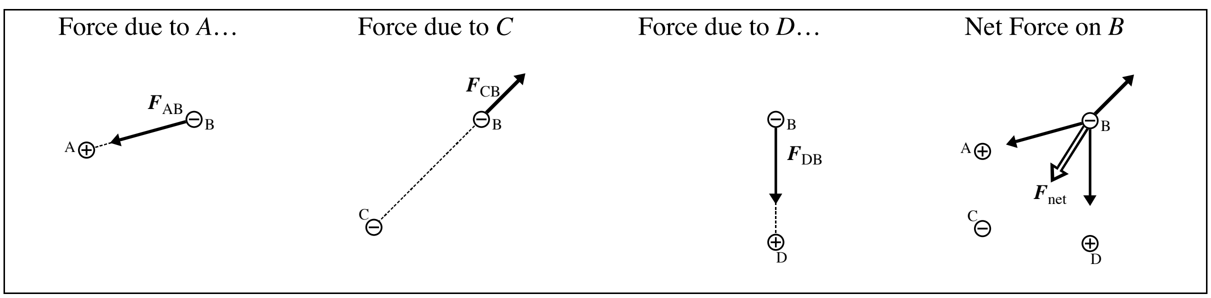
PHYS 2212   Lab Exercise 01: Force, Field, & Superposition

PRELIMINARY MATERIAL TO BE READ BEFORE LAB PERIOD

I. **Force & Coulomb's Law:** Coulomb's Law is an empirical rule describing the force between charged particles. (By "empirical", we mean that the law is a principle inferred from *experimental* observation and measurement.) The two most important preliminary issues we must realize are: **(1) Coulomb's Law only applies between *pairs* of charged particles**; and **(2) Coulomb's Law is only accurate if the charges involved are *pointlike*.** These issues will have a number of important consequences when we try to make practical use of Coulomb's Law. A third key point that we must keep in mind is: **(3) the forces implied by Coulomb's Law are always along the direct line joining the two charges.** If they are *like* charges, the force is repulsive, while if they are *unlike* charges, the force is attractive.

What impact do these constraints have in practice? First of all, when *more than two* point charges are present, we must compute the *net* force on any particular charge by *summing* all of the individual force vectors due to the pairwise interactions with each of the other charges. (This is a vector summation, not a simple scalar addition of magnitudes.) In order to accomplish this summation we take advantage of the fact that **the force between any pair of charges is *independent* of the possible presence of any other *charges* nearby.** This means, for example, that when we try to compute the net force on particle *B* in the diagram at right, we do so via three separate and utterly independent applications of Coulomb's Law: once to get the force vector $\boldsymbol{F}_{AB}$ (ignoring *C* and *D*), once to get the force vector $\boldsymbol{F}_{CB}$ (ignoring *A* and *D*), and once to get the force vector $\boldsymbol{F}_{DB}$ (ignoring *A* and *C*)





This process—determining a *net* effect due to multiple causes by first determining the separate effects and then adding them together—is the essence of the *Superposition Principle*. This principle allows us to turn complex problems into smaller, "bite-sized" chunks, which can be solved independently and their answers added back together to give a final result (in what would otherwise be an very difficult calculation).

We must also keep in mind that we are *only* allowed to apply Coulomb's Law when we are dealing with *"pointlike"* charges. What does this mean? Essentially, a charge is pointlike if it "looks" like a point particle—the size of the charge should be something so small as to be negligible. More practically, we say that charges can be treated as pointlike (and hence, Coulomb's Law can be applied) in situations where ***the sizes of the charged particles are very small compared to the distance that separates them.*** (In the diagrams above, the sizes of the charges are merely "small"—rather than "very small"—compared to the distances between the charges, so we are almost at the limit where Coulomb's Law begins to break down.) Requiring charges to be pointlike will make the calculation of electrostatic forces between *charged bodies* (e.g. a rod or ball) somewhat difficult when the bodies are *close together*—but if they are *very* far apart, each body will appear pointlike to the other, and Coulomb's Law *can* be applied in that limit!

II. Electric Fields: The ability for electric charges to interact with one another without *direct physical contact* is an example of "Action at a Distance". Although this may not be troubling to a non-scientist, Action at a Distance implies the ability for widely separated particles to exchange energy and momentum across the empty space between them. In order to avoid the conceptual difficulties that this would lead to, we introduce the idea of the "Field" as an *intermediary mechanism*, which allows the interaction (and the energy/momentum transfers) to "cross" empty space.

In electrostatics, we thus imagine that a given charge creates a "field" in the space around itself—which we interpret to mean a *distortion of the properties of space itself*. We then say that when other nearby charges experience a force, it is not due to the direct action of the original charge, but rather, it is the result of the field distortion *at the position of the new charge* which is actually exerting the force on that charge. This field is "invisible" to *us*—we cannot see or feel it, so it might seem to be just an imaginary thing that we "make up" to explain action at a distance. However, to *other charges* in the vicinity of the given charge, the field is *very* real, since it exerts a force on them. What's more, **we** can see and measure the force exerted upon those other charges, and thus we can *indirectly* infer the presence of the field that we cannot otherwise see.

To apply the field concept to electrostatics, we must first make a key distinction between **source charges** and **test charges**:

- A **source charge** is a charge that *creates* an electric field.
- A **test charge** is a charge that *experiences a force* due to an electric field that is *already present*.

Technically, *every* charge is simultaneously a source charge and a test charge, since every charge creates its own electric field, and every charge experiences forces due to the fields created by *other* charges. However, **no charge *ever* creates a field that exerts a force back on that same charge.** (Equivalently, "No charge ever exerts an electrostatic force on itself."—can you see *why* this must be true?) In practice, for any particular charge, it's an "either/or" distinction—*either* we are interested in the field which that charge creates, *or else* we are concerned with how that charge responds to a field that already exists.

The source/test charge distinction also gives us an "operational" means of defining the electric field in a rigorous mathematical sense. We only "see" fields via the force that they exert on test charges, so we will formally define the field in terms of the observed force. That is, we will use a test charge as a "probe" of the electric field. Before we do so, we must note a crucial requirement of the "probing" process: an electric field depends only upon the *source* charge (or charges) that created it. This means that:

- **A given field is *completely* independent of the properties of any test charges which might be used to probe it.**
- **A given field exists even if *no* test charges are around to probe it.**

Now, let us set about using a test charge to probe a pre-existing field. We do so by moving the charge around, and at each position we measure the force exerted on that charge. The only problem is that Coulomb's Law[1] tells us the force on the test charge is directly proportional to the amount of charge you are using as a probe! This violates the requirements of the first "bullet" above. We can get around this obstacle by "dividing out" the offending quantity: the amount of test charge being used. Hence, the **Electric Field** is formally defined as the **Force Per Unit Test Charge**. In particular, if a positive test charge q is placed at some point in space, and a resulting force $\vec{F}$ is found to act on that test charge, the electric field is:

$$\vec{E} \equiv \frac{\vec{F}}{q}$$

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[1] Question: we never said that the field we are probing is due to a *single*, *pointlike* source charge—so how do we know that conclusions based on Coulomb's Law are valid here?

It is conventional (although not strictly necessary) to use **positive** test charges when "probing" a particular field. This has the advantage of making the **vector** aspects of the problem more straightforward. Keep in mind that the electric field is a vector quantity, and hence the *direction* of the field is just as important as its *magnitude*. If we restrict ourselves to positive test charges, we conclude that:

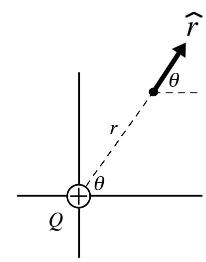
> **The direction of an electric field at a given point in space is the direction of the force that would be exerted on a positive test charge.**

A corollary to this is:

> **The direction of the electric field at a given point in space is *opposite* to the direction of the force that would be exerted on a *negative* test charge.**

Note that regardless of whether we use a *positive* test charge, or a *negative* test charge, or even *no test charge*[2], to probe a particular pre-existing field, we will always end up measuring the exact same values for the magnitude *and the direction* of the field. Finally, we should keep in mind that this "probing" process works on a point-by point basis: we *pick* a particular point in space (the "Field Point"—which is usually an *empty* point in space), and evaluate the field *at that point* by use of a test charge. If we want to know the field at some *other* point, we will have to separately probe *that* point in space, obtaining a *different* value for the magnitude and direction of the field at the new point.

III. Fields due to Multiple, Pointlike Source Charges: First of all, we note that if we have a *single* pointlike source charge, the force on a nearby test charge can be immediately computed via Coulomb's Law, and hence the electric field can be determined by simply *dividing out* the value of the unknown test charge. If we let "*Q*" be the source charge and "*q*" the test charge, we have:



$$|\vec{F}| = k\frac{|Q|\cdot|q|}{r^2} \quad \text{and} \quad \vec{E} = \frac{\vec{F}}{q} \quad \text{imply that} \quad \boxed{\vec{E} = k\frac{Q}{r^2}\hat{r}}$$

("**Coulomb's Law for the Electric Field**")

Here, $\hat{r}$ represents a unit vector **at the location where the field is being calculated,** pointing **directly away from the source charge *Q***. Note how this guarantees that the electric field would point away from a positive source charge (+*Q*) and/or towards a negative source charge (–*Q*). (In turn, that would imply a force on *positive test charges* that would be away from positive sources and/or towards negative sources. Likewise, a *negative test charge* would experience a force towards positive sources and/or away from negative sources. In *every* case, the rule of "opposites attract and likes repel" is satisfied.)

We can then expand this definition to encompass *multiple* pointlike source charges if we simply recognize that electric fields are subject to the Superposition Principle, in the same way that electrostatic forces are:

> **The electric field due to a collection of point source charges is the vector sum of the individual fields computed for each source charge separately.**

We can similarly consider the electric field of a continuous distribution of charges by breaking up the distribution into tiny sub-regions (which appear pointlike from our perspective). Then for the particular Field Point where we are computing $\vec{E}$, we first find the field vectors at that point due to *each* of the sub-regions separately (by treating them like point particles), then summing those results (vectorially) to get the net field at the point under consideration.

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[2] Recall that fields exist even if there are *no* test charges around to probe them. That means that we don't need to use *actual* charges to measure a field—we could simply image a "what if?" scenario, and determine the field by *pretending* that a test charge (of arbitrary and unknown charge "*q*") were present, and computing the force per unit of imaginary charge. Since we are dividing out the value of *q*, it won't even matter that *q* itself *doesn't actually exist!*