PHYS 2212 Problem-Solving Studio 11

Apr 18–21

SOLUTION

Danger Through Electromagnetism

Just for fun, you decide to build a powerful hand-held electromagnet that can-generate a field strength of 0.50 T. You realize that by using a solenoid with a ferromagnetic core, you can achieve the desired field using a current of only 0.20 A in the solenoid. You wind a copper wire several hundred times around an iron core, to build a solenoid with a resistance of 30 Ω and self inductance of 0.50 H. You hook up the solenoid in series with a suitable emf, along with a simple on/off toggle switch. When you flip the switch on, you realize that a 0.50 T magnet is MUCH too strong to handle safely, so you quickly switch it back off again—but when you do, you receive a really nasty electrical shock from the switch. On the way back from the ER, you wonder what average voltage was generated across the switch, just after it was opened. A quick internet search tells you that a typical current requires about 250 microseconds to die off to zero, after a toggle switch is opened.

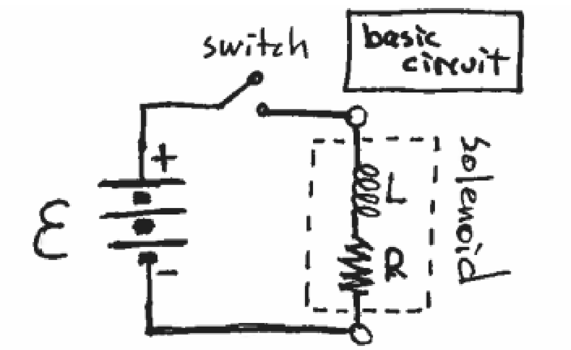
Instructions:

Construct a visual representation of the situation described, with all physical quantities represented by symbolic variables. Identify the concepts that will be needed to answer the question posed, as well as any simplifying assumptions that you will use. Outline a plan (that is, a series of analytical steps) that you will use solve the problem, and then follow those steps to solve the problem.

You may work as a group to complete this exercise, but each student is expected to submit an individual solution.

Assumptions and Observations:

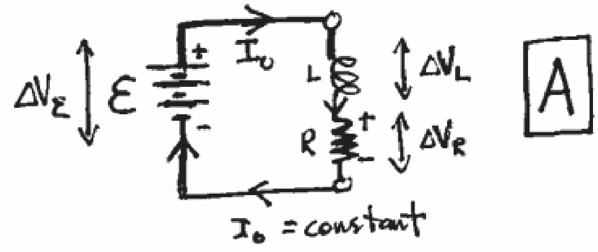
- Solenoid can be treated as an inductor and a resistance <u>in series</u>
- Switch is closed long enough for current to build up to steady-state value I_f
- When switch is reopened, I_f becomes "I_0", and drops to zero in time Δt – the average current during this interval is $I_0/2$
- Required emf $\mathcal{E}$ can be found by analyzing steady-state circuit with switch closed



Problem Formulation

Determine the voltage across an open switch in a series RL circuit with emf $\mathcal{E}$ and initial current I_0

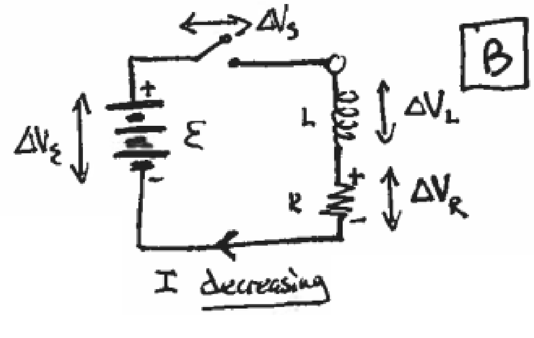
steady-state: switch closed:



Outline of solution

1. Apply loop rule to steady-state circuit A, to learn the required emf $\mathcal{E}$
2. Apply loop rule to the "dying" circuit B, to learn the ΔV across switch. Assume an average value $I_0/2$ for the current, in order to compute the average ΔV_R across the resistor.

Re-open switch:



① Steady-state circuit – see figure A

- I = constant tells us that $\Delta V_L = -L\frac{dI}{dt} = 0$ → inductor is "invisible" to loop rule
- loop rule gives: $\Delta V_\varepsilon + \Delta V_S + \Delta V_L + \Delta V_R = 0$

$$+\varepsilon + 0 + 0 + (-I_0 R) = 0$$

$$\boxed{\varepsilon = I_0 R = 6.0\text{V}}$$

(so, four 1.5-V batteries in series)

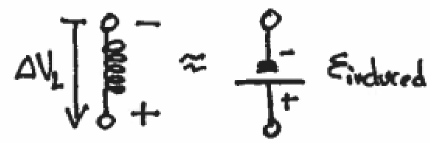
② Current die-off – see figure B

- Now, $\Delta I = I_f - I_0 = 0 - I_0 = (-I_0)$, in elapsed time Δt → $\Delta V_L = -L\frac{dI}{dt} \approx -L\frac{\Delta I}{\Delta t}$

$$= (-L)\frac{(-I_0)}{\Delta t} \rightarrow \boxed{\Delta V_L = +\frac{I_0 L}{\Delta t}} = 400\text{V}!!$$

Holy cow!

positive sign tells us that inductor is acting as "sustaining emf":



- using average value for current, we get $\Delta V_R \approx -\left(\frac{I_0}{2}\right)R = -3.0\text{V}$
- loop rule gives:

$$\Delta V_\varepsilon + \Delta V_S + \Delta V_L + \Delta V_R = 0$$

$$(+6\text{V}) + \Delta V_S + (+400\text{V}) + (-3\text{V}) = 0$$

again – Holy Cow!

so $\boxed{\Delta V_S = -403\text{V}}$