PHYS 2212 Problem-Solving Studio 09

Apr 04–07

SOLUTION

Electromagnetic Valves

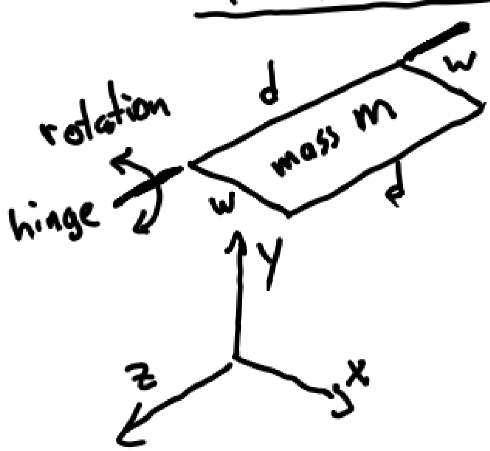
Your undergraduate research project involves the development of a new ultra-efficient internal combustion engine. Your particular project is to investigate electromagnetically controlled fuel-injection valves. A single valve is made of a thin rectangular piece of non-magnetic material with a loop of wire around it's perimeter. The rectangle (having mass 15 g and dimension 3.0 cm × 6.0 cm) is hinged along one of the long sides, and lies flat atop the engine manifold when closed. The valve is placed in a uniform magnetic field of magnitude 210 mT, oriented such that the field lies in the plane of the valve and is parallel to the short sides of the rectangle. The region with the magnetic field is slightly larger than the valve. You need to know what minimum current flowing around the perimeter of the wire loop will cause the valve to open.

Instructions:
Construct a visual representation of the situation described, with all physical quantities represented by symbolic variables. Identify the concepts that will be needed to answer the question posed, as well as any simplifying assumptions that you will use. Outline a plan (that is, a series of analytical steps) that you will use solve the problem, and then follow those steps to solve the problem.

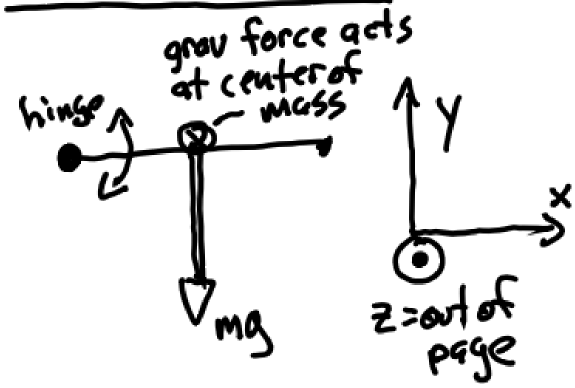
You may work as a group to complete this exercise, but each student is expected to submit an individual solution.

① Visualize the problem

perspective view



edge-on view



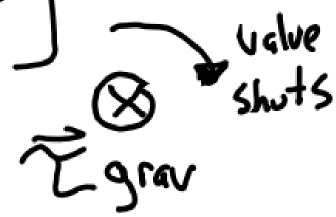
② This problem is about rotating the valve about its hinge

→ this is a torque problem

ⓐ Gravitational torque acts clockwise and holds the hinge shut

[for clockwise rotation, $\vec{\tau}_g$ is along the negative z-axis]

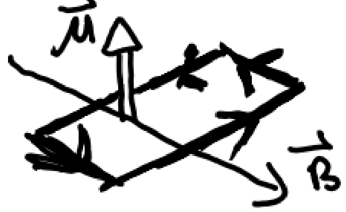
$$\vec{\tau}_{grav} = \vec{r} \times \vec{F} = \left(+\frac{w}{2}\hat{i}\right) \times \left(-mg\hat{j}\right)$$



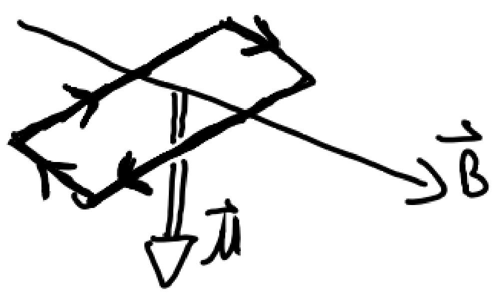
(2b) we need a counter-clockwise magnetic torque to <u>open</u> the valve, with $|\vec{\tau}_{mag}| \gtrsim |\vec{\tau}_{grav}|$

So — how do we <u>get</u> magnetic torque?
→ current around <u>perimeter</u> of valve creates a magnetic dipole moment $\vec{\mu}$, and $\vec{\tau}_{mag} = \vec{\mu} \times \vec{B}$

(3) More visualization



This situation (ccw current) creates $\vec{\mu}$ along $+\hat{j}$ direction
$\vec{\mu} \times \vec{B} \sim (-\hat{k})$ = into the page
→ THIS WON'T WORK



This situation (cw current) creates $\vec{\mu} \sim (-\hat{j})$ and resulting torque $\vec{\mu} \times \vec{B} \sim (+\hat{k})$ = ccw
→ THIS **WILL** work

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(4) Magnetic torque is $\vec{\tau}_{max} = \vec{\mu} \times \vec{B} = I A \hat{n} \times \vec{B}$
$= I(dW)(-\hat{j}) \times (B_0 \hat{i})$
$= I dW B_0 (+\hat{k})$

Now require net ccw torque:

$$\vec{\tau}_{net} = \vec{\tau}_{mag} + \vec{\tau}_{grav} > 0$$

$$(+I dW B_0)\hat{k} + \left(-mg\frac{W}{2}\right)\hat{k} > 0$$

$$I d \cancel{W} B_0 > \frac{mg \cancel{W}}{2} \Rightarrow \boxed{I > \frac{mg}{2 d B_0} = 5.83\,\text{A}}$$