PHYS 2212 Problem-Solving Studio 07

Mar 07–10

SOLUTION

Better Battery Life?

Your robotic pogo-stick project keeps failing because a critical electronic component is overheating—so you add a battery-powered cooling fan. Your calculations tell you that you need a minimum fan power of 0.34 W for proper cooling. The cooling circuit consists of two 1.5 V "AA" batteries in series with the fan itself. Plugging in a pair of old batteries that were lying around, you measure the total terminal potential across both batteries to be 2.25 V, and a current draw of 0.125 A, which is insufficient for your purposes. When you plug in a pair of fresh batteries, you measure 2.70 V and 0.150 A, respectively ... which would work. You wonder if you could get by with one fresh battery and one old battery, in order to get a longer overall battery life.

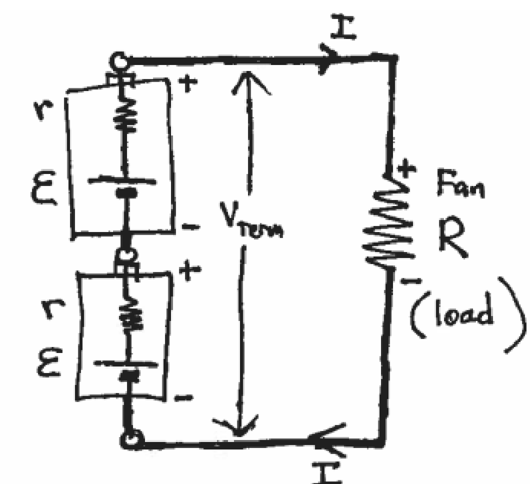
Instructions:

Construct a visual representation of the situation described, with all physical quantities represented by symbolic variables. Identify the concepts that will be needed to answer the question posed, as well as any simplifying assumptions that you will use. Outline a plan (that is, a series of analytical steps) that you will use solve the problem, and then follow those steps to solve the problem.

You may work as a group to complete this exercise, but each student is expected to submit an individual solution.

① Note that in both cases, V_{Term} < total emf. This tells us that we have to treat both the old and the fresh as real batteries, with internal resistance r

- Old batteries r_o result in current I_o and terminal potential V_o
- Fresh batteries r_f result in current I_f and terminal potential V_f



② Fan acts as a load resistance R

- current out of batteries = current through load
- Voltage rise V_{Term} in batteries = voltage drop in R

⇒ Ohm's law will allow us to deduce value for R

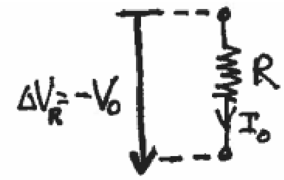
old batteries:
use r_o, V_o, I_o
fresh batteries:
use r_f, V_f, I_f

Formulation of Problem:

From the given information, deduce the internal resistances and load Resistance. Use this information to compute the power output when an old battery and a fresh battery are in series with the load.

Plan: ① Figure out the load resistance R using Ohm's Law
② Use loop rule twice to learn the internal resistances r_o and r_f
③ Use loop rule a third time, with o+f in series, to deduce the mixed current draw I_m and mixed terminal potential V_m
④ Mixed power output is found as the product $I_m \cdot V_m$

① with old batteries, voltage drop across load is $-V_0$ and current through load is I_0



$\to$ Ohm's Law says $\Delta V_R = -IR$

$$(-V_0) = -(I_0)R \Rightarrow R = \frac{V_0}{I_0}$$

$$\Rightarrow \boxed{R = 18.0\,\Omega}$$

(Note that you will get exactly the same value if you use V_f and I_f)

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② loop with old batteries (see diagram in Part One)

$\to$ loop CW, starting at (−) terminal of bottom battery

$$\underbrace{\left[(+\mathcal{E})+(-I_0 r_0)\right]}_{\text{battery-1}} + \underbrace{\left[(+\mathcal{E})+(-I_0 r_0)\right]}_{\text{battery-2}} + \underbrace{\left[-V_0\right]}_{\text{voltage drop across load}} = 0$$

$$\to 2\mathcal{E} - 2I_0 r_0 = V_0$$

$$\to 2\mathcal{E} - V_0 = 2I_0 r_0 \longrightarrow \boxed{r_0 = \frac{2\mathcal{E}-V_0}{2I_0} = 3.00\,\Omega}$$

exact same analysis for fresh batteries gives $\boxed{r_f = \frac{2\mathcal{E}-V_f}{2I_f} = 1.00\,\Omega}$

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③ Now consider same loop with one old / one fresh battery. let current drawn by load be I_m $\to$ Voltage drop across load is $\Delta V_R = (-I_m R)$

loop rule gives: $(+\mathcal{E} - I_m r_0) + (+\mathcal{E} - I_m r_f) + (-I_m R) = 0$

$$I_{mix} = \frac{2\mathcal{E}}{R + r_0 + r_f} = 0.13\overline{63}\text{ A}$$ (retain all decimals for now, round at end)

Then power consumed by load is

$$P_R = I_R \Delta V_R = -I_{mix}^2 R = \boxed{0.335\text{W}}$$

insufficient power!
We need 0.340 W!