PHYS 2212 Problem-Solving Studio 06

Feb 28–Mar 03

Capacitor Crash & Burn

SOLUTION

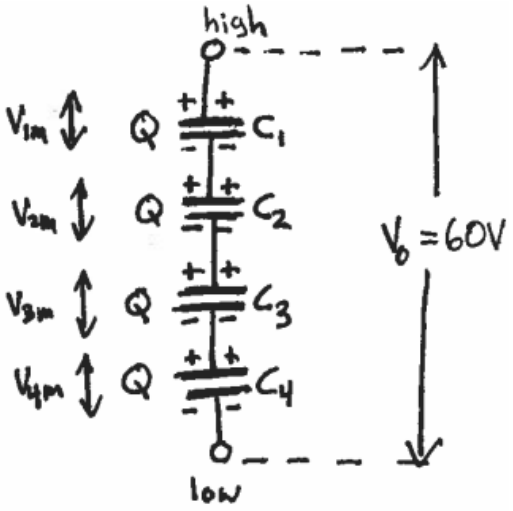
You are in the electronics lab, soldering together a capacitive voltage divider for your robotics project. The divider is a series arrangement of four capacitors, having capacitances of 4.0 μF, 8.0 μF, 12 μF and 6.0 μF (in that order). When you apply a test voltage of 60 V across the divider, you hear a "pop" and smell burning plastic. Looking closely at the network, you cannot see any visible clues that would indicate which capacitor popped. Looking up the specs on each capacitor, you see that they can withstand maximum internal voltages of 30 V, 20 V, 10 V, and 10 V (in the same respective order as above). You need to figure out which capacitor(s) you must replace, and what maximum voltage you can place across the repaired network.

Instructions:

Construct a visual representation of the situation described, with all physical quantities represented by symbolic variables. Identify the concepts that will be needed to answer the question posed, as well as any simplifying assumptions that you will use. Outline a plan (that is, a series of analytical steps) that you will use solve the problem, and then follow those steps to solve the problem.

You may work as a group to complete this exercise, but each student is expected to submit an individual solution.

- Capacitors are in series so all individual capacitors store the same charge Q
- Each capacitor C_i can maintain a maximum potential V_{im} without burning out
- Actual voltages across capacitor ΔV_i will be determined by the applied potential V_0



$C_1 = 4.0\mu F$, $V_{1m} = 30V$

$C_2 = 8.0\mu F$, $V_{2m} = 20V$

$C_3 = 12\mu F$, $V_{3m} = 10V$

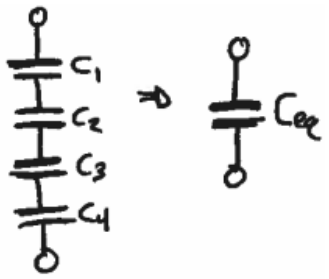
$C_4 = 6.0\mu F$, $V_{4m} = 10V$

Problem Formulation

Analyze the capacitor network at right, and determine the voltage across each capacitor when the full network has potential V_0 across it. Find all capacitors that have exceeded their max potential, then find a "safe" value V_s that will not cause any capacitor to exceed its max value.

Outline of solution

1. Reduce network to equivalent capacitance, to find total charge stored on network
2. Given stored charge, find ΔV across each capacitor in network
3. Find alternate applied potential V_s that will not burn out any capacitors



① Series capacitors:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}$$

$$= \frac{1}{4\mu F} + \frac{1}{8\mu F} + \frac{1}{12\mu F} + \frac{1}{6\mu F}$$ → Find common denominator

$$\frac{1}{C_{eq}} = \frac{6}{24\mu F} + \frac{3}{24\mu F} + \frac{2}{24\mu F} + \frac{4}{24\mu F}$$

$$\frac{1}{C_{eq}} = \frac{15}{24\mu F} \rightarrow C_{eq} = \frac{24}{15}\mu F = \frac{8}{5}\mu F = \boxed{1.6\mu F}$$

⇒ For applied potential $V_0 = 60V$, stored charge is $Q_{TOT} = C_{eq} V_0 = \boxed{96\mu C}$

② Series rule: all capacitors in this series share same charge: $Q_1 = Q_2 = Q_3 = Q_4 = Q_{TOT}$

ISO potential across any individual capacitor is found from: $\Delta V_i = Q_i / C_i = \frac{Q_{TOT}}{C_i}$

Tabulating results, and comparing to V_{max}

	actual ΔV	max ΔV	
C_1	24V	30V	okay!
C_2	12V	20V	okay!
C_3	8V	10V	okay
C_4	16V	10V	No!

$C_4 = 6.0\mu F$ blew out when 60V was applied

To find safe applied potential V_s:

(a) assume C_4 at max safe value, 10V

(b) $Q_4 = C_4 \Delta V_4 = 60\mu C$

(c) Total "safe" charge is $Q_{TOT} = 60\mu C$

(d) safe voltage is $V_s = \frac{Q_{TOT}}{C_{eq}} = \boxed{37.5V}$