PHYS 2212 Problem-Solving Studio 05

Feb 21–24

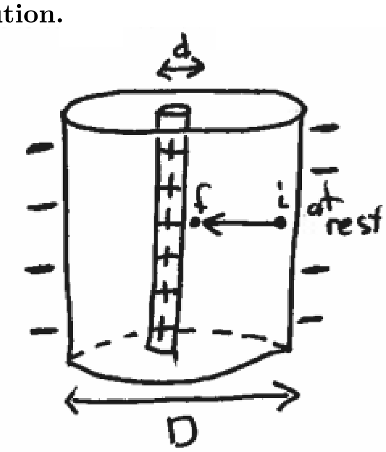
Geiger Counter

SOLUTION

As part of an undergraduate research project, you are re-designing a Geiger counter to detect a hypothetical new cosmic ray particle. The Geiger counter consists of a cylindrical conducting shell that is 12 cm long and 5.0 cm in diameter, that will be negatively charged to form a cathode. Along the central axis of the shell, a wire of diameter 2.5 mm will be positively charged to form an anode. The ends of the shell are sealed off with an insulating material (that is effectively transparent to the new particle), forming a closed tube. Cosmic particles entering the tube will collide with gas molecules inside, creating a cascade of free electrons that fall toward the anode and generate an electrical signal. Your task is to figure out the required surface charge density on the anode that will result in a potential difference of 750 V between the anode and cathode. You research advisor suggests that you start off by considering an electron that falls all the way from the cathode to the anode.

Instructions:
Construct a visual representation of the situation described, with all physical quantities represented by symbolic variables. Identify the concepts that will be needed to answer the question posed, as well as any simplifying assumptions that you will use. Outline a plan (that is, a series of analytical steps) that you will use solve the problem, and then follow those steps to solve the problem.

You may work as a group to complete this exercise, but each student is expected to submit an individual solution.



Assumptions:

① Let diameters of wire and shell be d and D (respectively)

② Assume field within tube can be modeled as axially symmetric / line charge field:

$$E(r) = \lambda / 2\pi\varepsilon_0 r$$

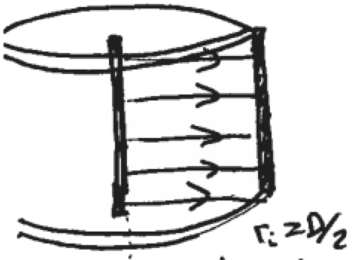
③ Assume edge effects near ends of tube can be ignored (as shown, electron falls near middle of tube)

④ Wire is small enough to be treated as a linear charge (λ) rather than surface charge (η)

Formulation of Problem: Find a relation between the linear charge density on the wire, and the potential difference between wire and shell, in terms of d, D, λ, ΔV, and permittivity constant ε_0

Steps

- let electron ($-e$) fall from $r_i = D/2$ (at rest) to $r_f = d/2$
- Find work done by field, $W_{i\to f}$
- Express PE change of electron as $\Delta U = -W_{i\to f}$
- Determine potential difference as $\Delta V = \Delta U / q = \Delta U/(-e)$
- Invert expression to solve for linear density λ (or, if you prefer, surface charge density η on wire)



Field inside is $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$ (radially outward ⊥ to wire)

Force on charge is $\vec{F} = q\vec{E} = (-e)\frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$

(radially inward)

so work by field is

$$W_{i\to f} = \int_{r_i}^{r_f} \vec{F}\cdot d\vec{s} = \int_{r_i}^{r_f}\left(\frac{-e\lambda}{2\pi\varepsilon_0 r}\right)(dr)$$

$$= \frac{-e\lambda}{2\pi\varepsilon_0}\int_{D/2}^{d/2}\frac{dr}{r} = \frac{-e\lambda}{2\pi\varepsilon_0}\left[\ln\frac{d}{2} - \ln\frac{D}{2}\right]$$

$$W_{elec} = \boxed{\frac{-e\lambda}{2\pi\varepsilon_0}\ln\left(\frac{d}{D}\right)}$$

Potential energy change is thus

$$\Delta U = -W = \boxed{\frac{+e\lambda}{2\pi\varepsilon_0}\ln\left(\frac{d}{D}\right)}$$

[this PE change is actually negative, because $\ln(u) < 0$ for $u < 1$]

Hence, $\Delta V = \frac{\Delta U}{-e} = \frac{-\lambda}{2\pi\varepsilon_0}\ln\left(\frac{d}{D}\right) = \frac{+\lambda}{2\pi\varepsilon_0}\ln\left(\frac{D}{d}\right)$ [using $-\ln\left(\frac{a}{b}\right) = +\ln\left(\frac{b}{a}\right)$]

so $\boxed{\Delta V = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{D}{d}\right) = \frac{\lambda}{2\pi\varepsilon_0}\ln(20)}$

$\to$ $\boxed{\lambda = \frac{2\pi\varepsilon_0 \Delta V}{\ln(20)} = 13.9\,nC/m}$ rounds to 14 nC/m

[or, in terms of surface density

$$\eta = \frac{\lambda}{2\pi r} = \frac{\lambda}{\pi d} = 1.77\,\mu C/m^2$$]