PHYS 2212 Problem-Solving Studio 04

Feb 14–17

SOLUTION

Fusion Studies

You have a research internship at Brookhaven National Laboratory, performing experiments to understand the nuclear fusion process that powers the Sun. Your experiment will shoot α-particles from a Van de Graaf accelerator at a sheet of palladium metal. An α-particle is the nucleus of a helium atom, having of 2 protons and 2 neutrons; a palladium nucleus consists of 46 protons and 62 neutrons. To assure that you are actually studying the effects of the nuclear force, the α-particle (having a radius of 1.6 fm) must make direct physical contact with the palladium nucleus (having a radius 3 times larger). Your research advisor wants you to make two calculations:

(a) What is the minimum launch speed that will allow an α-particle to contact a palladium nucleus?

(b) What potential difference within the Van de Graaf accelerator will generate such a speed for the α-particle?

Instructions:
Construct a visual representation of the situation described, with all physical quantities represented by symbolic variables. Identify the concepts that will be needed to answer the question posed, as well as any simplifying assumptions that you will use. Outline a plan (that is, a series of analytical steps) that you will use solve the problem, and then follow those steps to solve the problem.

You may work as a group to complete this exercise, but each student is expected to submit an individual solution.

Assumptions:

- Alpha particle begins at rest in accelerator, and emerges at other end traveling with max speed
- Exit point from accelerator is "very far" from target nucleus, so that we can use $r \approx \infty$
- Alpha has just enough energy to contact Pd nucleus $\rightarrow v_f \approx 0 \quad r_f \approx R_\alpha + R_{Pd} = 4R_\alpha$
- The Pd nucleus $\boxed{\text{remains} \approx \text{stationary}}$ during all this

Formulation

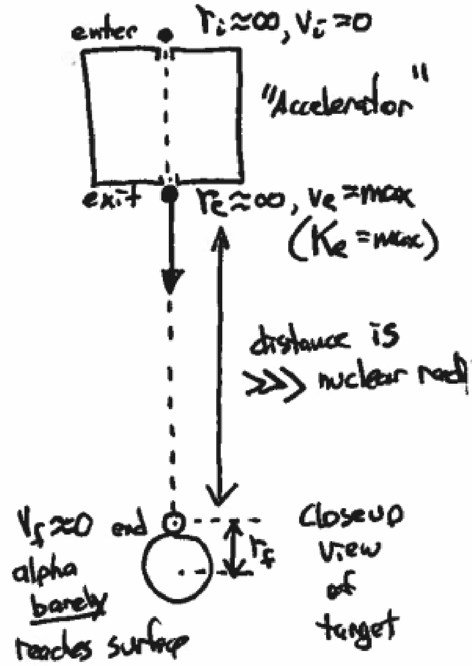
Determine the speed required to "fire" a positively charged ($q=+2e$) particle at a target nucleus with charge $Q=+46e$, starting from a long distance away and ending in direct contact with target. Then, find the accelerating electric potential required to achieve this.

Outline of solution: work backwards

① Apply conservation of energy $K+U_{elec}$
$\rightarrow$ given final separation, speed $\approx$ zero, find speed when very far away

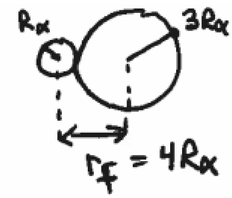
② Apply conservation of energy while inside accelerator, using $\Delta U_{elec} = q\Delta V$ (electric potential)

i = "initial", when particle enters accelerator
e = "exit", when particle leaves accelerator
f = "final", when particle contacts target nucleus



① At moment contact is made, $v_f \approx 0$ so $K_f \approx 0$

potential energy is $U_f = \frac{kQe}{r_f} = \frac{k(46e)(2e)}{4R_\alpha}$



② Alpha emerges from accelerator:

...with speed $v_e \to K_e = \frac{1}{2} m_\alpha v_e^2$

...at distance $r_e \approx \infty \to U_e \approx 0$

③ Conservation of energy: $K_e + \cancel{U_e}^{\,0} = \cancel{K_f}^{\,0} + U_f \to$

$$K_e = U_f$$

$$\boxed{\frac{1}{2} m_\alpha v_e^2 = \frac{23ke^2}{R_\alpha}}$$

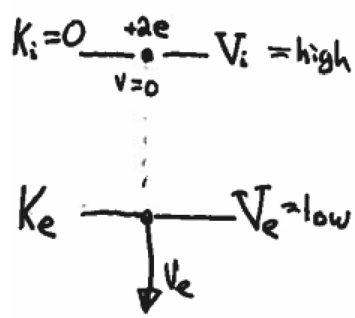
$$\boxed{v_e = \sqrt{\frac{46ke^2}{m_\alpha R_\alpha}}}$$

use $m_\alpha \approx 4 m_{proton}$ (since $m_{proton} \approx m_{neutron}$) where $m_p = 1.67 \times 10^{-27}$ kg

$$\boxed{v_e = 3.15 \times 10^7 \text{ m/s}} \; (\approx 0.1c!)$$

④ Acceleration while inside van de Graaf: let electric potential difference between entry and exit be $\Delta V = V_e - V_i$

then $\Delta U = q\Delta V = (2e)\Delta V$



Conservation of energy: $\Delta K + \Delta U = 0$ (since ΔK = pos, ΔU must be neg.)

$$(K_f - 0) + (2e\Delta V) = 0$$

$$\Delta V = \frac{-K_f}{2e} = \boxed{\frac{-23ke}{2R_\alpha}}$$

using result of step 3

$$\boxed{\Delta V = -10.4 \text{ MV} = -10.4 \times 10^6 \text{ volts}}$$