PHYS 2212 Problem-Solving Studio 02

Jan 25–31

Chip Diagnostics

SOLUTION

Your internship at a microelectronics firm involves the development of a diagnostic chip that uses a precisely calibrated electric field to perform quality checks on other, production-line microchips. The diagnostic chip consists of two semi-circular arcs of diameter 5.0 mm, separated by a tiny insulating gap, so that together they form a circle. When properly biased in the test circuit, the top arc will develop a uniformly-distributed positive charge, and the bottom arc will develop an equal but opposite charge (see schematic below). When biased, the probe point at the center of the circular cutout experiences a field of magnitude 25 mN/C. You wonder how many electrons were transferred from the top arc to the bottom, in order to generate such a field.

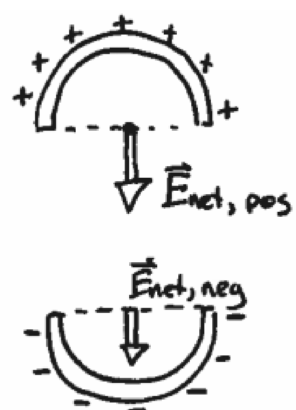
Instructions:
Construct a visual representation of the situation described, with all physical quantities represented by symbolic variables. Identify the concepts that will be needed to answer the question posed, as well as any simplifying assumptions that you will use. Outline a plan (that is, a series of analytical steps) that you will use solve the problem, and then follow those steps to solve the problem.

You may work as a group to complete this exercise, but each student is expected to submit an individual solution.

Note that left/right (ie mirror) symmetry of the charged arcs allows us to infer that the net field of either arc is purely y-directed

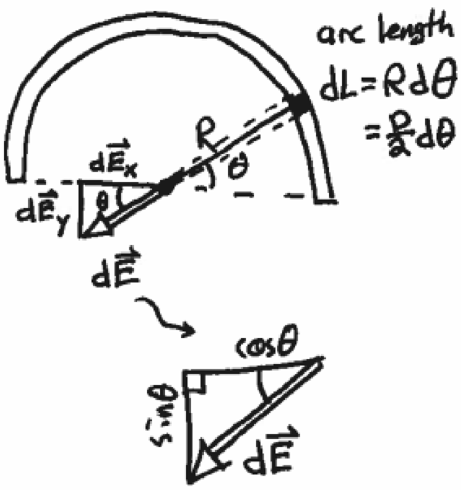
- for positive arc, <u>away</u> = <u>down</u>
- For negative arc, toward = <u>down</u>

] Net field is downward



Also: charges are equal in magnitude, so the fields they create are equal

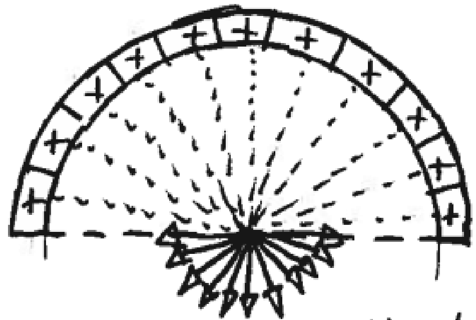
So: Net field can be found by finding the field for either arc, and <u>doubling</u>



- integration <u>around</u> arc using $dL = Rd\theta$ where $\theta: 0 \to \pi$
- charge on segment is $dQ = \lambda dL = \lambda R d\theta$
- charge density is $\lambda = Q/L = Q/\pi R$

Problem boils down to: compute expression for Electric Field at center of curvature for a uniform semicircular arc. Use this to determine the number of electron charges, e, on either arc

Symmetry analysis for positive arc



All these small field vectors will have the same magnitude (same distance from their source charges), and will be arrayed in a symmetric arc

→ When summed, all x-components will net to <u>zero</u> and all y-components will "reinforce" each other.

Negative arc works out the same way, although with the small field vectors all pointing <u>toward</u> their sources. You still get: NO x, downward y

Using figure from Part One, electric field of the tiny subsegment shown is:

$$d\vec{E} = \frac{k\,\delta Q}{r^2}\hat{r} = \frac{k(\lambda dL)}{R^2}\left[-\cos\theta\,\hat{i} - \sin\theta\,\hat{j}\right]$$

Since only y-component will survive integration, use $d\vec{E}_y = \dfrac{k\left(\frac{Q}{\pi R}\right)(R\,d\theta)}{R^2}\left(-\sin\theta\,\hat{j}\right)$

Net field due to positive arc is then found by integrating:

$$\vec{E}_+ = \int_{\theta=0}^{\theta=\pi} \frac{kQ}{\pi R^2}(-\hat{j})\sin\theta\,d\theta = \left(\frac{-kQ}{\pi R^2}\hat{j}\right)\left[-\cos\theta\right]_0^{\pi} = \boxed{-\frac{2kQ}{\pi R^2}\hat{j}}$$

Now note: problem gives diameter of arc, so $R \to D/2$

Also: Net field is <u>twice</u> this value, due to contribution of <u>negative</u> arc

$$\vec{E}_{net} = 2\left[-\frac{2kQ}{\pi(D^2/4)}\hat{j}\right] \quad \text{or} \quad |\vec{E}_{net}| = \frac{16kQ}{\pi D^2} = E_0$$

Finally, invert to solve for Q, and then note that # electrons is given by $Q = Ne$

$$Q = \frac{\pi D^2 E_0}{16k} = Ne$$

$$\boxed{N = \frac{\pi D^2 E_0}{16ke}} \approx \boxed{85 \text{ electrons}}$$