PHYS 2212 Problem-Solving Studio 01

Jan 18–24

SOLUTION

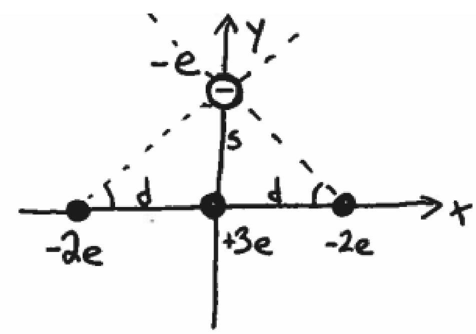
Ionic Equilibrium

You have a co-op job with a chemical company that is designing an ionic CO_2 scrubber. Your boss has asked you to determine where a chlorine ion having effective charge $-e$ would situate itself near a carbon dioxide ion. The carbon dioxide ion is composed of two oxygen ions each having an effective charge $-2e$ and a carbon ion having an effective charge $+3e$. These ions are arranged in a linear configuration, with the carbon ion sandwiched midway between the two oxygen ions. The distance between each oxygen ion and the carbon ion is 3.0×10^{-11} m. Assuming that the chlorine ion is on a line that is perpendicular to the axis of the carbon dioxide ion and that the line goes through the carbon ion, what is the equilibrium distance for the chlorine ion relative to the carbon ion on this line? For simplicity, you assume that the carbon dioxide ion does not deform in the presence of the chlorine ion.

Instructions:
Construct a visual representation of the situation described, with all physical quantities represented by symbolic variables. Identify the concepts that will be needed to answer the question posed, as well as any simplifying assumptions that you will use. Outline a plan (that is, a series of analytical steps) that you will use solve the problem, and then follow those steps to solve the problem.

You may work as a group to complete this exercise, but each student is expected to submit an individual solution.

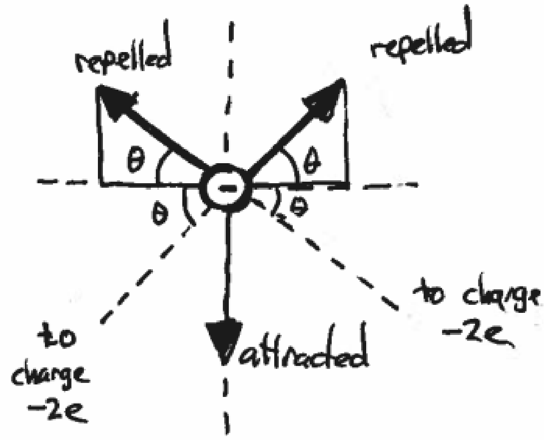
- Assume point particles so that Coulomb's Law can be applied
- Choose coord system (at right) with CO_2^+ ion along x-axis, Cl^- ion on y-axis
  - locations of O^{--} ions are $x = \pm d$
  - location of Cl^- ion is $y = +s$



⇒ This is an equilibrium problem:

$\boxed{\text{require forces on } Cl^- \text{ ion to sum to zero}}$

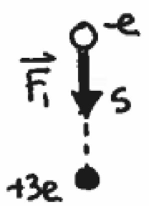
Steps will be:
- apply Coulomb's Law three times to find force magnitudes
- decompose all forces into x/y components
- require $\sum \vec{F}_x = 0$ and $\sum \vec{F}_y = 0$



Formulation: For the given orientation of the point charges, find the location "S" on the y-axis at which a charge '$-e$' will experience zero net force

Solution involves two major steps: Calculate three force vectors in decomposed form, then require those forces to sum to zero

① Force by +3e

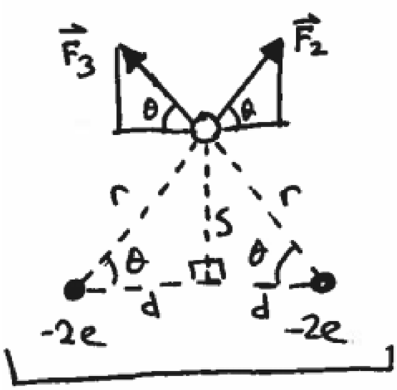


$$\vec{F}_1 = k\frac{(3e)(e)}{s^2}(-\hat{j}) \text{ for attractive force} \quad \text{or} \quad \boxed{\vec{F}_1 = 3\frac{ke^2}{s^2}(-\hat{j})}$$

② Force by charges -2e will be repulsive

Note that they will also be mirror images: $\vec{F}_{2y} = \vec{F}_{3y}$

$\vec{F}_{2x} = -\vec{F}_{3x}$



trig: $r^2 = s^2 + d^2$

and $\cos\theta = \dfrac{d}{\sqrt{s^2+d^2}}$

$\sin\theta = \dfrac{s}{\sqrt{s^2+d^2}}$

$$|\vec{F}_2| = |\vec{F}_3| = k\frac{(2e)(e)}{r^2} = \frac{2ke^2}{s^2+d^2}$$

Now, decompose: $\vec{F}_2 = \left[|\vec{F}_2|\cos\theta\right]\hat{i} + \left[|\vec{F}_2|\sin\theta\right]\hat{j}$

$\vec{F}_3 = \left[-|\vec{F}_3|\cos\theta\right]\hat{i} + \left[|\vec{F}_3|\sin\theta\right]\hat{j}$

$$\text{so} \quad \boxed{\vec{F}_2 = \left[+\frac{2ke^2}{s^2+d^2}\cdot\frac{d}{\sqrt{s^2+d^2}}\hat{i} + \frac{2ke^2}{s^2+d^2}\frac{s}{\sqrt{s^2+d^2}}\hat{j}\right]}$$

($\vec{F}_3$ is almost identical, but with sign of x-component being negative)

Okay — Now require forces to sum to zero $\sum \vec{F}_x = 0$ and $\sum \vec{F}_y = 0$

① $\sum \vec{F}_x = 0$ is automatically guaranteed, since $\vec{F}_{1x} \equiv 0$ and $\vec{F}_{2x} = -\vec{F}_{3x}$

② So — last step is to make sure $\sum \vec{F}_y = 0$:

$$\sum \vec{F}_y = -3\frac{ke^2}{s^2} + 2\frac{ke^2}{s^2+d^2}\cdot\frac{s}{\sqrt{s^2+d^2}} + 2\frac{ke^2}{s^2+d^2}\cdot\frac{s}{\sqrt{s^2+d^2}} = 0$$

Note that all terms have factor ke^2, which therefore cancels out

rearrangement gives $\quad 4\cdot\frac{s}{(s^2+d^2)^{3/2}} = 3\cdot\frac{1}{s^2}$

$$4s^3 = 3(s^2+d^2)^{3/2}$$

Take each side to $(2/3)$ power:

$$(4)^{2/3}s^2 = (3)^{2/3}(s^2+d^2)$$

$$\left(\frac{4}{3}\right)^{2/3}s^2 = s^2+d^2$$

$$\left[\left(\frac{4}{3}\right)^{2/3}-1\right]s^2 = d^2$$

$$s = \frac{d}{\sqrt{\left(\frac{4}{3}\right)^{2/3}-1}} \approx \boxed{2.17d}$$

Finally, plugging in $d = 3.0\times10^{-11}$ m, we get

$$\boxed{s = 6.5\times10^{-11}\,\text{m}}$$