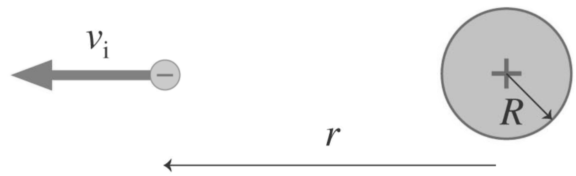

I. (16 points) An electron is at a distance r from the center of a spherical bead of radius $R < r$ and uniformly distributed positive charge $+Q$. It has an initial velocity directed straight away from the bead.

For what maximum initial speed v_{max} will the electron eventually strike the bead? If the electron will always or never strike the bead, prove that $v_{\text{max}} = 0$ or $v_{\text{max}} = \infty$. Otherwise, express your answer in terms of parameters defined in the problem, and physical or mathematical constants.

The maximum speed for which the electron will strike the bead is also the minimum speed for which it won't strike the bead. It won't strike the bead if the initial speed is great enough for the electron to escape to infinite distance. Use the Work-Energy Theorem.



$$W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{th}}$$

Choose a system consisting of the electron and the bead, so there are no external forces. There are no dissipative forces transforming energy within that system. The potential energy in the system is that of point charges. (The bead can be treated as a point in its own center when considering the potential energy outside it, due to its spherical symmetry.) Letting the potential energy be zero at infinite separation:

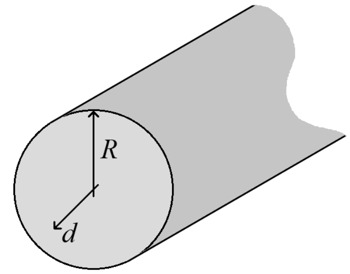
$$0 = \left(\tfrac{1}{2}m_e v_f^2 - \tfrac{1}{2}m_e v_i^2\right) + \left(K\frac{q_1 q_2}{r_f} - K\frac{q_1 q_2}{r_i}\right) + 0$$

If the electron has just enough initial speed to escape to infinite distance, the final speed and the final potential energy will be zero.

$$0 = \left(0 - \tfrac{1}{2}m_e v_{\text{max}}^2\right) + \left(0 - K\frac{(+Q)(-e)}{r}\right) \qquad \Rightarrow \qquad v_{\text{max}} = \sqrt{\frac{2KQe}{m_e r}}$$

II. (16 points) Consider an infinitely long solid insulating cylinder with radius R that carries a volume charge density ρ that depends on distance r from the central axis, according to

$$\rho(r) = \rho_0 r/R$$



where ρ_0 is a constant.

Find the magnitude of the electric field at a point inside the cylinder, that is, at a distance d from the axis, where $d < R$. Express your answer in terms of parameters defined in the problem, and physical or mathematical constants.

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Use Gauss' Law, $\epsilon_0 \Phi = q_{\text{in}}$. Choose a Gaussian surface that is a cylinder with length L and radius d, so it passes through the point at which the electric field magnitude is to be found.

First, find the flux through the surface. Note that, by symmetry, there is no flux through the ends of the cylindrical Gaussian surface. Furthermore, the field has constant magnitude over the curved part of the Gaussian surface, and is everywhere perpendicular to that surface. This is *why* that surface was chosen!

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \oint E \cos\theta \, dA = E \oint dA = E 2\pi d L$$

Next, the charge within the surface must be calculated. Choose an element of charge, dq, that is a thin cylindrical shell with length L, radius r, and thickness dr. This element of charge occupies an element of volume $dV = 2\pi r L \, dr$. The volume charge density, ρ, is dq/dV, so

$$q_{\text{in}} = \int dq = \int \rho \, dV = \int_0^d \left(\rho_0 \frac{r}{R}\right) 2\pi r L \, dr = \frac{\rho_0 2\pi L}{R} \int_0^d r^2 \, dr = \frac{\rho_0 2\pi L}{R} \left[\frac{r^3}{3}\right]_0^d = \frac{\rho_0 2\pi L d^3}{3R}$$

Putting these together

$$\epsilon_0 \Phi = q_{\text{in}} \qquad \Rightarrow \qquad \epsilon_0 E 2\pi d L = \frac{\rho_0 2\pi L d^3}{3R} \qquad \Rightarrow \qquad E = \frac{\rho_0 d^2}{3\epsilon_0 R}$$

1. (6 points) Consider a point outside the cylinder in the above problem, at a distance D from the cylinder axis, where $D > R$. On what distances does the magnitude of the electric field at that point depend?

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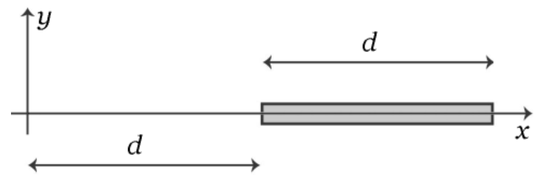
The field magnitude will depend on the radius of the Gaussian surface, which is D, as it must pass through the point at which the field is to be found. It also depends on the charge inside, which extends from zero to R. So the field will not depend on d, and since r is just a dummy variable, the field depends on distances . . .

D and R only.

III. (16 points) A thin rod of length d lies on the x axis with one end at $x = d$, as illustrated. The linear charge density, λ, of the rod depends on position, x, according to

$$\lambda = \lambda_0 \left(\frac{x}{d}\right)$$

where λ_0 is a positive constant.



What is the electric potential at the origin, with respect to zero at infinity? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants.

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Divide the rod into infinitesimal elements of length dx, each containing an infinitesimal element of charge dq. In terms of magnitudes, each element of charge contributes an element of electric potential dV. Since the elements of length are so small as to be point-like, the element of potential is

$$dV = \frac{K\,dq}{r}$$

We'd like to add up the contributions from each element of length (that is, integrate), but first both dq and r must be obtained in terms of position x. Since any element of length at a position x is also at a distance x from the origin, $r = x$. The element of charge can be related to the element of length through the linear charge density:

$$\lambda = \frac{dq}{dx} \qquad \Rightarrow \qquad dq = \lambda\,dx = \lambda_0 \left(\frac{x}{d}\right) dx$$

So

$$V = \int dV = \int \frac{K\,dq}{r} = \int_d^{2d} \frac{K\lambda\,dx}{x} = \int_d^{2d} \frac{K\lambda_0\,(x/d)\,dx}{x} = \frac{K\lambda_0}{d}\int_d^{2d} dx = \frac{K\lambda_0}{d}\Big[x\Big]_d^{2d} = \frac{K\lambda_0}{d}\Big[2d - d\Big] = K\lambda_0$$

2. (6 points) In the problem above, what is the direction of the electric potential at the origin?

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Electric potential is a scalar!

This is not a meaningful question.

3. (7 points) A plane with positive uniform area charge density η passes through the centers of three closed surfaces (a sphere of radius R, a cylinder of radius R and height $4R$, and a rectangular prism with height R and square base of edge $2R$), as illustrated. The cube with edge $2R$ does not intersect the plane. Rank the electric flux, Φ, through these surfaces from greatest to least.

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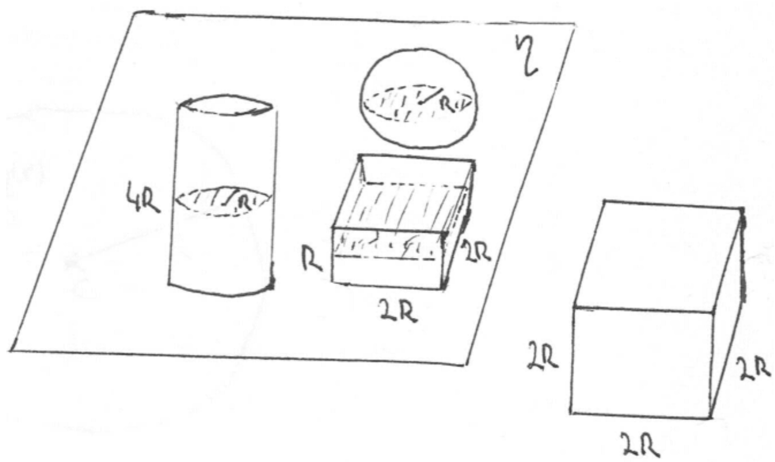

From Gauss' Law, the greatest flux will be through the surface that contains the greatest net charge. Since area charge density of the plane is uniform, the surface containing the greatest net charge will be that which contains the greatest area of the plane.

$$\Phi_{\text{prism}} > \Phi_{\text{cylinder}} = \Phi_{\text{sphere}} > \Phi_{\text{cube}}$$

4. (7 points) A system consists of three charged particles at the vertices of an equilateral triangle, as illustrated. Two of the particles have charge of equal magnitude, q, but opposite sign. The third particle has charge Q. How does the electric potential energy of the system change if the particle with charge Q is removed to infinitely far away?

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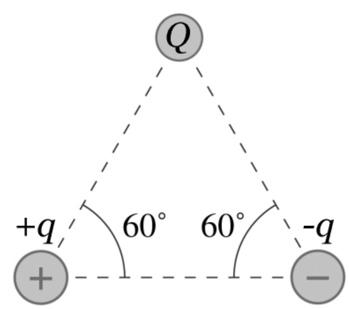

The particle with charge Q is equidistant from charged particles of equal magnitude but opposite sign. Therefore, it is at a position with an electric potential of zero, with respect to zero at infinite distance. So, when it is removed to infinitely far away, there is no change in its electric potential, and no change in the electric potential energy of the system.

It remains the same.

5. (7 points) A hollow conductor, illustrated in cross-section, carries a net charge of -3 nC. Within its void lies a particle with a charge of +5 nC. What is the net charge on the inner and outer surfaces of the conductor at equilibrium?

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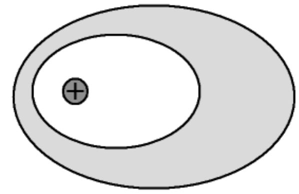

Consider a Gaussian Surface within the solid part of the conductor. The field in the solid part of a conductor at equilibrium is zero, so the flux through this Gaussian Surface is zero, so the net charge contained within the Gaussian Surface must be zero. There must be a charge of -5 nC on the inner surface of the conductor to balance the +5 nC charge on the particle.

Charge is conserved. If there is -5 nC on the inner surface of the conductor, but the conductor has a net charge of -3 nC, then there must be +2 nC on the outer surface.

$$Q_{\text{inner}} = -5\,\text{nC while } Q_{\text{outer}} = +2\,\text{nC}$$

6. (7 points) A thin insulating ring has total charge Q and radius R. What is the electric potential at its center, with respect to zero at infinity?

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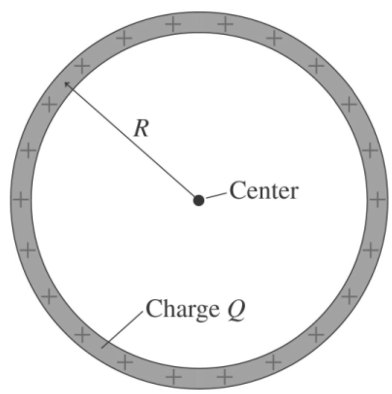
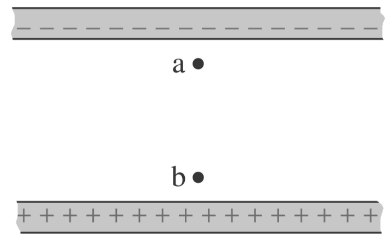


Electric potential is a scalar, so the direction of the charge from the point in question is irrelevant — all that matters is the distance. All the charge in the ring is the same distance from the point where the electric potential is to be found. Note that all the charge in a point charge is the same distance from any point at which electric potential is to be found. Therefore, the potential at the center of the ring must be the same as the potential due to a point charge at a distance equal to the radius.

$$KQ/R$$

7. (6 points) Two points, a and b, are pictured within an ideal parallel-plate capacitor. If an *electron* moves from point a to point b, how does the *electric potential energy* of the capacitor-electron system change?



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If the electron were simply released from rest at point a, it would move to point b spontaneously.

The electric potential energy of the system decreases.

8. (6 points) In the problem above, compare the *electric potential* at point a when the electron is located there, to the electric potential at point b when the electron is located there.

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Since electric potential describes points in space without regard to what object, if any, is located at those points, the electron is irrelevant. Imagining a positively charged particle moving from point a to point b can, however, be helpful. If such a positively charged particle were to start at rest at point a, an external force would have to be applied for it to move to point b. This external force would do positive work on the system, increasing its potential energy. Since the electric potential is the electric potential energy per unit charge, point b must be at a higher electric potential than point a.

The electric potential at point a is less than that at point b.