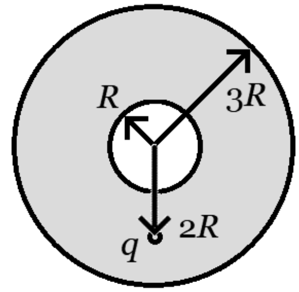

I. (16 points) A hollow sphere with inner radius R and outer radius $3R$ has uniform volume charge density ρ. A particle with charge q is embedded in the sphere, a distance $2R$ from the center. What is the magnitude of the electric force, if any, exerted by the particle on the sphere? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants.



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Imagine a sphere of radius $2R$, concentric with the sphere of charge. It passes through the particle with charge q. Charge outside this sphere exerts no force on the particle. Charge inside this sphere exerts a force as if it were a point charge in the center. Letting Q represent this charge inside the sphere of radius $2R$, use Coulomb's Law.

In this case, $r = 2R$. The charge inside the sphere of radius $2R$ is the volume of charge times the charge density. The volume of charge is that of a sphere having radius $2R$, less the volume of the hollow.

$$Q = \rho V = \rho \left[\frac{4}{3}\pi \left(2R\right)^3 - \frac{4}{3}\pi R^3 \right] = \frac{4}{3}\rho\pi \left[8R^3 - R^3\right] = \frac{4}{3}\rho\pi 7R^3$$

In terms of magnitudes,

$$F = K\frac{Qq}{r^2} = K\frac{\left(\frac{4}{3}\rho\pi 7R^3\right)q}{\left(2R\right)^2} = K\frac{\left(\frac{4}{3}\rho\pi 7R^3\right)q}{4R^2} = K\frac{7}{3}\rho\pi Rq$$

II. (16 points) Three charged particles lie on the y axis. Particles with negative charge $-2q$ lie at $\pm d$. A particle with positive charge $+q$ lies on the origin. Find an expression for the electric field $\vec{E}$ as a function of position, x, on the x axis. Express your answer in terms of parameters defined in the problem and physical or mathematical constants.

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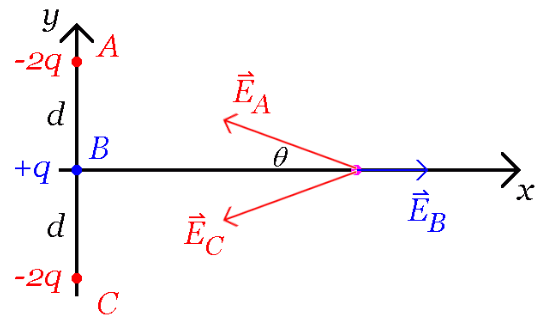
Sketch the problem. The three charged particles have been labelled A, B, and C. From the sketch, it can be seen that the y component of the electric field due to particle B with charge $+q$ is zero, and the y components of the electric fields due to particles A and C with charge $-2q$ add to zero. Therefore, only the x components need to be calculated. In terms of magnitudes,



$$E_{Bx} = E_B = \frac{Kq}{x^2}$$

And

$$E_{Cx} = E_{Ax} = E_A \cos\theta = \frac{K2q}{\sqrt{x^2+d^2}^2}\cos\theta = \frac{2Kq}{x^2+d^2}\left(\frac{x}{\sqrt{x^2+d^2}}\right) = \frac{2Kqx}{(x^2+d^2)^{3/2}}$$

Looking at the sketch once again, it can be seen that E_{Bx} is in the positive direction, while E_{Ax} and E_{Cx} are in the negative direction. So,

$$E_x = E_{Bx} - E_{Ax} - E_{Cx} = E_{Bx} - 2E_{Ax} = \frac{Kq}{x^2} - 2\frac{2Kqx}{(x^2+d^2)^{3/2}} = Kq\left(\frac{1}{x^2} - \frac{4x}{(x^2+d^2)^{3/2}}\right)$$

and

$$\vec{E} = Kq\left(\frac{1}{x^2} - \frac{4x}{(x^2+d^2)^{3/2}}\right)\hat{\imath}$$

1. (6 points) In the problem above, what is the direction, if any, of the electric field on the $+x$ axis?

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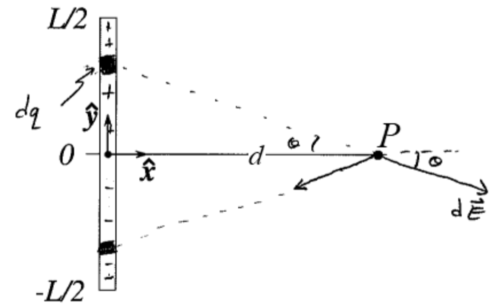
At small values of x the contributions to the field by the negative charges will tend to cancel, so the field will be in the positive direction. But at larger values of x the field will approach that of a net negative charge, and will be in the negative direction.

The direction depends on the particular value of x.

2. (6 points) A straight, insulating rod is placed on the y-axis, spanning from $y = -L/2$ to $y = +L/2$. The rod carries a nonuniform charge density,

$$\lambda = \lambda_0 \left(\frac{y}{L}\right)$$

where λ_0 is a positive constant. What is the direction of the electric field vector at point P, located on the x-axis at distance d from the rod?



For every element of positive charge dq on the upper half of the rod, there is a corresponding element with the same charge magnitude, but opposite sign, on the lower half of the rod. A positive probe charge placed at point P will be repelled by the upper half of the rod, and attracted to the lower half of the rod. The net force, and so the electric field, is directed toward

$$-\hat{y}$$

III. (16 points) What is the electric field magnitude at point P? If it is non-zero, express the field magnitude in terms of λ_0, y, L, d, and physical or mathematical constants, and **leave it as a definite integral; do not evaluate it!** If the field is zero, prove it.

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Consider the rod to consist of point-like bits of charge dq. Each makes a bit of electric field $d\vec{E}$. From symmetry, the x components of these bits of field cancel, so the electric field at point P is due to the y components contributed by all the bits of charge. In terms of magnitudes,

$$E = \int dE_y = \int dE \sin\theta = \int \frac{K\,dq}{r^2} \sin\theta$$

where

$$dq = \lambda\, dy = \lambda_0 \left(\frac{y}{L}\right) dy$$

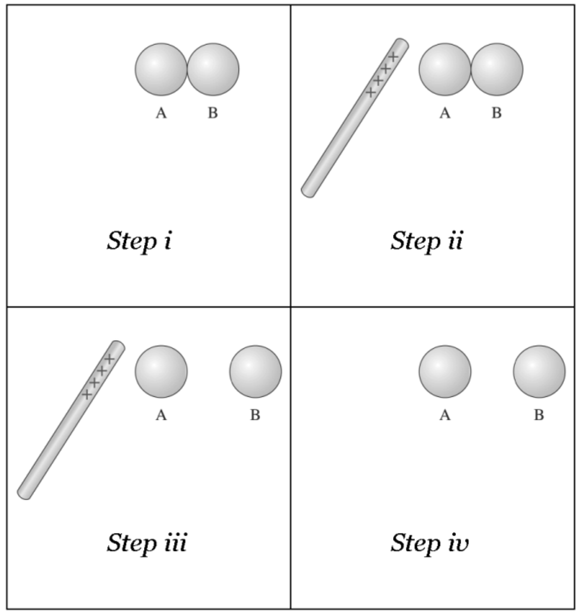
$$r = \sqrt{y^2 + d^2}$$

$$\sin\theta = \frac{y}{r} = \frac{y}{\sqrt{y^2 + d^2}}$$

so

$$E = \int \frac{K\lambda\, dy}{\left(\sqrt{y^2 + d^2}\right)^2} \frac{y}{\sqrt{y^2 + d^2}} = \int_{-L/2}^{+L/2} \frac{K\lambda_0 \left(\frac{y}{L}\right) y\, dy}{\left(y^2 + d^2\right)^{3/2}}$$

3. (7 points) Two neutral conducting spheres, A and B, are in contact (*Step i*). A positively charged rod is brought near, but not touching, sphere A (*Step ii*). The spheres are separated (*Step iii*), and then the rod is removed (*Step iv*). Describe the charge of each sphere after *Step iv*.



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The spheres are in contact in *Step i*, and so will act as a single conductor. In *Step ii* that conductor is polarized, with sphere A being the negative end (electrons are attracted to the positively charged rod), and sphere B being the positive end (as it is now deficient in electrons). Separating the spheres in *Step iii* traps those charges on the spheres, and they remain that way in *Step iv* when the rod is removed.

A is negative, B is positive.

4. (7 points) A positive point charge q is released near a positive fixed charge Q. As q moves further away from Q, it will move with:

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increasing velocity and decreasing acceleration.

Consider Newton's Second Law and Coulomb's Law

$$\sum F = K\frac{Qq}{r^2} = ma$$

so the acceleration decreases as r increases. However, the acceleration is always in the same direction as the velocity, so the magnitude of the velocity always increases.

5. (7 points) The figure shows electric field lines in a region of space. Rank in order, from greatest to least, the electric field magnitudes at points i through iv. FIX DUPLICATE ANSWERS

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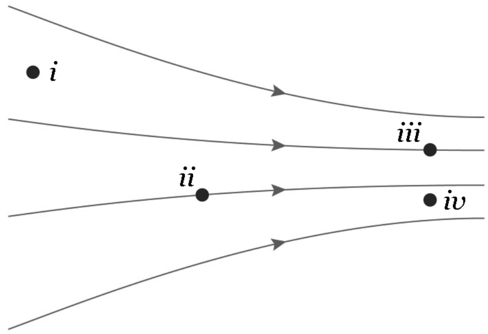
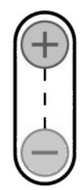


The density of field lines in a region represents the magnitude of the electric field in that region.

$$iv = iii > ii > i$$

6. (7 points) An electric dipole is released from rest in the orientation shown. It is observed to rotate 90° clockwise, then move to the right. If this motion is due to an electron, where is that electron located?



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A dipole will align with the field, then move toward a region of greater field magnitude. The field due to an electron is has greater magnitude near the electron, so the electron must be in the direction the dipole moves,

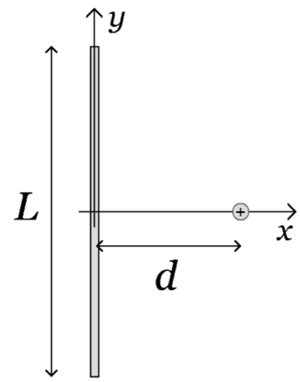
To the right of the dipole.

7. (6 points) A non-uniform line segment of charge with length L is centered on the y axis. Its linear charge density, λ, depends on position, y, according to

$$\lambda = \lambda_0 y^4$$

where λ_0 is a positive constant.

A proton is placed at a position $+d$ on the x axis. What is the direction of the electric field due to the line segment of charge at the location of the proton?



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The linear charge density is an even function of y. That is, for each point at a position $+y$ on the positive y axis, there is a point at a position $-y$ on the negative y axis that has a charge density of the same magnitude and same sign.

The electric field at a location is in the same direction as the electric force on a positive probe charge. The proton is such a probe charge, and it is repelled by the positively charged portion of the rod on the $+y$ axis, and also repelled by the positively charged portion of the rod on the $-y$ axis. From the symmetry of the problem, the y components of these forces add to zero, and only the x components, which all point in the $+x$ direction, remain.

The electric field at the proton is in the $+x$ or $+\hat{\imath}$ direction.

8. (6 points) In the question above, an electron is placed at a position $+d$ on the x axis, instead of a proton. What is the direction of the electric field due to the line segment of charge at the location of the electron?

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The linear charge density is an odd function of y. That is, for each point at a position $+y$ on the positive y axis, there is a point at a position $-y$ on the negative y axis that has a charge density of the same magnitude, but opposite sign.

The electric field at a location is in the same direction as the electric force on an imagined positive probe charge. If there were a positive probe charge at the location of the electron (the electron itself is irrelevant), it would be repelled by the positively charged portion of the rod on the $+y$ axis, and also repelled by the positively charged portion of the rod on the $-y$ axis. From the symmetry of the problem, the y components of these forces add to zero, and only the x components, which all point in the $+x$ direction, remain.

The electric field at the electron is in the $+x$ or $+\hat{\imath}$ direction.