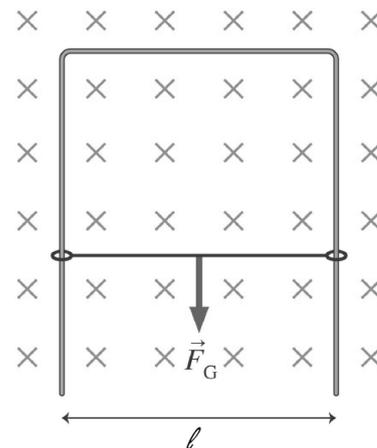


e	Fundamental charge	m_e	Mass of an electron
μ_0	Permeability constant	c	Speed of light
K	Coulomb constant = $1/4\pi\epsilon_0$	g	Magnitude of Free Fall Acceleration

Unless otherwise directed, drag should be neglected, and all circuit elements are ideal.

Any integrals in free-response problems must be evaluated. Questions about magnitudes will state so explicitly.

- I. (16 points) A U-shaped conducting rail is oriented vertically in a uniform, horizontal, magnetic field with magnitude B directed into the page. The rail has no electric resistance and does not move. A slide wire with mass m , length ℓ , and resistance R can slide up and down without friction while maintaining electrical contact with the rail. The slide wire is released from the rest. Find the expression for the terminal speed v of the slide wire in terms of other parameters defined in the problem and physical or mathematical constants. (*On Earth.*)



At the terminal speed, the acceleration of the wire, and thus the net force, must be zero. There is a gravitational force downward on the wire, so gravitational force must be upward. In terms of magnitudes, Newton's Second Law is

$$\sum F = F_G - F_B = ma = 0 \quad \Rightarrow \quad mg - I\ell B \sin \theta = 0$$

The direction of current flow through the slide wire is necessarily perpendicular to the magnetic field, so $\sin \theta = 1$. From the definition of resistance, $\Delta V = IR = \mathcal{E}$.

$$mg = I\ell B = \frac{\mathcal{E}}{R}\ell B$$

From Faraday's Law, the magnitude of the emf is

$$\mathcal{E} = \frac{d\Phi}{dt} = \frac{d}{dt} \int \vec{B} \cdot d\vec{A} = \frac{d}{dt} [BA \cos \phi] = B \cos \phi \frac{dA}{dt} = B \cos \phi \ell \frac{ds}{dt} = B\ell v \cos \phi$$

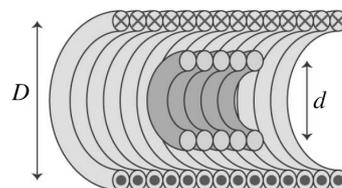
where s is the distance from the slide wire to the top of the "U". Note that the magnetic field and area vectors are parallel, so $\cos \phi = 1$.

$$\mathcal{E} = B\ell v \quad \Rightarrow \quad mg = \frac{B\ell v}{R}\ell B \quad \Rightarrow \quad v = \frac{mgR}{\ell^2 B^2}$$

II. (16 points) A five-turn coil with diameter d and resistance R is inside a solenoid with diameter D , as shown. The solenoid has N_{sol} turns, a length ℓ much greater than D , and carries a time-varying current

$$I_{\text{sol}} = at^2 + bt + c$$

where a , b and c are positive constants. Determine the magnitude of the current in the coil. Express your answer in terms of parameters defined in the problem and physical or mathematical constants.



Since the solenoid's length is much greater than its diameter, it can be treated as ideal. The magnetic field in an ideal solenoid is

$$B = \mu_0 I_{\text{sol}} n = \mu_0 (at^2 + bt + c) \frac{N_{\text{sol}}}{\ell}$$

Faraday's Law relates the time rate of magnetic flux change through the coil to the induced emf. The flux through one turn is

$$\Phi = \int \vec{B} \cdot d\vec{A} = B_{\text{sol}} A_{\text{coil}} = \left[\mu_0 (at^2 + bt + c) \frac{N_{\text{sol}}}{\ell} \right] \left(\frac{\pi}{4} d^2 \right) = \frac{\mu_0 N_{\text{sol}} \pi d^2}{4\ell} (at^2 + bt + c)$$

So the magnitude of the emf is

$$\mathcal{E} = N_{\text{coil}} \frac{d\Phi}{dt} = N_{\text{coil}} \frac{d}{dt} \left[\frac{\mu_0 N_{\text{sol}} \pi d^2}{4\ell} (at^2 + bt + c) \right] = \frac{N_{\text{coil}} \mu_0 N_{\text{sol}} \pi d^2}{4\ell} (2at + b)$$

Using the definition of resistance and remembering that $N_{\text{coil}} = 5$,

$$I_{\text{coil}} = \frac{\mathcal{E}}{R} = \frac{N_{\text{coil}} \mu_0 N_{\text{sol}} \pi d^2}{4\ell R} (2at + b) = \frac{5\mu_0 N_{\text{sol}} \pi d^2}{4\ell R} (2at + b)$$

1. (6 points) In the problem above, what is the direction of the induced magnetic field and induced current in the coil?

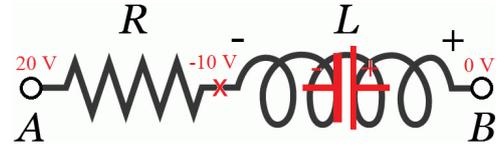
Using the Right-Hand-Rule, the field produced by the solenoid is to the left. Because the current

$$I_{\text{sol}} = at^2 + bt + c$$

increases with time, the field magnitude increases with time. Flux through the coil increases to the left with time. Lenz' Law tells us that the induced field must oppose the change in flux, so it must be to the right. Using the Right-Hand-Rule again,

Induced field is to the right, and induced current flows in the bottom and out the top of each coil.

III. (16 points) In the circuit segment shown, the inductance is 1.0 H and the resistance is 5.0 Ω. The potential difference is $\Delta V_{AB} = V_B - V_A = -20 \text{ V}$ at an instant when the rate of current change in the circuit has a **magnitude** of 10 A/s. The polarity of the inductor at this moment is indicated. Is the inductor supplying or storing energy? At what rate?



Since

$$\Delta V_{AB} = V_B - V_A = -20 \text{ V} \quad \Rightarrow \quad V_A = V_B + 20 \text{ V}$$

we know that the potential at A is 20 V greater than that at B. Only differences in potential matter, so I'll choose $V_B = 0 \text{ V}$, making $V_A = +20 \text{ V}$.

Since

$$\mathcal{E} = -L \frac{dI}{dt} \quad \Rightarrow \quad |\mathcal{E}| = L \left| \frac{dI}{dt} \right| = (1.0 \text{ H})(10 \text{ A/s}) = 10 \text{ V}$$

we know the potential difference across the inductor is 10 V. With the given polarity, the point marked "x" between the resistor and the inductor must be at a potential 10 V lower than point B, or -10 V.

We now see that there is a 30 V potential difference across the 5.0 Ω resistor. The current must be

$$\Delta V_R = IR \quad \Rightarrow \quad I = \frac{\Delta V_R}{R} = \frac{30 \text{ V}}{5.0 \Omega} = 6.0 \text{ A}$$

This current must flow to the right, as current flows from higher to lower potential through a resistor. If we imagine a battery with the same polarity as the inductor, we see that the current is flowing as if the battery were supplying energy. The inductor, therefore, is supplying energy. The rate at which it supplied energy is the power,

$$P = I \Delta V = (6.0 \text{ A})(10 \text{ V}) = 60 \text{ W}$$

So

The inductor is **supplying energy** at a rate of **60 W**.

2. (6 points) Considering, if necessary, the direction of current flow you found above, and the polarity of the inductor indicated in the figure, is the magnitude of the current through the circuit increasing or decreasing?

The emf in an inductor is induced to oppose the change in current. Imagining a battery with the same polarity as the inductor, one sees that the battery would cause current to flow to the right. For a rightward current to oppose the change, the current must either be flowing to the right and decreasing, or flowing to the left and increasing.

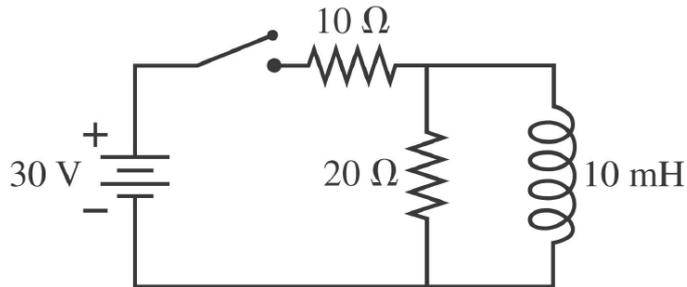
I found current flowing to the right, so it must be decreasing.

However, full credit (for consistency) was issued for

I found current flowing to the left, so it must be increasing.

3. (8 points) The switch in the circuit shown on the figure has been open for a long time. It is closed at time $t = 0$. What is the current through the $20\ \Omega$ resistor immediately after the switch is closed and after the switch has been closed for a long time?

After the switch has been open for a long time, no current is flowing through the inductor. When the switch is closed, the inductor opposes any change in current through it. At that instant, no current flows through the branch of the circuit with the inductor. The circuit is essentially a $30\ \text{V}$ battery with two resistors in series. Since the equivalent resistance is $30\ \Omega$, the current is $I = \Delta V/R = 1.0\ \text{A}$.



After the switch has been closed for a long time, there are no more changes in current for the inductor to oppose. The inductor has no induced emf, so the ends of the inductor are at the same potential. With no potential difference across the $20\ \Omega$ resistor, no current flows through it.

The current is $1.0\ \text{A}$ immediately after closing the switch, and $0\ \text{A}$ after a long time.

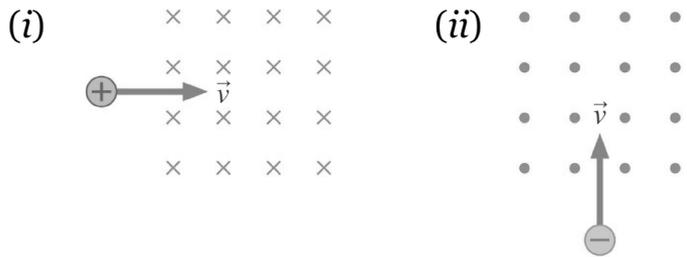
4. (6 points) What is the initial direction of deflection for the charged particles entering the magnetic fields as shown?

The magnetic force on a moving charge is

$$\vec{F} = q\vec{v} \times \vec{B}$$

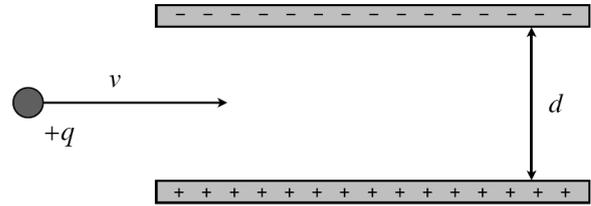
using the Right-Hand-Rule,

Particle i is deflected up the page and particle ii is deflected to the left.



5. (6 points) A positively charged particle with charge q travels with speed v between the two parallel charged plates shown in the figure. The plates are separated by a distance d . What magnetic field strength and direction will allow the charged particle to pass between the plates without being deflected?

If the particle is not to be deflected, its acceleration must be zero. Therefore (Newton's Second Law) the net force on it must be zero. In terms of magnitudes



$$F_E = F_B \Rightarrow qE = qvB \sin \theta \Rightarrow B = \frac{E}{v \sin \theta}$$

The choices for magnetic field direction were all perpendicular to the velocity, so $\sin \theta = 1$.

The electric field points from positive to negative charge, so is up the page in this case. Since q is positive, the electric force is up the page. The magnetic force must be down the page. As the magnetic force is $\vec{F} = q\vec{v} \times \vec{B}$, the Right-Hand-Rule shows that the magnetic field must be out of the page.

$$B = E/v \text{ directed out of the page.}$$

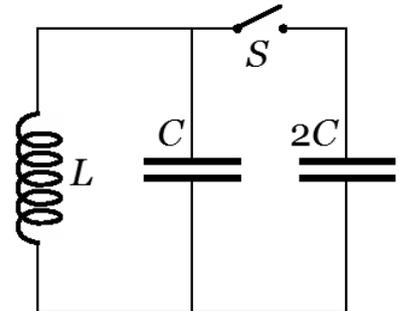
6. (6 points) The LC circuit oscillates with a frequency f_0 when the switch S is closed. What will be the frequency of oscillation when the switch is open?

The oscillation frequency of an LC circuit is

$$\omega = \frac{1}{\sqrt{LC_{\text{eq}}}} = 2\pi f$$

When the switch is open

$$C_{\text{eq}} = C_{\text{open}} = C \Rightarrow f_{\text{open}} = \frac{1}{2\pi\sqrt{LC}}$$



But when the switch is closed

$$C_{\text{eq}} = C_{\text{closed}} = C + 2C = 3C \Rightarrow f_0 = \frac{1}{2\pi\sqrt{L(3C)}} = \left(\frac{1}{\sqrt{3}}\right) \frac{1}{2\pi\sqrt{LC}} = \left(\frac{1}{\sqrt{3}}\right) f_{\text{open}} \Rightarrow f_{\text{open}} = f_0\sqrt{3}$$

7. (8 points) Consider two inductors that have the same length and cross section. The second inductor has three times as many coils as the first. The first inductor stores magnetic energy U_1 when a current I_1 flows through it. What energy U_2 is stored in the second inductor when the current through it is $I_2 = I_1/4$?

.....

Assuming the inductors can be treated as an ideal solenoids of length ℓ and cross-sectional area A , the magnetic fields within them are

$$B = \mu_0 I n = \mu_0 I \frac{N}{\ell}$$

As the flux through one turn is BA , the flux through N turns is

$$\Phi = NBA = N \left(\mu_0 I \frac{N}{\ell} \right) A = N^2 \mu_0 I A / \ell$$

Since inductance is the flux per unit current,

$$L = \frac{\Phi}{I} = \frac{N^2 \mu_0 I A / \ell}{I} = N^2 \mu_0 A / \ell$$

The energy stored in an inductor is $U_L = \frac{1}{2} L I^2$, so

$$U_1 = \frac{1}{2} L_1 I_1^2 = \frac{1}{2} \left(N_1^2 \mu_0 \frac{A}{\ell} \right) I_1^2 \quad \text{and} \quad U_2 = \frac{1}{2} L_2 I_2^2 = \frac{1}{2} \left(N_2^2 \mu_0 \frac{A}{\ell} \right) I_2^2 \quad \text{so} \quad \frac{U_1}{U_2} = \frac{N_1^2 I_1^2}{N_2^2 I_2^2}$$

Given $N_2 = 3N_1$ and $I_2 = I_1/4$,

$$\frac{U_1}{U_2} = \frac{N_1^2 I_1^2}{(3N_1)(I_1/4)^2} = \frac{16}{9} \quad \Rightarrow \quad U_2 = \frac{9}{16} U_1$$

8. (6 points) A conducting loop is halfway into a magnetic field pointing into the page. Suppose the magnetic field begins to decrease rapidly in strength. What happens to the loop? The loop is ...

.....

As the field decreases, there is less and less magnetic flux into the page through the loop. Lenz' Law tells us that current will flow to oppose the change. Therefore, using the Right-Hand-Rule, we see that current must flow clockwise in the loop, to generate some induced field (and therefore some magnetic flux) into the page.

Current flowing clockwise will be flowing up the page in the magnetic field on the left side of the loop. A current in a magnetic field experiences a force

$$\vec{F} = I \vec{L} \times \vec{B}$$

which, using the Right-Hand-Rule, is to the left. The loop is ...

pulled to the left, into the magnetic field.

