Final Exam date: **Period Thirteen**—Friday May 2, 8:00–10:50 AM.

Final Exam location: **Howey L3 and L4** (our usual locations).

Physics 2212G
Spring 2014
Test 5

Test form 556

Name Instructor Solutions

Recitation Section (see back of test):______

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".
2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.
4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**
5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**
6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

Numerical Constants:

$k = 8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}$

$\varepsilon_0 = 8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}$

$\mu_0 = 4\pi\times10^{-7}\ \mathrm{T\cdot m/A}$

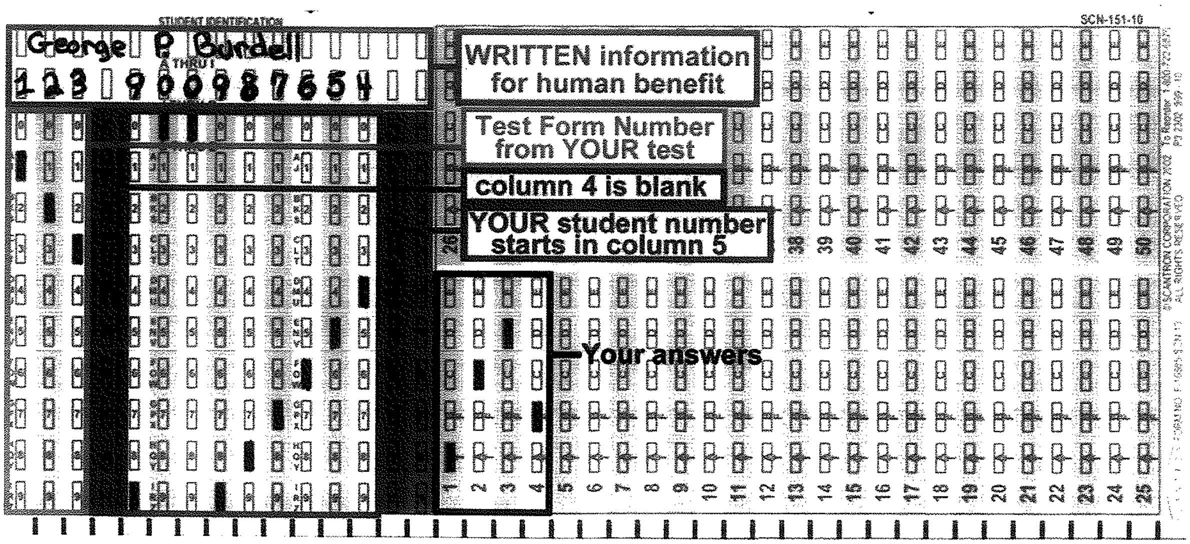
$e = 1.60\times10^{-19}\ \mathrm{C}$

$g = 9.81\ \mathrm{m/s^2}$

$m_e = 9.11\times10^{-31}\ \mathrm{kg}$

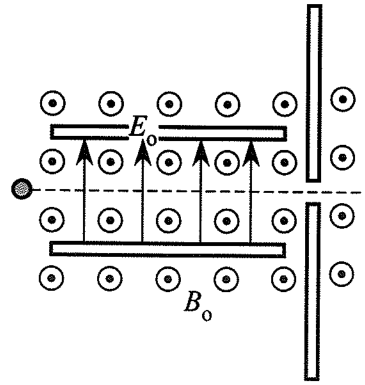
$m_p = 1.67\times10^{-27}\ \mathrm{kg}$

Your test form is: 556



The following problem will be hand-graded. <u>*Show all your work for this problem.*</u> *Make no marks and leave no space on your answer card for it.*

[I] A negatively-charged hydrogen ion (mass m, charge $-e$) is travelling through a *velocity selector* that consists of: (1) a uniform magnetic field directed out of the page (magnitude B_o), and (2) a charged capacitor generating an upward-directed electric field (magnitude E_o). When the hydrogen ion enters the apparatus from the left with a horizontally-directed speed v_1, it passes straight through without deflection.



(A) *(8 points)* A doubly-positive helium ion (mass $4m$, charge $+2e$) is fired horizontally through the same apparatus with unknown speed v_2, and *also* experiences no deflection. Determine v_2, expressed as a fraction or multiple of v_1.

Undeflected charge: electric and magnetic forces cancel $\vec{F}_E + \vec{F}_B = 0$

or $|q\vec{E}| = |q\vec{v}\times\vec{B}| \longrightarrow E_o = vB_o$ (because $\vec{v} \perp \vec{B}$)

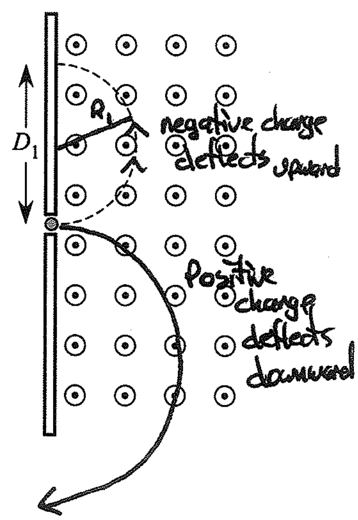
$\Rightarrow$ Note well: mass of particle is never relevant to this calculation

charge of particle does appear, but on both sides $\rightarrow$ charge drops out

For either particle, $v = E_o/B_o$ so $v_2 = E_o/B_o = v_1$ $\boxed{v_2 = v_1}$

[well, duh — it is called a "velocity selector", after all — not a mass/charge selector...]

(B) *(8 points)* After emerging from the capacitor, both charges continue to travel through the uniform magnetic field. The trajectory of the first ion is displayed; it strikes the screen at a distance D_1 above its entry point, after an elapsed time Δt_1. Where and when will the second ion strike the screen (relative to D_1 and Δt_1)?



For either charge: magnetic force $q\vec{v}\times\vec{B}$ is directed $\perp$ to motion, and results in a circular trajectory governed by Newton's 2nd law

$\Sigma \vec{F}_{radial} = m\vec{a}_{radial} \longrightarrow |q|vB_o = m\frac{v^2}{R}$

or: $R = \frac{mv}{|q|B_o}$ or $\boxed{D = 2R = \frac{2mv}{|q|B_o}}$, for diameter of circle

Also: time to complete half-circle is found from $\Delta s = v\Delta t \rightarrow \frac{\pi}{2}D = v\Delta t$

$\Delta t = \frac{\pi D}{2v} = \frac{\pi}{2v}\left[\frac{2mv}{|q|B_o}\right] \rightarrow$ simplifying: $\boxed{\Delta t = \frac{\pi m}{|q|B_o}}$

note that both D and Δt involve the ratio $\frac{m}{|q|}$

$\rightarrow$ for 1st particle, $\frac{m}{|q|} = \frac{m}{e}$

for 2nd particle, $\frac{4m}{2e} = 2\frac{m}{e}$ $\Rightarrow$ we conclude $\boxed{\begin{array}{c} D_2 = 2D_1 \\ \Delta t_2 = 2\Delta t_1 \end{array}}$

(remember — both particles have same speed)

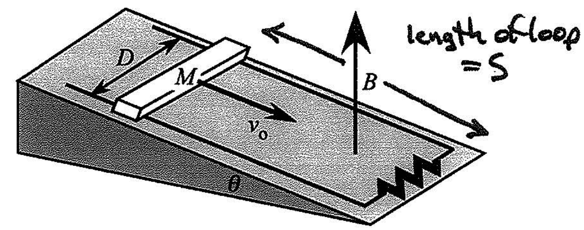
also: charge of 2nd particle is opposite to 1st particle

$\Rightarrow$ #2 experiences opposite deflection

$\boxed{\text{\#2 deflects downward}}$

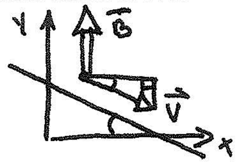
The following problem will be hand-graded. <u>*Show all your work for this problem*</u>*. Make no marks and leave no space on your answer card for it.*

[II] A ramp lies in a vertically upward magnetic field of magnitude B. The surface of the ramp is inclined at an angle θ relative to the horizontal. A bar of mass M slides down frictionless conducting rails that are separated by a distance D. The rails are connected at the bottom of the ramp by a load resistance R.



(A) *(8 points)* Find an expression for the induced emf in the bar, when it is sliding down the rails with a speed v_o. Express your answer entirely in terms of the parameters listed above.

Method ①: Motional emf



- magnetic force on electrons:

$(-e)\vec{v}\times\vec{B}$

$= (-e)\left[V_0\cos\theta\,\hat{i} - V_0\sin\theta\,\hat{j}\right]\times(B\hat{j})$

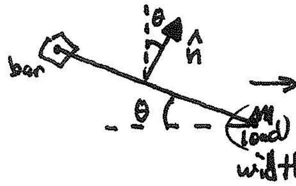
$= -eV_0B\cos\theta\,\hat{k}$ (into page)

- equilibrium when $\vec{E}_{induced}$ balances $\vec{B}$:

$|\vec{F}_E| = |\vec{F}_B| \rightarrow \cancel{(-e)}E_{ind} = \cancel{(-e)}V_0B\cos\theta$

- $\mathcal{E}_{ind} = \Delta V_{bar} = E_{ind}\cdot D = \boxed{V_0BD\cos\theta}$

Method ② Faradays law



loop = bar + rails + load

→ seen edge-on, area vector is:

$\vec{A} = D\cdot S\cdot\hat{n}$

width ← → length

so $\Phi = \vec{B}\cdot\vec{A} = BA\cos$(angle between vectors)

$= B\cdot D\cdot S\cdot\cos(\theta)$

$\mathcal{E}_{ind} = \left|\frac{d\Phi}{dt}\right| = BD\cos\theta\frac{ds}{dt} = \boxed{BD\cos\theta\,V_0}$

(direction doesn't matter, so take absolute value)

(B) *(12 points)* Find an expression for the *terminal speed* with which the bar will slide down the rails. (That is, if released from rest, what maximum speed will it attain?) Express your answer in terms of the parameters listed above, as well as the <u>symbols</u> for any necessary physical constants (such as e, g, μ_0 or ε_0).

Method ①: Power Balance

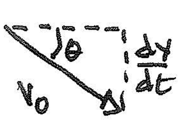
→ equilibrium when rate of grav PE loss matches rate of resistive loss in load:

$P_{grav} = P_{load} \rightarrow \frac{dU_{grav}}{dt} = I_{load}\Delta V_{load}$

but $U_{grav} = Mgy$

$I_{load} = \frac{\mathcal{E}_{ind}}{R}$ and $\Delta V_{load} = (-\mathcal{E}_{ind})$ (loop rule: voltage drop in load)

So: $Mg\frac{dy}{dt} = -\frac{\mathcal{E}_{ind}^2}{R} = -\frac{V_0^2B^2D^2\cos^2\theta}{R}$



but $\frac{dy}{dt} = -V_0\sin\theta$

so:

$-MgV_0\sin\theta = -\frac{V_0^2B^2D^2\cos^2\theta}{R}$

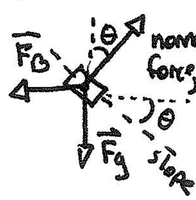
solving for V_0:

$\boxed{V_0 = \frac{MgR}{B^2D^2}\frac{\sin\theta}{\cos^2\theta}} = \frac{MgR}{B^2D^2}\sec\theta\tan\theta$

Method ② Force Balance

(tricky, because $\vec{F}_B$ = horizontal, $\vec{F}_g$ = vertical)

1st note: induced current is out of page, through bar



$\rightarrow \vec{F}_B = I\vec{L}\times\vec{B} = IDB$, to right

- $\sum F_x = 0$

$-F_B + N_x = 0$

$-I_{ind}DB + N\sin\theta = 0$

$N = \frac{I_{ind}DB}{\sin\theta}$

- $\sum \vec{F}_y = 0$

$+N_y - Mg = 0 \Rightarrow N\cos\theta = Mg$

$\frac{I_{ind}DB}{\sin\theta}\cos\theta = Mg$

but $I_{ind} = \frac{\mathcal{E}_{ind}}{R} = \frac{V_0BD\cos\theta}{R}$

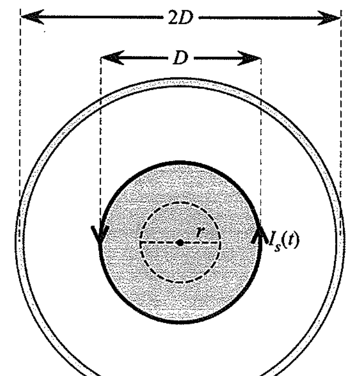
so

$\frac{V_0BD\cos^2\theta}{R\sin\theta} = Mg$

$\boxed{V_0 = \frac{MgR\sin\theta}{B^2D^2\cos^2\theta}}$

The following problem will be hand-graded. <u>*Show all your work for this problem*</u>. *Make no marks and leave no space on your answer card for it.*

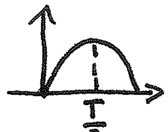
[III] The figure at right displays an end-on view of a solenoid of diameter D. The solenoid has N turns of wire, extending over a length L. It is surrounded by a copper ring of diameter $2D$, having resistance R. There is a time-dependent, counterclockwise current in the solenoid given by the expression:



$$I_s(t) = I_1\left(\frac{t}{T} - \frac{t^2}{T^2}\right) \quad \text{for} \quad 0 \le t \le T$$

(A) *(8 points)* Determine the <u>direction</u> of the induced current in the copper ring, at all times during the interval $t \in [0, T]$. Specifically, when (if ever) the induced current is clockwise? Counterclockwise? <u>*Zero*</u>?

① Plot $I(t)$:



quadratic plot = parabola
$I = 0$ at $t = 0, t = T$
$I = \max$ at $t = T/2$

② from zero to $T/2$: current increasing ⇒ magnetic flux out of page is increasing
→ ring self-generates flux into page ⇒ $\boxed{I_{ind} = \text{CW for } t \in [0, T/2)}$

③ At $t = T/2$: $I_s = \max$ implies $\frac{dI}{dt} = 0$ → $\frac{d\Phi}{dt} = 0$ at this moment: $\boxed{I_{ind} = 0 \text{ for } t = T/2}$

④ for $t > T/2$, I_s is decreasing ⇒ magnetic flux out of page is decreasing
→ ring self-generates flux out of page ⇒ $\boxed{I_{ind} = \text{CCW for } t \in (T/2, T]}$

(B) *(12 points)* Consider a point <u>inside</u> the solenoid, at a distance $r = D/4$ from the axis. What is the <u>maximum</u> induced electric field at point r (magnitude only; ignore direction) during the interval $t \in [0, T]$?

Faraday's law, in terms of induced E-field: $\oint \vec{E}_{ind} \cdot \vec{ds} = \frac{-d}{dt}\left[\int \vec{B} \cdot \vec{dA}\right]$

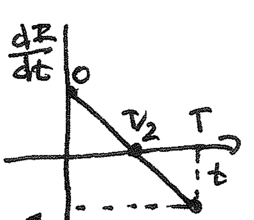
path integral around closed loop

surface integral over area bounded by loop

→ for circular loop of radius $r = D/4$:

$$\oint \vec{E}_{ind} \cdot \vec{ds} = E_{ind}(r) \cdot 2\pi r = E_{ind} \cdot \frac{\pi D}{2}$$

$$\int \vec{B} \cdot \vec{dA} = B_{sol} \cdot \pi r^2 = B_s \cdot \frac{\pi D^2}{16} \quad \text{where } B_s = B_s(t) = \mu_0 \frac{N}{L} I(t)$$



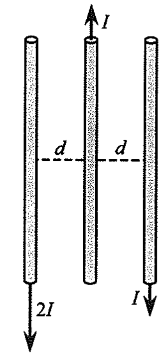
so $\int \vec{B} \cdot \vec{dA} = \frac{\mu_0 N}{L} I_1 \left(\frac{t}{T} - \frac{t^2}{T^2}\right) \frac{\pi D^2}{16}$

and $(-)\frac{dB}{dt} = \frac{(-)\mu_0 N}{L} I_1 \left(\frac{1}{T} - \frac{2t}{T^2}\right) \frac{\pi D^2}{16}$ [since we ask about magnitude, the minus sign (−) does not matter]

$$E_{ind} \cdot \frac{\pi D}{2} = \left| \frac{\mu_0 N \pi D^2}{16 L} \cdot \frac{I_1}{T} \underbrace{\left(1 - \frac{2t}{T}\right)}_{\text{max when } t = 0, T} \right| \rightarrow \max \boxed{E_{ind} = \frac{\mu_0 N D I_1}{8 L T} \text{ at } t = 0, t = T}$$

Question value 8 points

(1) Three long wires are aligned parallel to one another, with a uniform spacing d between each wire. Each wire carries the current indicated in the figure. Which (if any) of the wires experiences a net magnetic force to the left?



(a) Only the wire in the center.

(b) None of the wires.

(c) Both the wires on the left and right.

(d) Only the wire on the left.

(e) Only the wire on the right.

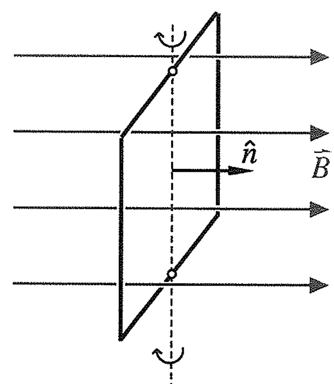
• opposite currents repel
→ center is repelled by left and right, but repulsion by left wire is greater ($2I > I$) → force to right on C

• right wire is attracted to left, repelled by center
BUT: at right wire, $|\vec{B}_L| = \frac{\mu_0 (2I)}{2\pi(2d)}$ and $|\vec{B}_c| = \frac{\mu_0 I}{2\pi d}$
→ forces will cancel on R

• left wire is repelled by C, attracted by R. Both C and R have same current (I), but R is further away – $|\vec{B}_R| < |\vec{B}_c|$, so $|\vec{F}_R| < |\vec{F}_c|$
⇒ net force on L is $\boxed{\text{to the left}}$

Question value 8 points

(2) A simple electrical generator consists of a square loop of wire rotating in a uniform magnetic field. The loop begins with its normal aligned parallel to the field, and rotates about a vertical axis with period T. At what time during the rotation will you detect the maximum induced current flowing clockwise around $\hat{n}$ (i.e. clockwise when viewed from a perspective where $\hat{n}$ points directly at you)?



(a) At time $T/4$.

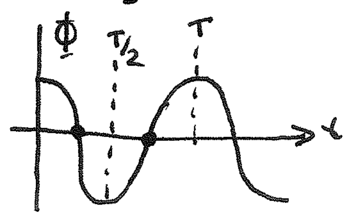
(b) At time $T/2$.

(c) At time $t = 0$.

(d) At time T.

(e) At time $3T/4$.

① start by plotting flux as a function of time: starts at max, positive, when aligned with $\vec{B}$:



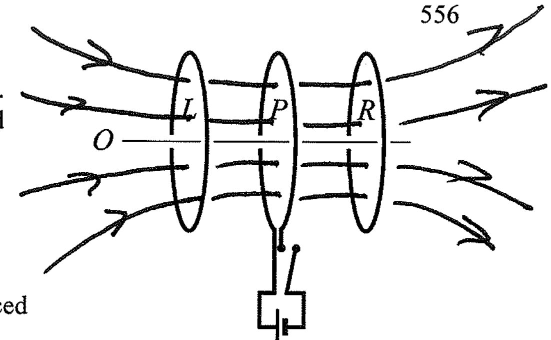
② induced emf (and hence current) is found as the time derivative of flux = slope of graph

③ max slope is at either $T/4$ or $3T/4$, from graph – but which?

④ "Ind is clockwise around $\hat{n}$" → implies self-generated flux is opposite to $\hat{n}$
→ implies external flux opposite to $\hat{n}$ is decreasing
OR external flux along $\hat{n}$ is increasing
OR external flux changing from "opposite to $\hat{n}$" to "along $\hat{n}$"

→ true at $\boxed{t = 3T/4}$

The next two questionsboth involve the following situation:
Primary loop P is coaxial with nearby loops L (on the left) and R (on the right). If the switch in the primary circuit closed, current will flow clockwise around loop P, as seen by an observer at position O.



Magnetic flux generated BY P, when switch is closed

Question value 4 points

(3) If the initially *closed* switch is opened, what will be the nature of the induced current in loop R (as seen by an onbserver at O)?

(a) There will be a steady, clockwise current in loop R, for as long as the switch remains ~~closed.~~ open

(b) There will be a brief, counterclockwise current in loop R, at the moment the switch ~~closes.~~ opens

(c) There will be a brief, clockwise current in loop R, at the moment the switch ~~closes.~~ opens

(d) There will be a steady, counterclockwise current in loop R, for as long as the switch remains ~~closed.~~ open

(e) There will be no current in loop R.

closed → open : primary current dies : rightward primary flux goes away
R self-induces rightward flux → requires CW induced current in R
→ change is temporary, so I_{ind} is temporary

Question value 4 points

(4) If the initially *open* switch is closed, what will be the nature of the induced current in loop L (as seen by an observer at O)?

(a) There will be a steady, clockwise current in loop L, for as long as the switch remains closed.

(b) There will be a brief, counterclockwise current in loop L, at the moment the switch closes.

(c) There will be a steady, counterclockwise current in loop L, for as long as the switch remains closed.

(d) There will be a brief, clockwise current in loop L, at the moment the switch closes.

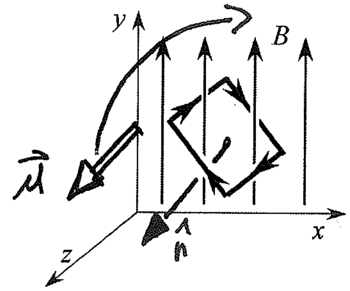
(e) There will be no current in loop L.

open → closed : primary current starts up : no flux becomes rightward flux
L self-induces leftward flux, to oppose change
→ requires CCW induced current, again, temporary only

align $\vec{\mu}$ with $\vec{B}$

Question value 8 points

(5) The loop of wire placed in the uniform magnetic field shown at right will experience



(a) a torque about the <u>negative</u> x-axis.

(b) a torque about the <u>positive</u> x-axis.

~~(c) *zero* torque, because the field is uniform.~~

(d) a torque about the <u>negative</u> z-axis.

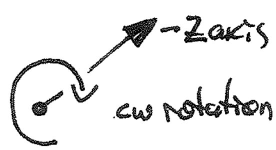
(e) a torque about the <u>positive</u> z-axis.

Note that normal to loop — and thus, dipole moment $\vec{\mu}$ — points "away" from us (down and left) in this perspective view

In a uniform field, torque will rotate dipole to be <u>parallel</u> to $\vec{B}$

① Rotation around $\pm X$ axis will <u>never</u> accomplish this: $\vec{\mu}$ has an X-component, but rotation <u>around</u> x-axis doesn't change $\vec{\mu}_x$ — it only converts $\vec{\mu}_y$ to $\vec{\mu}_z$ or vice versa (try it...)

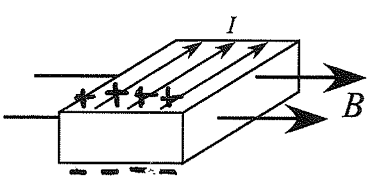
② To align $\vec{\mu}$ <u>with</u> $\vec{B}$, a <u>CW</u> rotation in the page will succeed



↳ equivalent to rotation about negative z-axis

Question value 8 points

(6) A slab of conducting material lies in a rightward-directed magnetic field, and carries a current directed into the page. Which two faces of the slab will develop a Hall voltage across them, and in particular, which of those two faces will be at the <u>higher</u> Hall potential?



(a) The front face will be at a higher Hall potential than the back.

(b) There will be no Hall potential, in this conficguration.

(c) The bottom face will be at a higher Hall potential than the top.

(d) The top face will be at a higher Hall potential than the bottom.

(e) The right face will be at a higher Hall potential than the left.

<u>electrons</u> drift <u>out of page</u>

Magnetic force on electrons is

$$\vec{F}_B = q\vec{v}\times\vec{B} = \underbrace{(-e)\,\underbrace{\vec{v}_{out}\times\vec{B}_{right}}_{\text{upwards}}}_{\text{downwards}}$$

→ electrons preferentially deflect <u>downward</u>
residual (+) charges left behind on top

Top is at <u>high</u> potential, bottom is at <u>low</u> potential