Printed Name

Solutions

Nine-digit GT ID

signature

Fall 2021 **PHYS 2212 G** **Test 04**

Test Form:

4A

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on yourtest, and fill the bubble for your answer **on this front page**.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

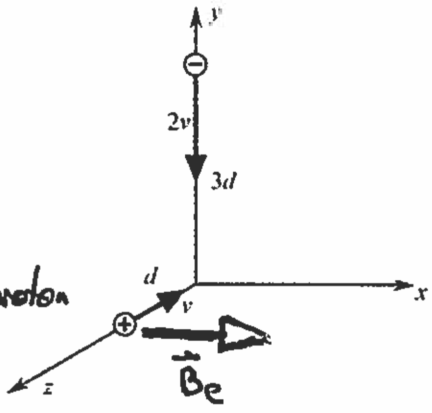
Fill in bubbles for your Multiple Choice responses HERE
Mark answers answers darkly and neatly.
If you wish to change an answer, draw a clear "X" through the non-answer!

1. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ
2. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ
3. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ
4. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ
5. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ
6. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ

The following problem will be hand-graded. *Show all supporting work for this problem.*

[I] *(20 points)* At a particular moment in time, a proton is located on the positive z-axis at a distance d from the origin, and is moving with speed v in the negative z-direction. At the same moment, an electron is located on the positive y-axis at a distance $3d$ from the origin, moving in the negative y-direction with speed $2v$.

Determine the magnetic force on the proton by the electron. Be sure to specify magnitude and direction.

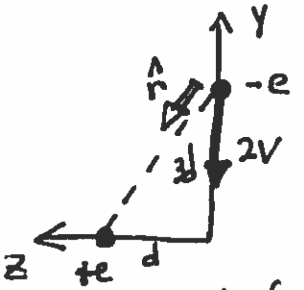


① Use Biot-Savart to find $\vec{B}_{electron}$ at position of proton

② Find force on proton as $\vec{F}_p = (q\vec{v})_p \times \vec{B}_e$

$$\vec{B}_e = \frac{\mu_0}{4\pi}\frac{(q\vec{v})_e \times \hat{r}}{r^2}$$

Side view:



moving out of page

where $q = (-e)$ $\vec{v} = (-2v\hat{j})$

$r = \sqrt{(3d)^2 + d^2} = \sqrt{10}\,d$

$\hat{r} = -\frac{3}{\sqrt{10}}\hat{j} + \frac{1}{\sqrt{10}}\hat{k}$

so $$\vec{B}_e = \frac{\mu_0}{4\pi}\frac{(-e)(-2v\hat{j}) \times \left[-\frac{3}{\sqrt{10}}\hat{j} + \frac{1}{\sqrt{10}}\hat{k}\right]}{10d^2}$$

$$= \frac{+2ev\mu_0}{4\pi \cdot 10d^2}\left[\cancel{\left(\hat{j}\times\frac{-3}{\sqrt{10}}\hat{j}\right)}\to 0 + \left(\hat{j}\times\frac{1}{\sqrt{10}}\hat{k}\right)\right]$$ but $\hat{j}\times\hat{k} = +\hat{i}$

$$\boxed{\vec{B}_e = +\frac{ev\mu_0}{20\pi d^2}\frac{1}{\sqrt{10}}\hat{i}}$$ ie to the right, in original figure

Then $$\vec{F}_p = (q\vec{v})_p \times \vec{B}_e = (+e)(-v\hat{k}) \times \left[\frac{ev\mu_0}{20\pi d^2\sqrt{10}}(+\hat{i})\right]$$

$$= \frac{e^2v^2\mu_0}{20\sqrt{10}\pi d^2}\left[-\hat{k}\times\hat{i}\right]$$ but $\hat{k}\times\hat{i} = +\hat{j}$
so $-\hat{k}\times\hat{i} = -\hat{j}$

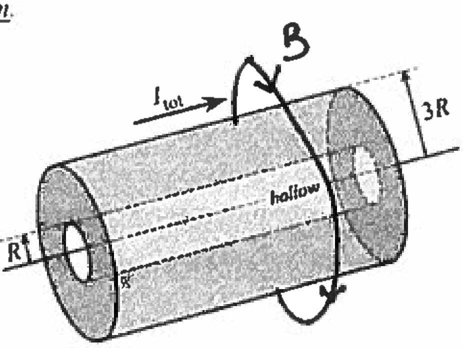
$$\boxed{\vec{F} = \frac{e^2v^2\mu_0}{20\sqrt{10}\pi d^2}(-\hat{j})}$$

The following problem will be hand-graded. Show all supporting work for this problem.

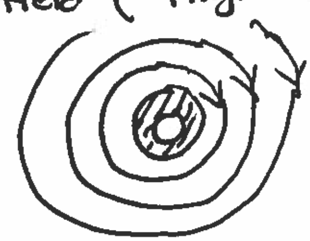
[II] *(20 points)* A long straight current-carrying wire is hollow, with an inner (cavity) radius R and and outer (surface) radius $3R$. A non-uniform but symmetric current flows in the wire, with a current density give by

$$J(s) = \frac{J_0\, s}{R}, \quad \text{for} \quad R \le s \le 3R,$$

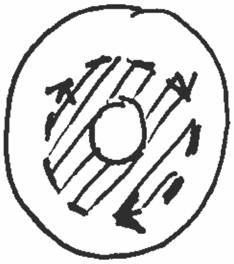
where J_0 is a positive constant and s is the distance from the central axis of the wire. Determine the magnitude of the magnetic field at a distance $2R$ from the central axis of the wire.



Apply Ampere's Law → exploit symmetry/pattern of $\vec{B}$-field (= rings around wire)



- assume we view wire end-on, with current flowing away
- choose a path: CW circle of radius 2R



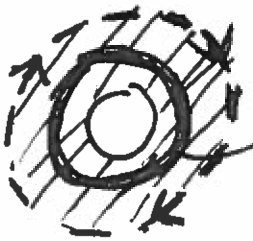
I = into page

$$\Rightarrow \oint_{path} \vec{B}\cdot d\vec{s} = \oint_{path} B(2R)\,ds = B(2R)\oint_{path} ds = \boxed{B(2R)\cdot 2\pi(2R)}$$

So, next step is to find current passing through our loop

= all current flowing between $s=R$ and $s=2R$ (but no further out!)

⇒ Find this current by summing over thin rings radius s thickness ds $R \le s \le 2R$



This ring has area $dA = 2\pi s\cdot ds$

if density on ring is $J(s) = \frac{J_0 s}{R}$,

then $dI = J(s)\,dA = \left(\frac{J_0 s}{R}\right)(2\pi s\,ds)$ = current through this ring

→ sum over all rings with $s \in [R, 2R]$

$$I_{Through} = \int_{s=R}^{s=2R} \left(\frac{2\pi J_0}{R}\right) s^2\,ds = \frac{2\pi J_0}{R}\left[\frac{s^3}{3}\right]_R^{2R} = \frac{2\pi J_0}{3R}\left[8R^3 - R^3\right] = \frac{14\pi J_0 R^2}{3}$$

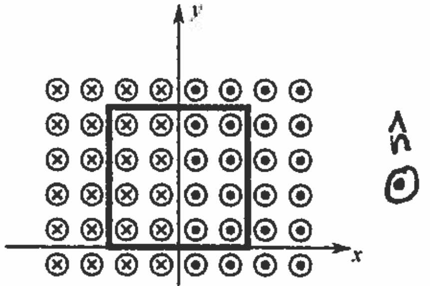
Plug in to Ampere's Law

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{Through} \rightarrow B\cdot 4\pi R = \mu_0 \frac{14\pi J_0 R^2}{3}$$

$$\boxed{B = \frac{7\mu_0 J_0 R}{6}}$$

The following problem will be hand-graded. Show all supporting work for this problem.

[III] *(20 points)* A square wire loop of dimension L lies in the xy-plane, extending from $x = -L/2$ to $x = +L/2$. The loop lies in a magnetic field that points in the negative z-direction in the left half-plane (i.e. for $x < 0$), and points in the positive direction in the right half-plane (i.e. for $x > 0$).



The magnetic field in both regions varies with time, according to:

$$\vec{B}(t) = \begin{cases} (B_0 - at)(+\hat{k}) & x > 0 \\ (B_0 + at)(-\hat{k}) & x < 0 \end{cases}$$

Determine the magnitude of the induced emf in the loop at time $t = 0$, and determine the direction in which the induced current (if any) would flow around the loop. You must support the latter answer with quantitative or qualitative reasoning, to receive any credit for your response.

let normal to loop be out of page $\boxed{\hat{n} = +\hat{k}}$

Faraday's Law $|\mathcal{E}_{ind}| = \left|-\frac{d\Phi}{dt}\right|$ where $\Phi = \int_{loop} \vec{B}\cdot d\vec{A}$

First, express flux

$$\Phi = \Phi_{left} + \Phi_{right} = \vec{B}_L\cdot\vec{A}_L + \vec{B}_R\cdot\vec{A}_R$$

[break loop into two subregions]

$$= \left[B_0 + at\right](-\hat{k})\cdot\left(L\tfrac{L}{2}\hat{k}\right) + \left[B_0 - at\right](+\hat{k})\cdot\left(L\tfrac{L}{2}\hat{k}\right)$$

[each half-square has area $A = l\cdot w = L\cdot L/2$]

$$= \frac{L^2}{2}\left(-B_0 - at\right) + \frac{L^2}{2}\left(+B_0 - at\right)$$

$$\boxed{\Phi_{net,loop} = -L^2at}$$

hence $|\mathcal{E}_{ind}| = \left|\frac{d\Phi}{dt}\right| = aL^2$ = constant

evaluate at $t = 0$: $\boxed{|\mathcal{E}_{ind}| = aL^2}$

Lenz's Law:

- in left half plane, flux into page is increasing: left half-loop self-generates flux out of page
- in right half plane, flux is out of page, and decreasing: right half self-generates flux out of page

→ Both sides are working together to make $\boxed{\phi_{self} = \text{out of page}}$

$$\Downarrow$$

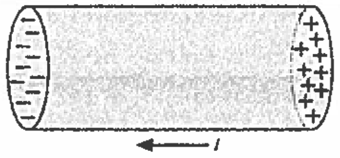
$\boxed{\text{induced current is CCW}}$

RH curl around ϕ_{self} = CCW

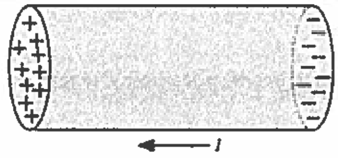
Question value 4 points

(01) A cylindrical wire of length L and diameter D carries a right-to-left current I. Which of the figures below best represents the distribution of charges on the wire that "shepherd" the flow of current in the wire?

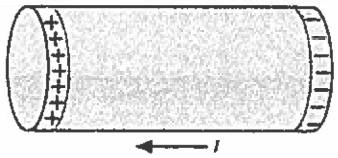
61% success rate



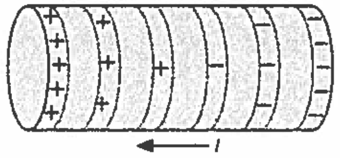
(a)



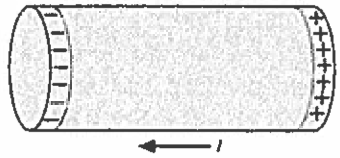
(b)



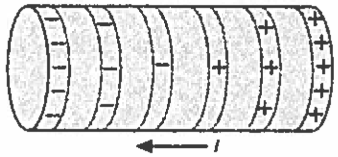
(c)



(d)



(e)



[✓ (f)]

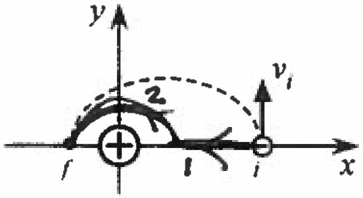
Current Model: I is caused by an $\bar{E}$-field that is right-to left, here $\bar{E}$ is caused by bands of surface charge

Bands vary continuously from very positive on far right, to smaller positive, then neutral, then small negative, then very negative on far left

Question value 4 points

(2) In the figure at right, a positive source charge is held fixed at the origin, and a negative test charge is initially at location i, moving vertically with speed v_i. The test charge follows the dotted trajectory, reaching position f—at which point it is moving vertically *downward*. No external forces act during this process.

37% success rate



What can you say about the work done by the electric field of the (positive) source charge, and the change in the electric potential experienced by the (negative) test charge, as the test charge moves from i to f?

[✓ (a) The field has done positive work and the charge has moved to higher electric potential.]
(b) The field has done zero work and the charge has moved to lower electric potential.
(c) The field has done negative work and the charge has moved to higher electric potential.
(d) The field has done negative work and the charge has moved to lower electric potential.
(e) The field has done positive work and the charge has moved to l~~ower electric potentia~~l.

- W_{field} is path independent — follow solid arc
  W_1 = positive [Force ∥ radial displacement]
  W_2 = zero [Force ⊥ tangential displacement]

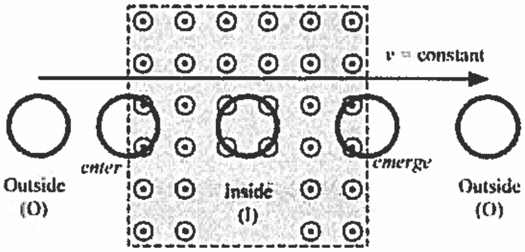
$\boxed{W_{field} = \text{positive}}$

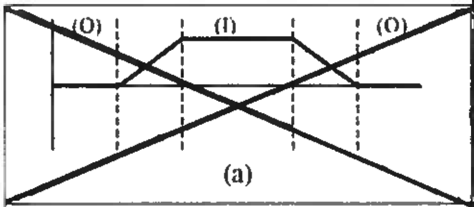
- ⊕ creates a Potential Hill : $\boxed{\text{closer to} ⊕ = \boxed{\text{higher potential}}}$

Question value 8 points

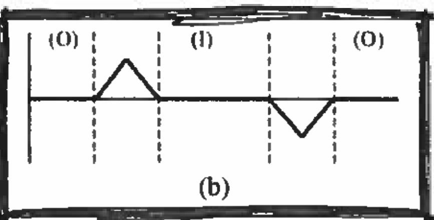
(03) A region of space has a uniform magnetic field $\vec{B}$ directed out of the page. A circular copper ring is pulled through the field at constant speed from left to right, with the plane of the ring oriented perpendicular to $\vec{B}$. Assume a sign convention that a *positive* induced current in the ring denotes a *clockwise* flow of positive charge. Which of the plots below *best* denotes the induced current in the ring, as a function of time?

67% success rate

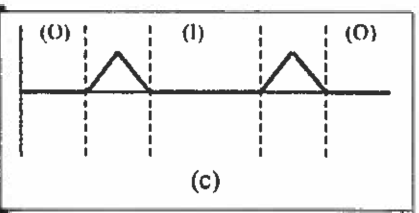




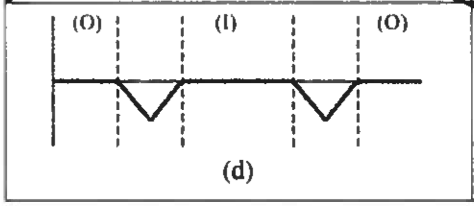
(a)



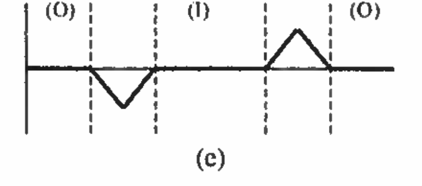
(b)



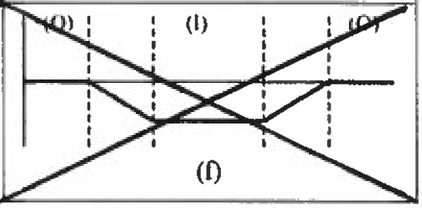
(c)



(d)



(e)



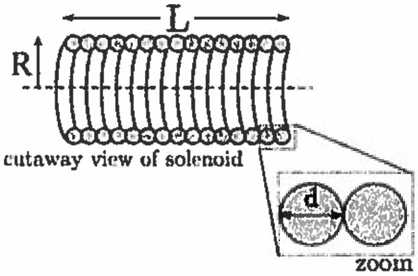
(f)

- when ring is fully inside, Φ = constant, so no induced current (eliminate a, f)
- When entering: Flux out of page ↑, so induce flux into page: $\otimes$ RH curl
  ⇒ $\boxed{\text{clockwise current = positive "blip"}}$
- when exiting: Flux out of page ↓, so induce flux out of page: $\odot$ RH curl
  $\boxed{\Rightarrow \text{CCW current = negative}}$

Question value 8 points

(04) You have constructed a solenoid out of a wire having diameter d, by winding a single layer of wire with the coils as close together as possible, around a cylinder of length L and radius R. When a current I flows through the wire, what will be the magnitude of the magnetic field inside the solenoid?

56% success rate



cutaway view of solenoid

(a) $B = \mu_0 I \cdot \pi R^2$

[✓ (b) $B = \mu_0 I \cdot (1/d)$]

(c) $B = \mu_0 I \cdot L$

(d) $B = \mu_0 I \cdot (1/R)$

(e) $B = \mu_0 I \cdot (R/d)$

(f) $B = \mu_0 I \cdot (d/\pi R^2)$

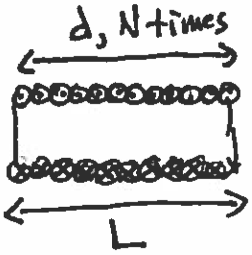
From formula sheet:

$$B_{sol} = \mu_0 n I = \mu_0 \frac{N}{L} I$$

N = # turns of wire
L = length of solenoid

From figure: length L of solenoid has N windings
each winding extends distance d along sol.

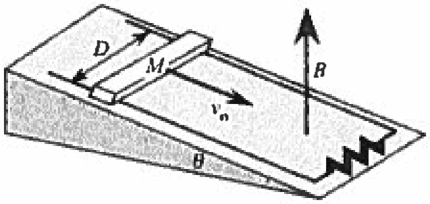
$$L = N \cdot d \quad \text{or} \quad \boxed{\frac{N}{L} = \frac{1}{d}}$$



$$\boxed{B_{sol} = \mu_0 I \cdot \left(\frac{1}{d}\right)}$$

Question value 8 points

(05) A conducing bar of length D slides at constant speed v_0 down a ramp inclined at an angle θ below the horizontal. The bar is in a uniform verical magentic field of magnitude B. What is the magnitude of the motional emf induced in the bar?



80% success rate

(a) $\mathcal{E}_{ind} = v_0BD/\sin\theta$

(b) $\mathcal{E}_{ind} = v_0BD/\cos\theta$

$\boxed{\text{(c)}\ \mathcal{E}_{ind} = v_0BD\cdot\cos\theta}$

(d) $\mathcal{E}_{ind} = v_0BD$

(e) $\mathcal{E}_{ind} = v_0BD\cdot\sin\theta$

$$\mathcal{E}_{ind} = |\Delta V_{bar}|$$

$$= \underbrace{E_{induced}} \cdot D$$

↳ induced field is found by requiring electrical and magnetic forces to balance

$$|\vec{F}_e| = |\vec{F}_B| \rightarrow |e\vec{E}| = |e\vec{v}\times\vec{B}|$$

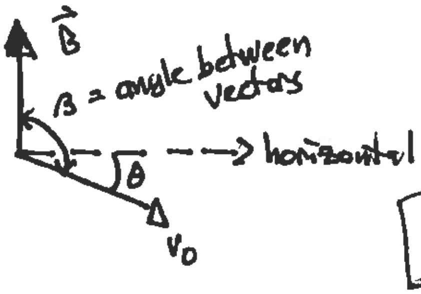
or $E_{ind} = |\vec{v}\times\vec{B}|$

$$= |v_0 B \sin\beta|$$

$$= |v_0 B \sin(90° + \theta)|$$

$$= v_0 B \cos\theta$$

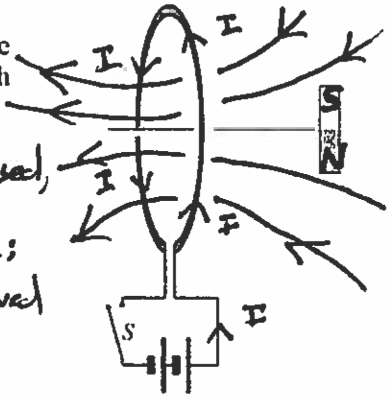
$$\boxed{\mathcal{E}_{ind} = v_0 B D \cos\theta}$$



Question value 8 points

(06) When the switch in the diagram at right is closed, the emf will drive a current through the large loop. What effect (if any) will there be on the nearby bar magnet when the switch is closed?

61% success rate

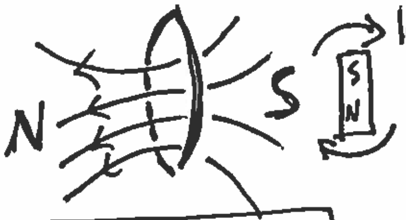


with switch closed, current flows as shown; CCW, when viewed from left

- (a) The magnet will rotate so that the north pole is on the left and the south pole on the right.
- (b) The magnet as a whole will be pulled toward the loop, without rotating.
- (c) The magnet will rotate so that the south pole is on the left and the north pole on the right.
- (d) There will be no effect, because an electrically charged ring will not influence a magnet.
- (e) The magnet will flip so that the north pole is on top and the south pole is on the bottom.

⇒ loop acts as Magnetic Dipole, with $\vec{B}$ pointing $\boxed{\text{right-to-left}}$

⇓



loop acts as magnet with S pole on right

⇓

attracts N pole of actual magnet

torque on real magnet rotates it clockwise, so that N is on left and S is on right