Printed Name	Nine-digit GT ID
Solutions	

signature

Fall 2021 **PHYS 2212 G** **Test 03**

Test Form:

3A

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on yourtest, and fill the bubble for your answer **on this front page**.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

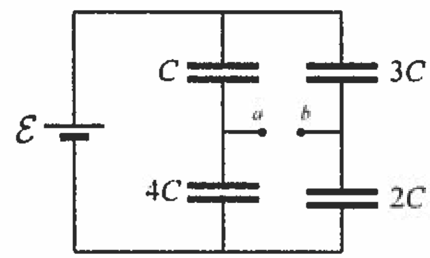
Fill in bubbles for your Multiple Choice responses HERE
Mark answers answers darkly and neatly.
If you wish to change an answer, draw a clear "X" through the non-answer!

1. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ
2. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ
3. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ
4. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ
5. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ
6. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ

The following problem will be hand-graded. Show all supporting work for this problem.

[I] *(20 points)* Four capacitors—$C, 2C, 3C$, and $4C$—are arranged in the network shown at right, and attached to an ideal battery having emf $\mathcal{E}$. Determine the potential difference across gap $a - b$. Be sure to specify which side of the gap, a or b, is at high potential. Express your answer as a fraction of $\mathcal{E}$.

Hint: analyze the network and then consider a loop rule expression for a loop including the gap.



① C and 4C are in series:

$$(C_{14})^{-1} = \frac{1}{C} + \frac{1}{4C} = \frac{5}{4C}$$

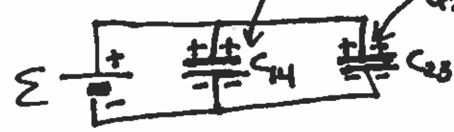
so $\boxed{C_{14} = \frac{4}{5}C}$

2C and 3C are in series

$$(C_{23})^{-1} = \frac{1}{2C} + \frac{1}{3C} = \frac{5}{6C}$$

so $\boxed{C_{23} = \frac{6}{5}C}$

⇒ Network is now



② Both reduced capacitors have full emf across them: $\Delta V_{14} = \Delta V_{23} = \mathcal{E}$

hence $Q_{14} = C_{14}\,\Delta V_{14} = \left(\frac{4}{5}C\right)(\mathcal{E}) = \frac{4}{5}C\mathcal{E}$

$Q_{23} = C_{23}\,\Delta V_{23} = \left(\frac{6}{5}C\right)(\mathcal{E}) = \frac{6}{5}C\mathcal{E}$

We do not need to reduce circuit any further because:

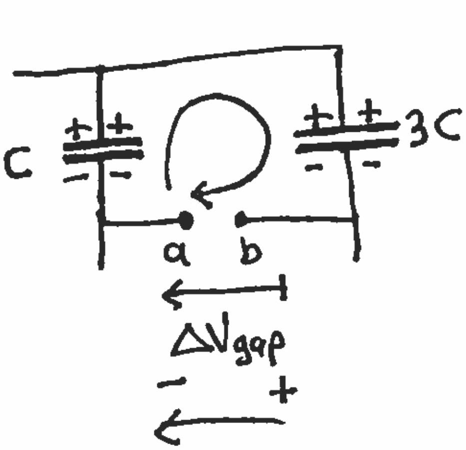
- C and 4C in series: same charge as C_{14} : $\frac{4}{5}C\mathcal{E}$

  Therefore potential across capacitor C is $\Delta V_1 = Q_1/C = \boxed{\frac{4}{5}\mathcal{E}}$ with top = high bottom = low

- 3C and 2C in series: same charge as C_{23} : $\frac{6}{5}C\mathcal{E}$

  Therefore, potential across 3C is: $\Delta V_3 = \frac{Q_3}{3C} = \boxed{\frac{2}{5}\mathcal{E}}$ top = high bottom = low

③ With two known potentials, we can examine a simple loop



loop clockwise from a:

$$\Delta V_1 + \Delta V_3 + \Delta V_{gap} = 0$$

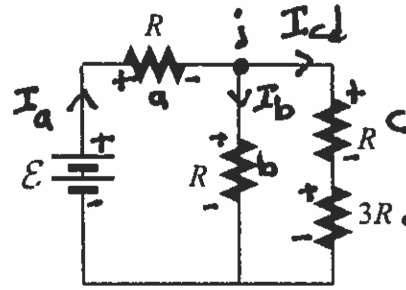
$$\left(+\frac{4}{5}\mathcal{E}\right) + \left(-\frac{2}{5}\mathcal{E}\right) + \Delta V_{gap} = 0$$

$$\boxed{\Delta V_{gap} = -\frac{2}{5}\mathcal{E}}$$

minus sign tells us: $\boxed{\text{b higher than a}}$

The following problem will be hand-graded. *Show all supporting work for this problem.*

[II] *(20 points)* In the resistor network at right, determine the power consumed by resitor $3R$. Express your answer in terms of $\mathcal{E}$ and R.



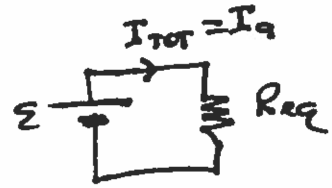
Label resistors a–d we want P_d

① c and d in series: $R_{cd} = 4R$

b and (cd) in parallel: $(R_{bcd})^{-1} = \frac{1}{R} + \frac{1}{4R} = \frac{5}{4R}$

$R_{bcd} = \frac{4}{5}R$

a and (bcd) in series: $R_{eq} = R + \frac{4}{5}R = \boxed{\frac{9}{5}R}$



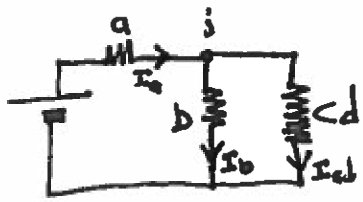
② Find total current:

$$(+\mathcal{E}) + (-I_{TOT}R_{eq}) = 0 \qquad I_{TOT} = \frac{\mathcal{E}}{R_{eq}} = \boxed{\frac{5}{9}\frac{\mathcal{E}}{R}}$$

(Note that I_{TOT} and I_a are the same)

③ At junction j, currents split

But: b and cd in parallel: same ΔV



- $\Delta V_b = \Delta V_{cd} \rightarrow I_b R = I_{cd}(4R)$

  $\boxed{I_b = 4I_{cd}}$

- junction rule:

  $I_b + I_{cd} = I_a = \frac{5}{9}\frac{\mathcal{E}}{R}$

  $(4I_{cd}) + I_{cd} = \frac{5}{9}\mathcal{E}/R \Longrightarrow \boxed{I_{cd} = \frac{1}{9}\frac{\mathcal{E}}{R}}$

④ Knowing current through (c+d), we know current through d

$$I_d = I_{cd} = \frac{\mathcal{E}}{9R} \rightarrow \text{Ohm's Law } \Delta V_d = (-)I_d(3R) = \boxed{-\frac{\mathcal{E}}{3}}$$

Hence, power consumed by d is

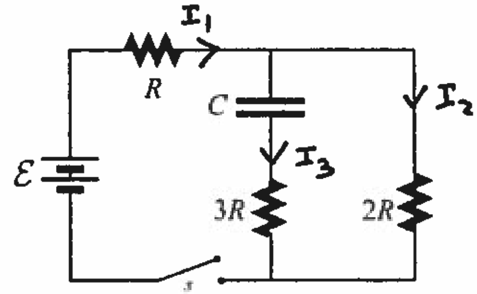
$$P_d = I_d \Delta V_d = \left(\frac{1}{9}\frac{\mathcal{E}}{R}\right)\left(\frac{-\mathcal{E}}{3}\right)$$

$$\boxed{P_d = (-)\frac{\mathcal{E}^2}{27R}}$$

negative sign reinforces our expectation of a power LOSS

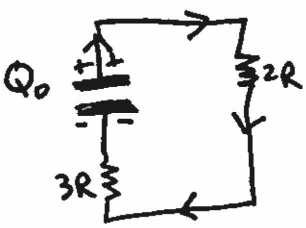
The following problem will be hand-graded. Show all supporting work for this problem.

[III] *(20 points)* In the resistor-capacitor circuiot at right, switch S has been closed for a very long time. At time $t = 0$, the switch is opened and the capacitor begins to discharge. How much time must elapse in order for the potential difference across the capacitor to drop to $\mathcal{E}/6$? Express your answer in terms of $\mathcal{E}$, R, and C, as needed.



①

- Switch closed — Capacitor changes
  - → After a long time: C is fully charged
    - ⇒ I_3 goes to zero at full charge
    - ⇒ Current flows around the perimeter $I_1 = I_2$
- Switch open — capacitor discharges through 2R and 3R

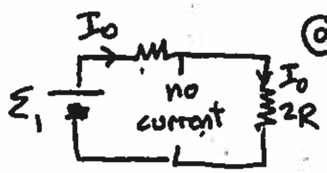


Time constant is $\tau = R_{eq} C = 5RC$

$q(t) = Q_0 e^{-t/\tau}$

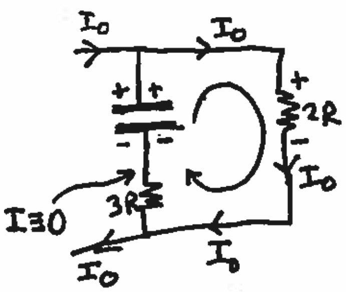
since $V_c(t) = \frac{q(t)}{C}$, we also know $\boxed{V_c(t) = V_{c0}\, e^{-t/\tau}}$

② Analyze circuit with switch closed to find ΔV_c at time switch opens



ⓐ Loop rule around perimeter: $+\mathcal{E} - I_0 R - I_0 2R = 0$

$I_0 = \frac{\mathcal{E}}{3R}$



ⓑ loop rule around RH loop $(+V_{c0}) + (-I_0 2R) + \cancel{\Delta V_{3R}} = 0$

zero because no current flows through 3R

$\Rightarrow V_{c0} = 2 I_0 R = 2\left(\frac{\mathcal{E}}{3R}\right)R \Rightarrow \boxed{V_{c0} = \frac{2}{3}\mathcal{E}}$

use this in exp decay equation above

③ So $V_c(t) = \frac{2}{3}\mathcal{E}\, e^{-t/\tau}$ with $\tau = 5RC$

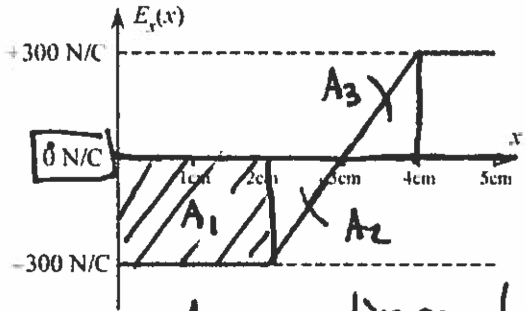
require $V_c(t) = \frac{\mathcal{E}}{6}$: $\frac{\mathcal{E}}{6} = \frac{2}{3} e^{-t/\tau} \rightarrow \frac{3\cancel{\mathcal{E}}}{12} = \cancel{\mathcal{E}}\, e^{-t/\tau}$

$e^{-t/\tau} = \frac{1}{4} \rightarrow e^{+t/\tau} = 4 \rightarrow \frac{t}{\tau} = \ln 4 \rightarrow t = \tau \ln(4)$

or $\boxed{\begin{aligned} t &= 5RC\ln(4) \\ &= 6.93RC \end{aligned}}$

Question value 4 points

(1) The graph at right depicts the x-component of the electric field in the vicinity of the origin. If the electric potential at the origin is –3 volts, what is the electric potential at $x = 4$ cm?



63% success rate

(a) –6 volts
(b) –9 volts
(c) 0 volts
(d) +6 volts
$\boxed{\text{(e) +3 volts}}$
(f) –3 volts

$V_f = V_i + \Delta V$

where $\Delta V = -\int_i^f \vec{E}\cdot d\vec{s}$

$= -(\text{area under curve})$

$= -(A_1 + \underbrace{A_2 + A_3})$ → from graph, it is clear that A_2 and A_3 are equal and opposite $\Rightarrow A_2 + A_3 \cong 0$

A_1 = negative area!

$\Delta V = -(A_1) = -(-h \cdot w)$

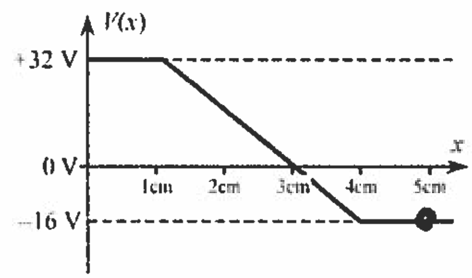
$= +(300\ N/C)(+2cm)$

$= +(300\ V/m)(0.02m) = \boxed{+6V}$

Therefore $V_f = V_i + \Delta V = (-3V) + (+6V) = \boxed{+3V}$

Question value 4 points

(2) The graph at right depicts the electric potential as a function of x, in the vicinity of the origin. What is the x-component of the electric field at $x = 5$ cm?



70% success rate

(a) 1600 N/C, to the right
$\boxed{\text{(b) zero}}$
(c) 2400 N/C, to the left
(d) 1600 N/C, to the left
(e) 2400 N/C, to the right

$\vec{E}_x = -\frac{dV}{dx} = -(\text{slope of V-vs-x graph})$

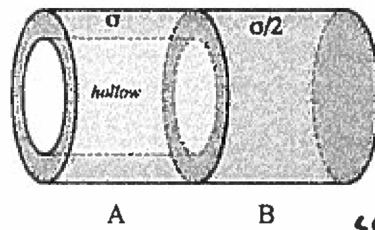
→ at $x = 5cm$, $V(x) = \text{constant}$

→ slope = zero

$\boxed{\vec{E}_x = \text{zero}}$

Question value 8 points

(3) Wire A (conductivity σ) is hollow, with inner radius R and outer radius $1.5R$. It is spliced to solid wire B (conductivity $\sigma/2$), with radius $1.5R$. Determine the relative electric field strengths in the two wires when a current flows through the junction.



same I

62% success rate

(a) $E_B = \frac{5}{2} E_A$

(b) $E_B = \frac{5}{9} E_A$

(c) $E_B = \frac{9}{10} E_A$

(d) $E_B = \frac{9}{2} E_A$

$\boxed{\text{(e)} \quad E_B = \frac{10}{9} E_A}$

Same current on each side

$I_A = I_B$

$J_A A_A = J_B A_B$

$(\sigma_A E_A)(\pi[(1.5R)^2 - R^2]) = \sigma_B E_B (\pi (1.5R)^2)$

$\sigma E_A \pi \left(\frac{5}{4} R^2\right) = \left(\frac{\sigma}{2}\right) E_B \pi \left(\frac{9}{4} R^2\right)$

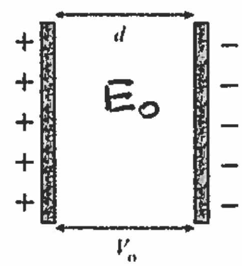
$5 E_A = \frac{9}{2} E_B$

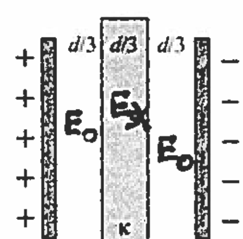
$\boxed{E_B = \frac{10}{9} E_A}$

Question value 8 points

(4) An isolated, vacuum-filled capacitor has a charge Q placed on it, resulting in a potential difference V_0 across the plates. An insulating slab having thickness $d/3$ and dielectric constant $\kappa = 1.5$ is inserted between the plates, as shown at bottom right. In terms of the original vacuum potential V_0, what is the new potential difference across the plates?

one step





Three steps

39% success rate

~~(a) $V = \frac{3}{2} V_0$~~

$\boxed{\text{(b)} \quad V = \frac{8}{9} V_0}$

(c) $V = \frac{2}{9} V_0$

(d) $V = \frac{2}{3} V_0$

~~(e) $V = \frac{9}{8} V_0$~~

~~(f) $V = \frac{9}{4} V_0$~~

① $\Delta V = -\int \vec{E} \cdot d\vec{s}$

② insertion of dielectric reduces $|\vec{E}|$:

$E_\kappa = \frac{1}{\kappa} \underbrace{E_0}_{\text{vacuum field}}$

$\Rightarrow$ New ΔV must be less than V_0

Formally:

- Vacuum: $V_0 = \left| -\int \vec{E} \cdot d\vec{s} \right| = |-E_0 d| \longrightarrow \boxed{V_0 = E_0 d}$

- with dielectric:

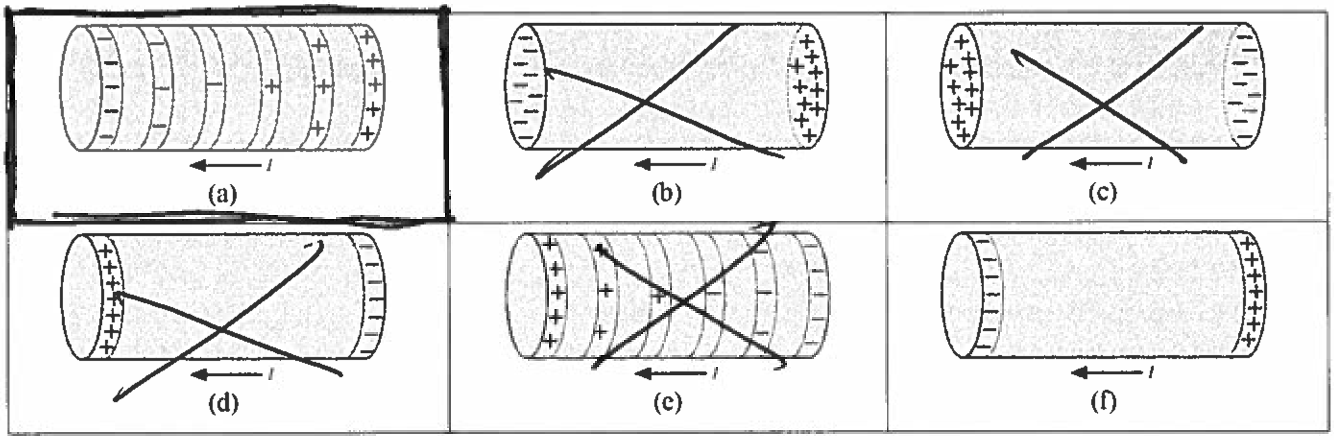
$V = \left| -\int \vec{E} \cdot d\vec{s} \right| = E_0 \cdot \frac{d}{3} + \frac{E_0}{\kappa} \cdot \frac{d}{3} + E_0 \cdot \frac{d}{3} = \frac{E_0 d}{3}\left[2 + \frac{1}{\kappa}\right] = \frac{E_0 d}{3}\left[2 + \frac{2}{3}\right]$

$V = \frac{E_0 d}{3}\left[\frac{8}{3}\right] = \frac{8}{9}\left[E_0 d\right] = \boxed{\frac{8}{9} V_0}$

Question value 8 points

(5) A cylindrical wire of length L and diameter D carries a right-to-left current I. Which of the figures below best represents the distribution of charges on the wire that "shepherd" the flow of current in the wire?

47% success rate

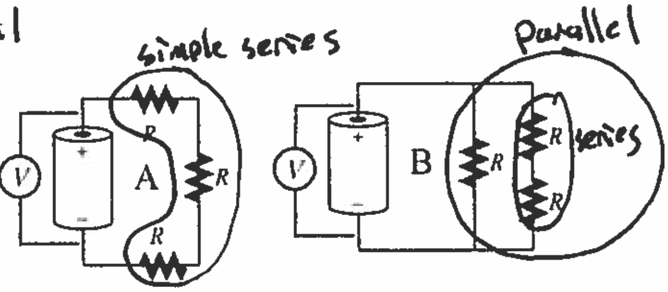


Charge model was discussed at length in text and lecture:

- surface charge bands, in a gradient from most positive → less positive → least positive → least negative → most negative
- current flow is from most positive/high potential to most negative/low potential

Question value 8 points

(6) A real battery having emf $\mathcal{E}$ and internal resistance r is connected in turn to each of the three 3-resistor networks shown at right. Rank, from greatest to least, the terminal potential across the battery when placed connected to each network.



57% success rate

(a) $V_A > V_C > V_B$

(b) $V_A > V_B = V_C$

(c) $V_A = V_C = V_B$

(d) $V_B > V_C > V_A$

(e) $V_B = V_C > A$

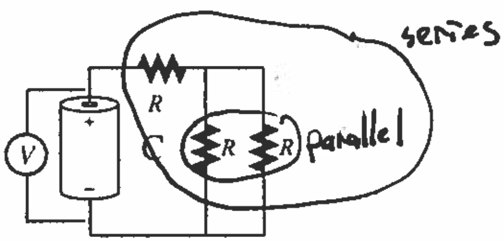
(f) $V_C > V_B > A$

$V_{Term} = \Delta V_{battery}$

$= \mathcal{E}$ (no current)

$= \mathcal{E} - Ir$ (with current)

⇒ greatest V_{Term} when the current drawn is least



also: loop rule for circuit is $\mathcal{E} - Ir - IR_{eq} = 0 \Rightarrow I = \dfrac{\mathcal{E}}{r + R_{eq}}$

so: greatest R_{eq} → smallest I → greatest V_{Term} : $\boxed{\text{compare } R_{eq}}$

A: $R_{eq} = 3R$

B: $R_{eq} = \left(\frac{1}{R} + \frac{1}{R+R}\right)^{-1} = \frac{2}{3}R$

C: $R_{eq} = R + \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = \frac{3}{2}R$

$R_A > R_C > R_B$

so $\boxed{V_A > V_C > V_B}$