Printed Name
Solutions

Nine-digit GT ID

signature

Fall 2021 **PHYS 2212 G** **Test 02**

Test Form: **2B**

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on yourtest, and fill the bubble for your answer **on this front page**.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Fill in bubbles for your Multiple Choice responses HERE
Mark answers answers darkly and neatly.
If you wish to change an answer, draw a clear "X" through the non-answer!

1. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ
2. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ
3. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ
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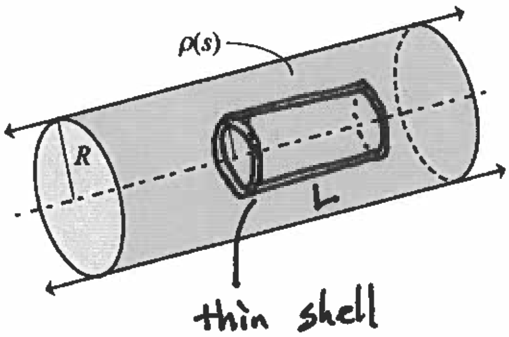
6. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ

The following problem will be hand-graded. *Show all supporting work for this problem.*

[I] *(20 points)* A very long insulating cylinder of radius R has a non-uniform but axially symmetric charge density, given by the expression:

$$\rho(s) = \rho_0(1 - s/R),$$

where s is the distance from the central axis of the rod ($0 \le s \le R$) and ρ_0 is a positive constant.



(i) What is the total charge per unit length, λ_{tot}, on the rod? (Hint: start by finding the total charge Q on a sublength L of the rod.) Express you answer entirely in terms of ρ_0 and R, along with any appropriate numerical factors.

(ii) What is the magnitude of the electric field *inside* the rod, at a distance r from the axis (where $r < R$)? Express your answer in terms of ρ_0, r, R, and ε_0.

When summing charge on rod, one must proceed layer by layer

→ each layer is a thin cylindrical shell: length L, radius s, radial thickness

Volume of such a layer is $\delta V = L \cdot 2\pi s \cdot ds$

(i) Charge on a length L is found by summing over all layers: $s=0 \rightarrow s=R$

$$Q = \int \delta Q = \int \rho\, \delta V = \int_{s=0}^{s=R} \rho_0\left(1-\frac{s}{R}\right)\cdot L\cdot 2\pi s\cdot ds = 2\pi L \rho_0 \int_0^R \left(s - \frac{s^2}{R}\right) ds$$

$$= 2\pi L \rho_0 \left[\frac{s^2}{2} - \frac{s^3}{3R}\right]_0^R = 2\pi L \rho_0\left[\frac{R^2}{2} - \frac{R^2}{3}\right] = \frac{2\pi L \rho_0 R^2}{6}$$

so $\lambda_{TOT} = \dfrac{Q}{L} = \boxed{\dfrac{\pi \rho_0 R^2}{3}}$

(ii) Apply Gauss's law, with a cylindrical gaussian surface of radius r

$$\Phi_{gs} = \int \vec{E}\cdot d\vec{A} \longrightarrow E_{in}(r)\cdot 2\pi r \cdot L \quad \text{due to radial symmetry of } \vec{E}$$

→ so, find charge inside this GS: same integration as above, except upper limit is r, not R

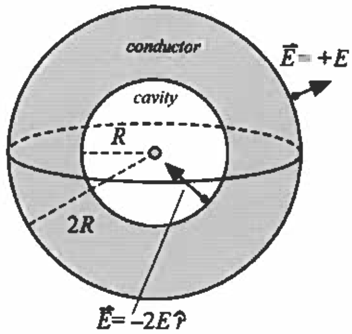
so $$Q_{inside} = 2\pi L \rho_0\left[\frac{s^2}{2} - \frac{s^3}{3R}\right]_0^r = \frac{\pi L \rho_0}{3}\left[3r^2 - 2\frac{r^3}{R}\right]$$

Thus: $E_{in}(r)\cdot 2\pi r L = \dfrac{Q_{in}}{\varepsilon_0} \rightarrow E_{in}(r) = \boxed{\dfrac{\rho_0}{6\varepsilon_0}\left[3r - 2\dfrac{r^2}{R}\right]}$

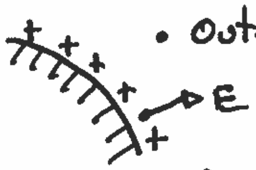
The following problem will be hand-graded. *Show all supporting work for this problem.*

[II] *(20 points)* A hollow, charged conducting sphere has an inner (cavity) radius R and an outer (surface) radius $2R$. An unknown point charge resides at the center of the cavity. Just outside the surface of the sphere (i.e. at $r = 2R$), the electric field is directed radially outward with a magnitude E. Just inside the cavity wall (i.e. at $r = R$), the electric field is directed radially inward with a magnitude $2E$.

Determine (i) the magnitude and sign of the charge that was placed within the cavity, and (ii) the magnitude and sign of the total charge placed on the sphere. In each case, express your anser in terms of R, E, and ε_0.



Method #1 Field next to conductor is related to charge on conductor by: $E = \frac{\eta_{surface}}{\varepsilon_0}$ or $\eta_{surface} = \varepsilon_0 E$



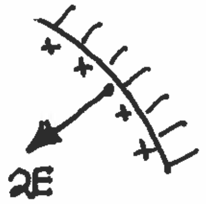
• Outer surface (2R): $\eta_{2R} = +\varepsilon_0 E$ so $Q_{2R} = 4\pi(2R)^2 \eta_{2R}$

$\boxed{Q_{2R} = +16\pi R^2 \varepsilon_0 E}$

• inner surface $\vec{E} = -2E$ (inward)

→ implies positive charges on surface (E points away from surface)

$\eta_R = +\varepsilon_0(2E)$ so $Q_R = 4\pi R^2 \eta_R = \boxed{+8\pi R^2 \varepsilon_0 E = Q_R}$



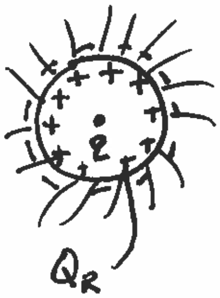
(ii) Total charge on sphere is thus $Q = Q_{2R} + Q_R$

$\boxed{Q = +24\pi R^2 \varepsilon_0 E}$

(i) to find point charge within cavity (at the center), use Gaussian surface barely larger than cavity, $r = R + \varepsilon$ (ε = "small")

$\Phi_{GS} = 0$ because $E \equiv 0$ inside conductor

→ GS contains zero net charge. Letting "q" denote charge at center



$Q_{inside} = 0 = q + Q_R$

so $q = -Q_R$

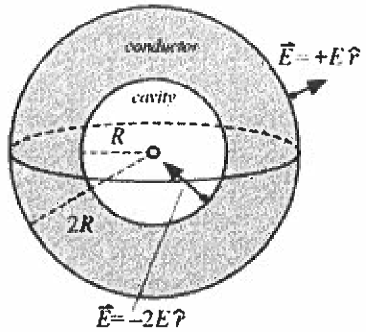
$\boxed{q = -8\pi R^2 \varepsilon_0 E}$

Form 2B

The following problem will be hand-graded. Show all supporting work for this problem.

[II] *(20 points)* A hollow, charged conducting sphere has an inner (cavity) radius R and an outer (surface) radius $2R$. An unknown point charge resides at the center of the cavity. Just outside the surface of the sphere (i.e. at $r = 2R$), the electric field is directed radially outward with a magnitude E. Just inside the cavity wall (i.e. at $r = R$), the electric field is directed radially inward with a magnitude $2E$.

Determine (i) the magnitude and sign of the charge that was placed within the cavity, and (ii) the magnitude and sign of the total charge placed on the sphere. In each case, express your anser in terms of R, E, and ε_0.



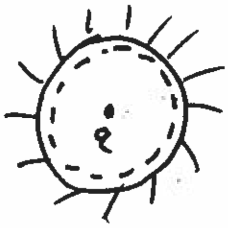
Method #2

(i) Apply Gauss's law using GS with radius barely smaller than R

→ charges on the conducting sphere are outside the GS, and do not contribute to field on our GS

→ Field is entirely due to central point charge q

(since $\vec{E}$ is radially inward, q must be negative



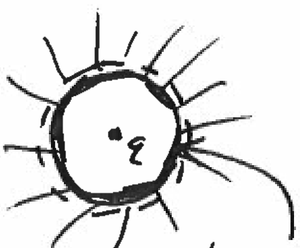
$$\vec{E}_{\text{at }R} = -2E\hat{r} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r} \quad \text{with } r \simeq R$$

$$q = -8\pi\varepsilon_0 r^2 E = \boxed{-8\pi\varepsilon_0 R^2 E}$$

(ii) Now let GS be barely larger than R

GS is within the conductor where $E \equiv 0$,

so GS contains zero charge:



$$Q_{\text{inside}} = q + Q_R \rightarrow Q_R = -q = -(-8\pi\varepsilon_0 R^2 E)$$

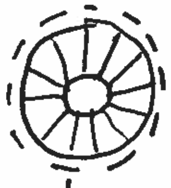
$$\boxed{Q_R = +8\pi\varepsilon_0 R^2 E}$$

let Q_R = charge on cavity surface

To find $Q_{\text{conductor}}$, use GS all the way outside

$$\Phi_{gs} = \frac{Q_{in}}{\varepsilon_0}$$

$$E \cdot 4\pi(2R)^2 = \frac{Q_{2R} + Q_R + q}{\varepsilon_0} \quad \text{but } Q_R + q = 0 \,!$$



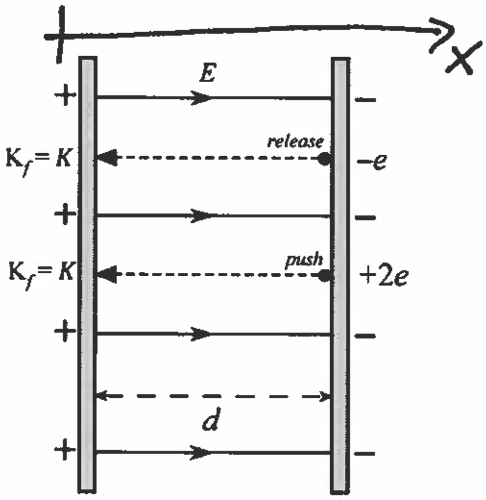
GS radius = 2R

Field on GS is $(+E\hat{r})$

$$Q_{2R} = \varepsilon_0 E\, 4\pi(4R^2) \qquad Q_{2R} = 16\pi R^2 \varepsilon_0 E$$

$$\text{so } Q_{\text{conductor}} = Q_R + Q_{2R} = \boxed{+24\pi R^2 \varepsilon_0 E}$$

The following problem will be hand-graded. *Show all supporting work for this problem.*



[III] *(20 points)* A capacitor has plate separarion d and a uniform internal field of magnitude E. An electron (charge $-e$) is released from the negative plate, and strikes the positive plate with kinetic energy K. Next, an alpha-particle (charge $+2e$) is *pushed* from the negative plate to the positive plate, starting from rest and ending with the same kinetic energy K as the electron.

(i) How much work was done by the electric field on the electron? How much work was done by the field on the alpha-particle?

(ii) How much work was done by the pushing agent on the alpha-particle?

For each part, express your answer in terms of d, E, and e.

(i) work by field is: $W_{elec} = \int \vec{F}_{elec} \cdot \vec{ds}$

since field is uniform, $\vec{E} = +E\hat{i}$ = constant, this becomes

$$W_{field} = \vec{F}_{elec} \cdot \vec{\Delta s} = (q\vec{E}) \cdot \vec{\Delta s} = (+q E\hat{i}) \cdot \vec{\Delta s}$$

in both cases, displacement is $\vec{\Delta s} = (-d\hat{i})$, given our coord system

electron: $q = (-e) \rightarrow W_e = (-e)(+E\hat{i}) \cdot (-d\hat{i}) = \boxed{+eEd}$

[pos work converts PE to KE, electron speeds up]

alpha: $q = (+2e) \rightarrow W_\alpha = (+2e)(+E\hat{i}) \cdot (-d\hat{i}) = \boxed{-2eEd}$

[neg work: field converts KE to PE]

(ii) First, note that electron = "free fall problem"

That conserves mechanical energy $\Delta E_{sys} = 0 = \Delta K + \Delta U$

$\rightarrow \Delta K = -\Delta U = -(-W_{field}) = -(-eEd) = +eEd$

since it started at rest, $\Delta K = K_f - K_i = K - 0$

$\rightarrow \boxed{K = +eEd}$ at impact

(this is important because it is also the final KE of alpha...)

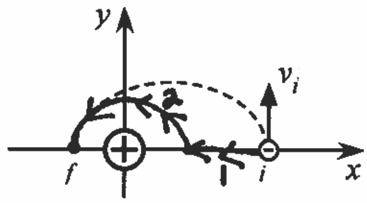
Pushing alpha does not conserve energy! $W_{ext} = W_{push} = \Delta E_{sys} \neq 0$

From above, $\Delta K_\alpha = \Delta K_{electron} = +eEd$

also, $\Delta U_\alpha = -W_\alpha = +2eEd$

$\Delta E_{sys} = \Delta K + \Delta U = 3eEd$ so $\boxed{W_{push} = +3eEd}$

The next two questions both involve the following situation:
In the figure at right, a positive source charge is held fixed at the origin, and a negative test charge is initially at location i, moving vertically with speed v_i. The test charge follows the dotted trajectory, reaching position f—at which point it is moving vertically *downward*. No external forces act during this process.



Question value 4 points

(1) What can you say about the work done by the electric field of the (positive) source charge, and the change in the electric potential experienced by the (negative) test charge, as the test charge moves from i to f?

42% correct

(a) The field has done zero work and the charge has moved to lower electric potential.
(b) The field has done negative work and the charge has moved to higher electric potential.
(c) The field has done negative work and the charge has moved to lower electric potential.
(d) The field has done positive work and the charge has moved to higher electric potential.
(e) The field has done positive work and the charge has moved to lower electric potential.

Work by field is path independent, so go straight in, then in circular arc (see figure)

on 1: Force and displacement are radially in: W = positive
on 2: path is tangential: W = zero

$\rightarrow$ $\boxed{\text{Field does positive work}}$

Positive point source: $V = \frac{K(+Q)}{r}$ = "hill"
moving closer to charge = "moving uphill"
= $\boxed{\text{higher electric potential}}$

Question value 4 points

(2) What can you say about the energy changes of the test charge, as it moves from i to f?

92% correct

(a) The charge has lost kinetic energy and lost potential energy.
(b) The kinetic and potential energies of the charge have remained unchanged.
(c) The charge has gained kinetic energy and lost potential energy.
(d) The charge has gained kinetic energy and gained potential energy.
(e) The charge has gained kinetic energy and lost potential energy.

From (1): Field has done positive work, so $\Delta U = -W_{field}$ is negative

$\boxed{\text{charge has lost potential energy}}$

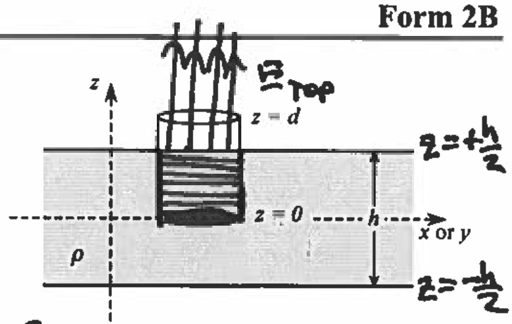
No external work was done, so system energy is conserved $E_{sys} = const$

PE loss $\rightarrow$ KE gain

$\boxed{\text{charge has gained kinetic energy}}$

Question value 8 points

(3) A thick insulating slab having thickness h and uniform volume charge density ρ lies in the xy-plane. It is seen in cross-section at right. The figure also displays a cylindrical Gaussian surface with one face at $z = 0$ (the midline of the slab) and the other outside the slab at a distance $z = d$ (where $d > h/2$). Determine the magnitude of the electric field on the top face of the Gaussian cylinder.



58% correct

(a) $E = \frac{\rho d}{\varepsilon_0}$

(b) $E = \frac{\rho h}{\varepsilon_0}$

(c) $E = 0$

(d) $E = \frac{\rho d}{2\varepsilon_0}$

(e) $\boxed{E = \frac{\rho h}{2\varepsilon_0}}$

By symmetry, $E \equiv 0$ at $z=0$
[equal charges above/below generate cancelling fields]

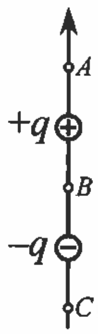
$\Rightarrow$ For GS shown $\Phi_{bottom} = 0$, $\Phi_{sides} = 0$, $\Phi_{Top} = E_{Top} A$

Meanwhile, charge enclosed by GS is
$Q_{in} = \rho V_{in} = \rho \cdot A \cdot \frac{h}{2}$

$\Rightarrow$ Gauss's Law gives
$E_{Top} \cdot A = \frac{\rho \cdot A \cdot \frac{h}{2}}{\varepsilon_0} \Rightarrow \boxed{E_{Top} = \frac{\rho h}{2\varepsilon_0}}$

Question value 8 points

(4) Two point charges having equal magnitudes but opposite signs are placed as shown at right. Compare the electric potentials at positions A, B, and C.



73% correct

(a) $V_A = V_B = V_C$

(b) $\boxed{V_A > V_B > V_C}$

(c) $V_C > V_B > V_A$

(d) $V_A = V_C > V_B$

(e) $V_B > V_A = V_C$

$V = \sum_i \frac{kQ_i}{r_i} = \frac{k(+Q)}{r_+} + \frac{k(-Q)}{r_-}$

A is closer to $+Q$ than $-Q$
$\Rightarrow$ $\boxed{V_A \text{ is positive}}$

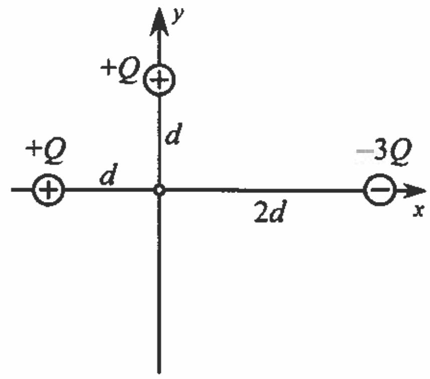
B is equidistant from $+Q$ and $-Q$
$\Rightarrow$ $\boxed{V_B \text{ is zero}}$

C is closer to $-Q$ than $+Q$
$\boxed{V_C \text{ is negative}}$

$\Rightarrow \boxed{V_A > V_B > V_C}$

Question value 8 points

(5) Charges $+Q$, $+Q$, and $-3Q$ are fixed on the coordinate axes as shown at right. How much work must be done by en external agent, to place a fourth charge $-2Q$ at the origin? Assume the charge begins and ends at rest.



55% correct

(a) $W_{ext} = +\frac{3kQ^2}{2d}$

(b) $W_{ext} = +\frac{kQ^2}{d}$

(c) $W_{ext} = +\frac{2kQ^2}{d}$

(d) $\boxed{W_{ext} = -\frac{kQ^2}{d}}$

(e) $W_{ext} = -\frac{3kQ^2}{2d}$

(f) $W_{ext} = -\frac{2kQ^2}{d}$

$\Delta K = 0$ by definition

Hence,

$W_{ext} = \Delta E_{sys} = \cancel{\Delta K}^{\,0} + \Delta U$

$W_{ext} = \Delta U = q\,\Delta V$

$= (-2Q)\left[V_f - \cancel{V_i}\right]$ $V \equiv 0$ at ∞

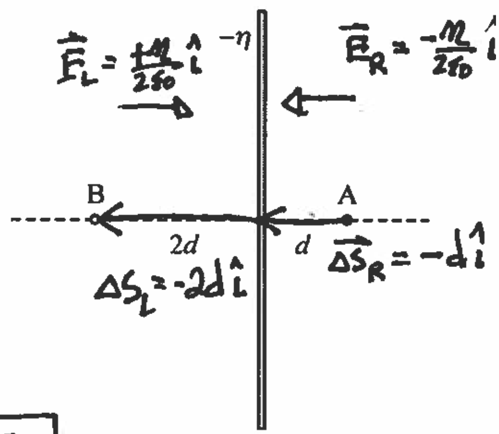
so $W_{ext} = (-2Q)\left[\frac{kQ}{d} + \frac{kQ}{d} + \frac{k(-3Q)}{2d}\right]$

$= (-2Q)\left[\frac{4kQ}{2d} - \frac{3kQ}{2d}\right] = (-2Q)\left[+\frac{kQ}{2d}\right]$

$\Rightarrow \boxed{W_{ext} = -\frac{kQ^2}{d}}$

Question value 8 points

(6) A very large, flat insulating sheet has uniform *negative* surface density, $-\eta$. (Here the symbol η is inherently *positive*.) Point A lies to the right of the sheet, at a distance d. Point B lies to the left of the sheet, at a distance $2d$. What is the potential difference $\Delta V_{A\to B} = V_B - V_A$?



32% correct

(a) $\boxed{\Delta V = +\frac{\eta d}{2\varepsilon_0}}$

(b) ~~$\Delta V = -\frac{\eta d}{\varepsilon_0}$~~

(c) $\Delta V = +\frac{\eta d}{\varepsilon_0}$

(d) ~~$\Delta V = -\frac{\eta d}{2\varepsilon_0}$~~

(e) $\Delta V = +\frac{3\eta d}{2\varepsilon_0}$

(f) ~~$\Delta V = -\frac{3\eta d}{2\varepsilon_0}$~~

- negative sheet = "valley"
- B is further from sheet than A

$\Rightarrow$ B is "further out of valley"

= <u>higher up</u>

$\boxed{\Delta V_{A\to B} \text{ MUST BE POSITIVE}}$

Moreover, in a uniform field, $\Delta V = -\vec{E}\cdot\vec{\Delta S}$

We must go in two stages: on the Right, on the Left

$\Delta V = \left(-\vec{E}_R\cdot\vec{\Delta S}_R\right) + \left(-\vec{E}_L\cdot\vec{\Delta S}_L\right)$

$= \left[-\left(-\frac{\eta}{2\varepsilon_0}\right)(-d)\right] + \left[-\left(+\frac{\eta}{2\varepsilon_0}\right)(-2d)\right] = \frac{-\eta d}{2\varepsilon_0} + \frac{2\eta d}{2\varepsilon_0}$

52% of class chose <u>negative</u> answers...

$\boxed{\Delta V = +\frac{\eta d}{2\varepsilon_0}}$