Printed Name	Nine-digit GT ID
Solutions	

signature

Fall 2021 **PHYS 2212 G** **Test 01**

Test Form: **1A**

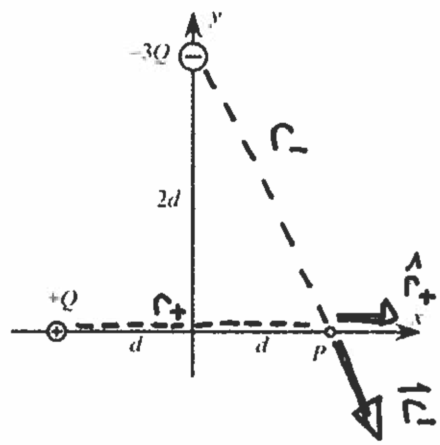
- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on yourtest, and fill the bubble for your answer **on this front page**.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Fill in bubbles for your Multiple Choice responses HERE
Mark answers answers darkly and neatly.
If you wish to change an answer, draw a clear "X" through the non-answer!

1. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ
2. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ
3. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ
4. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ
5. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ
6. ⓐ ⓑ ⓒ ⓓ ⓔ ⓕ

The following problem will be hand-graded. *Show all supporting work for this problem.*

[I] *(20 points)* In the figure at right, a source charge $+Q$ has been placed on the negative x-axis and a charge $-3Q$ has been placed on the positive y-axis. (The symbol "Q" denotes a *magnitude* and is inherently positive.) Determine the electric field at Point P, indicate, on the positive x-axis. Express the magnitude in terms of k, Q, and d. Express the direction as a numerical angle measured relative to a coordinate axis, to three-digit precision.

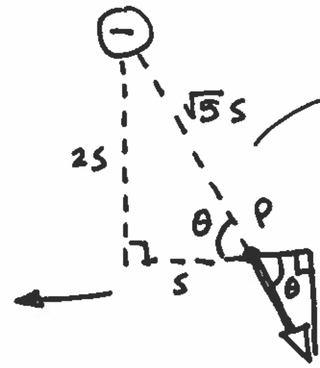


distances from sources to P are:

- $r_+ = 2d$
- $r_- = \sqrt{(2d)^2 + d^2} = \sqrt{5}\,d$

unit vectors at P, away from sources:

⊕ - - - - - - P ⟹ $\boxed{\hat{r}_+ = +\hat{\imath}}$



$\cos\theta = \frac{adj}{hyp} = \frac{s}{\sqrt{5}\,s} = \frac{1}{\sqrt{5}}$

$\sin\theta = \frac{opp}{hyp} = \frac{2s}{\sqrt{5}\,s} = \frac{2}{\sqrt{5}}$

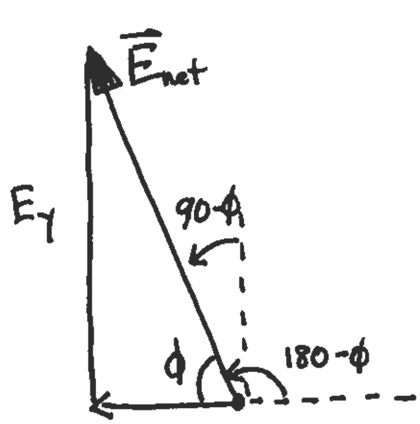
$\hat{r}_- = (+\cos\theta)\,\hat{\imath} + (-\sin\theta)\,\hat{\jmath}$

$\boxed{\hat{r}_- = \left(+\frac{1}{\sqrt{5}}\hat{\imath}\right) + \left(-\frac{2}{\sqrt{5}}\hat{\jmath}\right)}$

Net electric field at P is thus $\vec{E}_{net} = \vec{E}_+ + \vec{E}_- = \frac{k(+Q)}{r_+^2}\hat{r}_+ + \frac{k(-3Q)}{r_-^2}\hat{r}_-$

$$\Rightarrow \vec{E}_{net} = \frac{kQ}{4d^2}\hat{\imath} + \underbrace{\left[\frac{3kQ}{5d^2}\left(-\frac{1}{\sqrt{5}}\hat{\imath} + \frac{2}{\sqrt{5}}\hat{\jmath}\right)\right]}_{\text{note how this actually does point } \underline{\text{toward}} \ (-3Q)!}$$

$$= \frac{kQ}{d^2}\left[\left(\frac{1}{4} - \frac{3}{5\sqrt{5}}\right)\hat{\imath} + \left(\frac{6}{5\sqrt{5}}\right)\hat{\jmath}\right] = \frac{kQ}{d^2}\left[\underbrace{\left(-0.01833\,\hat{\imath} + 0.53666\,\hat{\jmath}\right)}_{\text{almost vertical, but a little to the left}}\right]$$



E_x

(exaggerated)

$$|\vec{E}| = \frac{kQ}{d^2}\sqrt{(0.01833)^2 + (0.53666)^2}$$

$$\boxed{\vec{E} = 0.5370\,\frac{kQ}{d^2}}$$

direction angle is $\phi = \tan^{-1}\left(\frac{|E_y|}{|E_x|}\right) = \boxed{88.0^\circ}$

so: either "88.0° above negative x-axis"

or "1.96° left of positive x-axis"

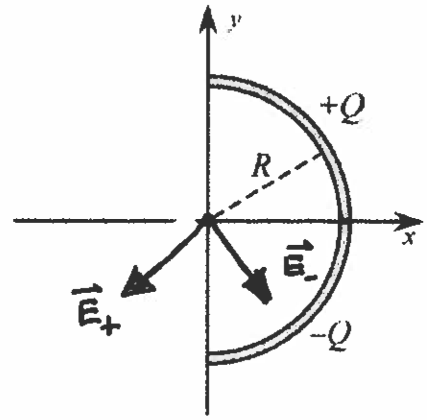
or "92.0° ccw from pos. x-axis"

(etc...)

The following problem will be hand-graded. *Show all supporting work for this problem.*

[II] *(20 points)* A semi-circular arc of radius R lies in the positive-x half-plane. Charge $+Q$ is uniformly distributed on the upper quadrant, while an equal but opposite charge $-Q$ is uniformly distributed on the lower quadrant.

Determine the electric field at the center of curvature of the arc. Express your answer as a cartesia component vector, in terms of k, Q, R, $\hat{\imath}$, and/or $\hat{\jmath}$.



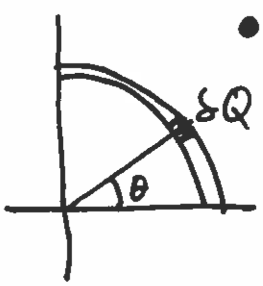
① By symmetry:

+ arc creates field $\vec{E}_+ = 45°$ down/left (away from midpoint of arc)

− arc creates field $\vec{E}_- = 45°$ down/right (toward midpoint of arc

$\Rightarrow$ $\vec{E}_{net}$ will be <u>straight</u> downward → $\vec{E}_{+x}$ and $\vec{E}_{-x}$ <u>cancel</u>, $\vec{E}_{+y}$ and $\vec{E}_{-y}$ are identical

So, look at <u>just</u> the upper quadrant: $\boxed{\vec{E}_{net} = 2\,\vec{E}_{+,y}}$ ↳ because $\vec{E}_{-y} = \vec{E}_{+y}$



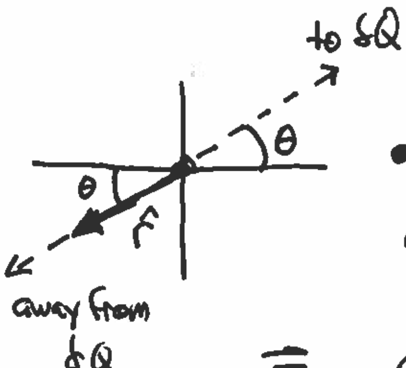
- On pos arc: $0 \leq \theta \leq \frac{\pi}{2}$

for small segment <u>at</u> θ, arc length is $ds = R\,d\theta$

charge on segment is $\delta Q = \lambda\, ds = \lambda R\, d\theta$,

where where $\lambda = \frac{Q_{arc}}{L_{arc}} = \frac{+Q}{\pi R/2} = \frac{2Q}{\pi R}$

$$\Rightarrow \delta Q = \left(\frac{2Q}{\pi R}\right)(R\,d\theta) = \frac{2}{\pi} Q\, d\theta$$



- at origin, unit vector <u>away</u> from δQ is $\hat{r} = -\cos\theta\,\hat{\imath} - \sin\theta\,\hat{\jmath}$

$$\text{so: } \delta\vec{E}_+ = \frac{k\,\delta Q}{R^2}\hat{r} = \frac{2kQ}{\pi R^2}\, d\theta \left(\underbrace{-\cos\theta\,\hat{\imath}}_{\text{from above, we can ignore this}} - \sin\theta\,\hat{\jmath}\right)$$

$$\vec{E}_{net} = 2\vec{E}_{+,y} = 2\int \delta\vec{E}_{+,y} = \frac{4kQ}{\pi R^2}\int_0^{\pi/2} (-\sin\theta\, d\theta)\,\hat{\jmath}$$

$$\left[\int \sin u\, du = -\cos u + C\right]$$

$$\vec{E}_{net} = \frac{4kQ}{\pi R^2}\Big[+\cos\theta\Big]_0^{\frac{\pi}{2}}\hat{\jmath}$$

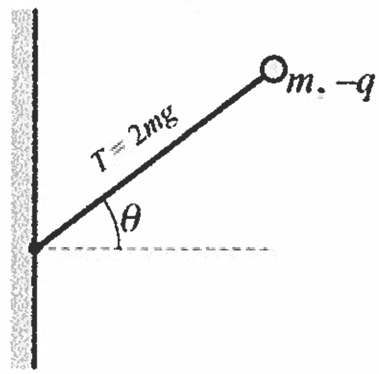
$$= \frac{4kQ}{\pi R^2}\Big[0 - (+1)\Big]\hat{\jmath}$$

$$\boxed{E_{net} = \frac{-4kQ}{\pi R^2}\hat{\jmath}}$$

The following problem will be hand-graded. Show all supporting work for this problem.

[III] *(20 points)* In the figure at right, a tiny insulating ball having mass m and negative charge, $-q$, is attached to a vertical wall by a string of length L. It lies in an uniform electric field $\vec{E}$ that causes the string to pull taut at an angle $\theta = 36.9°$ above the horizontal. A force meter measures the tension in the cord to be twice the weight of the ball (i.e. $T = 2mg$).

Determine the electric field (magnitude and direction) that causes the ball to "levitate" so. Express the magnitude in terms of the parameters m, q, L, and/or g, as needed. Express the direction as a numerical angle measured relative to the horizontal, to three-digit precision.



Ball is in equilibrium

$\Rightarrow$ F_{elec} must be comparable in magnitude to other forces in problem

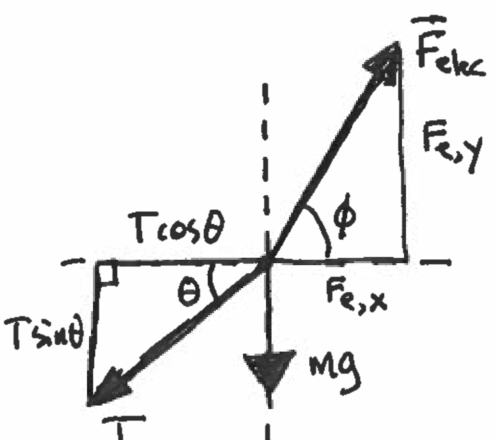
so $|\vec{F}_{elec}| \sim T$ and hence, $|\vec{F}_{elec}| \sim mg$

In other words, gravity cannot be neglected here!

Procedure: ① create free body diagram

② require $\Sigma \vec{F}_x = 0$ and $\Sigma \vec{F}_y = 0$

③ Figure out $\vec{F}_{elec}$, and hence $\vec{E} = \dfrac{\vec{F}_{elec}}{-q}$



$$\Sigma \vec{F}_x = 0 = \langle +F_{e,x} \rangle + \langle -T\cos\theta \rangle$$

$$F_{e,x} = T\cos\theta = (2mg)\left(\tfrac{4}{5}\right) = \boxed{\tfrac{8}{5} mg}$$

$$\Sigma \vec{F}_y = 0 = \langle +F_{e,y} \rangle + \langle -mg \rangle + \langle -T\sin\theta \rangle$$

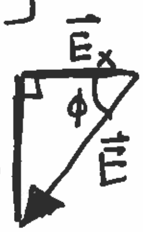
$$F_{e,y} = T\sin\theta + mg = (2mg)\left(\tfrac{3}{5}\right) + \left(\tfrac{5}{5} mg\right) = \boxed{\tfrac{11}{5} mg}$$

$$\left[\begin{array}{l} \cos\theta = \cos 36.9° = \tfrac{4}{5} \\ \sin\theta = \sin 36.9° = \tfrac{3}{4} \end{array}\right]$$

hence, $\vec{E} = \dfrac{\vec{F}_{elec}}{-q} = \boxed{\left(-\dfrac{8}{5}\dfrac{mg}{q}\hat{\imath}\right) + \left(-\dfrac{11}{5}\dfrac{mg}{q}\right)\hat{\jmath}}$

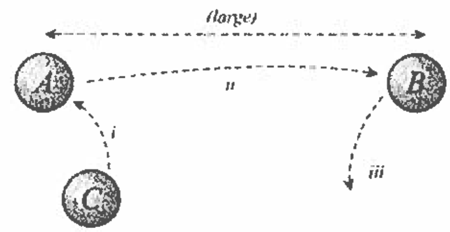
$|\vec{E}| = \dfrac{mg}{q}\sqrt{\left(\tfrac{8}{5}\right)^2 + \left(\tfrac{11}{5}\right)^2}$ $\quad \boxed{|\vec{E}| = \dfrac{\sqrt{185}}{5}\dfrac{mg}{q}}$

$\dfrac{\vec{F}_{elec}}{-q} = \vec{E}_y$



direction is

$\phi = \tan^{-1}\left(\dfrac{E_y}{E_x}\right) = \tan^{-1}\left(\dfrac{11}{8}\right) = \boxed{54.0°}$ below negative x-axis



The next two questions involve the following situation:
Two identical spheres A and B are separated by a distance D that is much greater than their radii. The spheres have identical charge Q, and the (repulsive) force between them is measured to have magnitude F_o. A third identical sphere C is uncharged. It is briefly touched to sphere A (i), then to sphere B (ii), then removed (iii).

Question value 4 points

(1) Assume that all three spheres are conductors. In terms of the original force F_o, what will be the new force between spheres A and B, after the actions with sphere C?

↳ charge transfers easily, on contact

(a) $\frac{1}{2}F_o$

(b) $\frac{3}{8}F_o$

(c) $\frac{3}{4}F_o$

(d) $\frac{1}{4}F_o$

(e) F_o

① Just before C touches A: A C ←-- ; Q zero

② Just after C touches A: A C ; Q/2 Q/2

③ Just before C touches B: --→ C B ; Q/2 Q

④ Just after C touches B: C B ; $\frac{3Q}{4}$ $\frac{3Q}{4}$

⇒ after C is removed, $Q_A = \frac{Q}{2}$, $Q_B = \frac{3Q}{4}$

Force is $k\frac{(\frac{Q}{2})(\frac{3Q}{4})}{D^2} = \frac{3}{8}\frac{kQ^2}{D^2} = \boxed{\frac{3}{8}F_0}$

Question value 4 points

(2) Assume instead that all three spheres are insulators. In terms of the original force F_o, what will be the new force between spheres A and B, after the actions with sphere C?

↳ charge does not transfer by simple contact

(a) $\frac{3}{8}F_o$

(b) $\frac{1}{4}F_o$

(c) $\frac{1}{2}F_o$

(d) F_o

(e) $\frac{3}{4}F_o$

① A C ←-- ; Q 0

② A C ; Q 0

③ --→ C B ; 0 Q

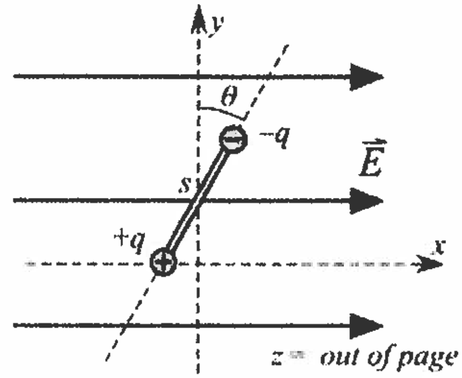
④ C B ; 0 Q

⇒ A and B both end up with the same charge Q

⇒ Force between A and B is still $\boxed{F_0}$

Question value 8 points

(3) In the figure at right, an electric dipole consists of two charges $\pm q$ separated by a fixed distance s. The dipole is placed in a uniform electric field of magnitude E that lies along the positive x-direction. What is the torque on the dipole when oriented at the angle shown relative to the y-axis?



$\boxed{\text{(a)} \quad +qsE\cos\theta\,\hat{k}}$

(b) $-qsE\cos\theta\,\hat{k}$

~~(c) $+qsE\sin\theta\,\hat{j}$~~

~~(d) $-qsE\cos\theta\,\hat{\imath}$~~

(e) $+qsE\sin\theta\,\hat{k}$

~~(f) $+qsE\sin\theta\,\hat{j}$~~

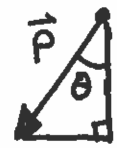
$\vec{\tau} = \vec{p} \times \vec{E}$

where $\vec{E} = +E\hat{\imath}$

and

$\vec{p} = (-p\sin\theta)\hat{\imath} + (-p\cos\theta)\hat{\jmath}$

$= qs[-\sin\theta\,\hat{\imath} - \cos\theta\,\hat{\jmath}]$



$\vec{P}_y = -p\cos\theta$

$\vec{P}_x = -p\sin\theta$

Note: $\vec{p}$ and $\vec{E}$ are in the xy-plane

$\Rightarrow \vec{\tau}$ must be $\perp$ to xy-plane

$\boxed{\vec{\tau} \text{ must be along z-axis}}$

$\vec{\tau} = qsE[-\sin\theta\,\hat{\imath} - \cos\theta\,\hat{\jmath}] \times (+\hat{\imath})$

$= qsE(-\cos\theta)\underbrace{(\hat{\jmath}\times\hat{\imath})}_{-\hat{k}}$ [recall $\hat{\imath}\times\hat{\imath} \equiv 0$]

so $\boxed{\vec{\tau} = +qsE\cos\theta\,\hat{k}}$

Question value 8 points

(4) A thin insulating disk of radius R has charge distributed *non-uniformly* on its surface, with a density given by the function $\eta(r) = A(2r - R)$, where A is a positive constant and $0 \le r \le R$. What is the total charge on the disk?

(a) $Q_{tot} = +\pi A R^3$

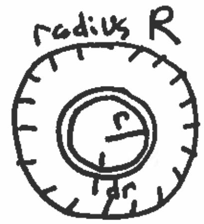
(b) $Q_{tot} = 0$

(c) $Q_{tot} = -\frac{\pi}{6} A R^4$

(d) $Q_{tot} = -\frac{\pi}{3} A R^3$

(e) $Q_{tot} = +\frac{\pi}{6} A R^4$

$\boxed{\text{(f)} \quad Q_{tot} = +\frac{\pi}{3} A R^3}$



consider thin ring somewhere on disk

- radius $= r$
- thickness $= dr$
- small area $\boxed{\delta A = 2\pi r \cdot dr}$ (circumference · radial thickness)

so, charge on this ring is

$\delta Q = \eta(r)\,\delta A = A(2r - R)\cdot 2\pi r\,dr$

Total charge on disk is $Q_{TOT} = \int_{disk} \delta Q = \int_{r=0}^{r=R} \eta\,\delta A$

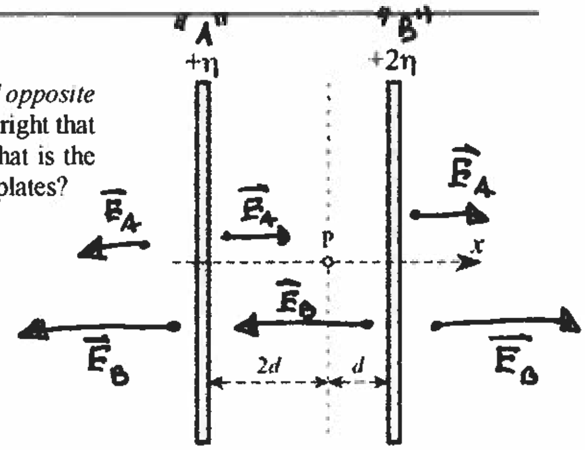
so $Q_{TOT} = 2\pi A \int_0^R (2r - R)\,r\,dr$

$= 2\pi A \int_0^R (2r^2 - Rr)\,dr = 2\pi A\left[\frac{2r^3}{3} - \frac{Rr^2}{2}\right]_0^R = 2\pi A\left[\frac{2R^3}{3} - \frac{R^3}{2}\right]$

$= 2\pi A\left[\frac{4}{6} - \frac{3}{6}\right]R^3 = \boxed{+\frac{\pi A R^3}{3}}$

Question value 8 points

(5) A typical capacitor consists of two conducting plates that have *equal and opposite* charge densities on their surfaces, $+\eta$ and $-\eta$. Consider the capacitor at right that has *unequal and like* charge densities on its surfaces, $+\eta$ and $+2\eta$. What is the electric field (magnitude and direction) at point P between the capacitor plates?



(a) $\vec{E} = +\frac{\eta}{2\varepsilon_0}\hat{\imath}$

(b) $\vec{E} = -\frac{3\eta}{2\varepsilon_0}\hat{\imath}$

(c) $\vec{E} = -\frac{\eta}{2\varepsilon_0}\hat{\imath}$

(d) $\vec{E} = +\frac{3\eta}{\varepsilon_0}\hat{\imath}$

(e) $\vec{E} = +\frac{3\eta}{2\varepsilon_0}\hat{\imath}$

(f) $\vec{E} = -\frac{3\eta}{\varepsilon_0}\hat{\imath}$

Both sheets are positive
→ individual fields point <u>away from</u> the source sheet

• <u>Between</u> sheets $\vec{E}_A$ is in <u>positive</u> direction with magnitude $\frac{\eta}{2\varepsilon_0}$ → $\boxed{\vec{E}_{A,in} = +\frac{\eta}{2\varepsilon_0}\hat{\imath}}$

• <u>Between</u> sheets, $\vec{E}_B$ is in <u>negative</u> direction with magnitude $\frac{(2\eta)}{2\varepsilon_0}$ $\boxed{\vec{E}_{B,in} = -\frac{2\eta}{2\varepsilon_0}\hat{\imath}}$

Net field is the sum of these two terms:

$$\vec{E}_{net,in} = \vec{E}_{A,in} + \vec{E}_{B,in} = \left(+\frac{\eta}{2\varepsilon_0}\hat{\imath}\right) + \left(-\frac{2\eta}{2\varepsilon_0}\hat{\imath}\right) = \boxed{-\frac{\eta}{2\varepsilon_0}\hat{\imath}}$$

Question value 8 points

(6) A silk cloth is rubbed against a plastic rod, charging the rod. The rod is then held near conducting sphere A, while that sphere is in contact with conducting sphere B. The spheres are separated while the rod is held near A. The rod is then removed. What interactions (if any) will the silk cloth have with sphere A and with sphere B?

(a) The cloth will be attracted by both A and B.

(b) The cloth will be repelled by both A and B.

(c) The cloth will be repelled by A and attracted by B.

(d) The cloth will feel no interaction with either A or B.

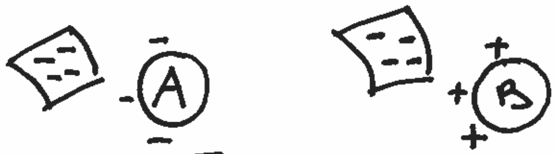
(e) The cloth will be attracted by A and repelled by B.

Assume rubbing makes rod positive
→ cloth must be negative:

Hold <u>rod</u> near spheres:

spheres polarize while in contact

separate spheres, interact with cloth



$\boxed{\text{repel A}}$ $\boxed{\text{attract B}}$