Solutions

Printed Name

Nine-digit GT ID

signature

Spring 2021 **PHYS 2212 G** **Test 04**

Test Form:

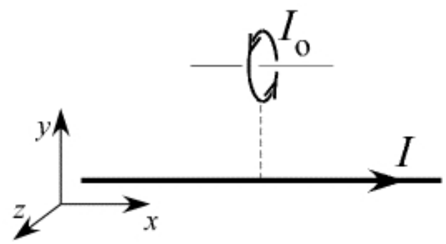
4A

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on your test, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- A standard formula sheet is provided as the cover page for this test. Please remove it from the test before you submit it to the proctor.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Question value 8 points

(01) A small circular wire loop carrying current I_o is placed near a very long straight wire, in the orientation shown at right. The long wire carries a rightward current I. If the loop is released and allowed to move freely, how will it move? You may assume that the size of the loop is small in comparison to the distance from loop to wire.

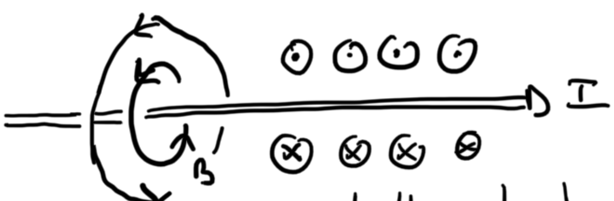
$\vec{B} \approx$ uniform, at loop



(a) The loop will move in a straight line, in the positive y-direction.
(b) The loop will rotate around the negative y-axis.
(c) The loop will rotate around the negative z-axis.
(d) The loop will not move.
(e) The loop will rotate around the negative x-axis.
(f) The loop will move in a straight line, in the negative y-direction.

Loop:

$\vec{\mu} = (+\mu\hat{i})$

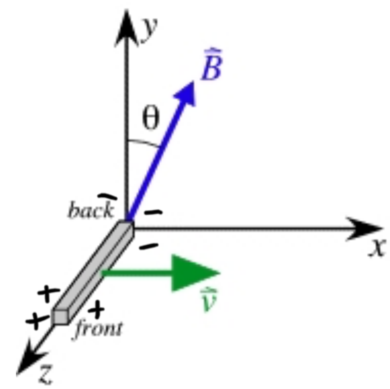


torque on loop is $\vec{\tau} = \vec{\mu} \times \vec{B}$
$= \mu B(\hat{i} \times \hat{k})$
$\boxed{= \mu B(-\hat{j})}$

straight wire: at all points above wire, $\vec{B}$ = out of page $= (+B\hat{k})$

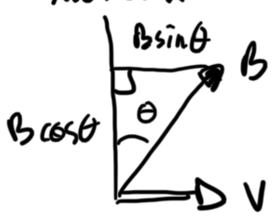
Question value 8 points

(02) The figure below shows an end-on view of a bar of length L that lies along the z-axis. It is pulled at constant speed v in the positive x-direction through a uniform magnetic field of magnitude B, that lies in the xy-plane oriented at an angle θ relative to the y-axis. What is the induced emf across the length of the bar?



(a) $\mathcal{E} = vBL\cos\theta$ with the back of the bar being at high potential.
(b) $\mathcal{E} = vBL\cos\theta$ with the front of the bar being at high potential.
(c) $\mathcal{E} = vBL\sin\theta$ with the back of the bar being at high potential.
(d) $\mathcal{E} = vBL$ with the front of the bar being at high potential.
(e) $\mathcal{E} = vBL\sin\theta$ with the front of the bar being at high potential.
(f) $\mathcal{E} = vBL$ with the back of the bar being at high potential.

Motional emf: $\mathcal{E} = |\vec{v} \times \vec{B}|L = vB_\perp L$
where $B_\perp$ = component of $\vec{B} \perp$ to $\vec{v}$
$= B\cos\theta$



so $\boxed{\mathcal{E}_{ind} = vBL\cos\theta}$

which end is at high potential?
→ Mag force on mobile electrons $\vec{F} = (-e)\vec{v} \times \vec{B}$
= into page

⇒ neg charges accumulate at back
⇒ pos charges accumulate at front

$\boxed{\text{Front is at high potential}}$

Question value 8 points

(03) A long straight wire having radius R carries a non-uniform current density, given by the expression $J(r) = A \cdot r^2$ where A is a positive constant and r is the radial distance from the center of the wire. What is the magnetic field strength at the surface of the wire?

(a) $B = \frac{\mu_0 A R^3}{2}$

(b) $B = \frac{\mu_0 A R^4}{3}$

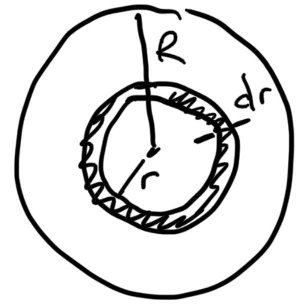
(c) $B = \frac{\mu_0 A R^4}{2}$

(d) $B = \frac{\mu_0 A R^3}{4}$

(e) $B = \mu_0 A R^3$

(f) $B = \frac{\mu_0 A R^4}{4}$

cross-sectional view of wire:
— consider thin ring of radius r and thickness dr



Area of ring is $dA = 2\pi r\,dr$
current through ring is
$dI = J(r)\,dA$
$= Ar^2 \cdot 2\pi r\,dr$

$\Rightarrow$ Total current through wire is $I = \int_{r=0}^{r=R} J(r)\,dA$
$= 2\pi A \int_0^R r^3\,dr$

$\boxed{I = 2\pi A \frac{R^4}{4}}$

Ampere's Law for loop of radius $r = R$:
$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{Through}$
$B \cdot \cancel{2\pi} R = \mu_0 \frac{\cancel{2\pi} A R^4}{4} \Rightarrow \boxed{B = \frac{\mu_0 A R^3}{4}}$

Question value 8 points

(04) You have designed a 4.0 GHz microprocessor "clock" based on a simple LC oscillator circuit. Your design specs call for an inductance $$L$$ = 0.25 nH, but a careless co-worker accidentally transcribes the value as "0.52 nH" when sending the design off to the manufacturer. What will be the actual frequency of thew manufactured processor clock?

~~(a) 8.3 GHz~~
(b) 3.3 GHz
(c) 2.8 GHz
~~(d) 7.0 GHz~~
(e) 1.9 GHz
~~(f) 5.8 GHz~~

angular frequency of oscillator: $\omega = \frac{1}{\sqrt{LC}}$

$\Rightarrow$ if L is too large, ω will end up being too small
$\Rightarrow$ actual frequency is < 4.0 GHz
(Half of possible choices can be eliminated)

Frequency in Hz is $f = \frac{\omega}{2\pi}$

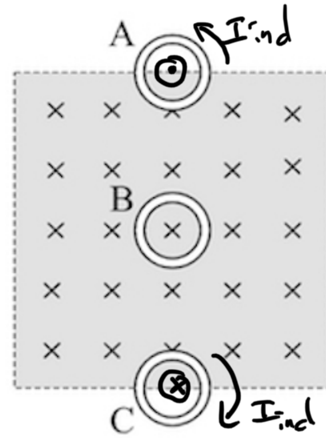
compare design frequency f_0 to actual frequency f

$\frac{f}{f_0} = \frac{\omega/2\pi}{\omega_0/2\pi} = \frac{\omega}{\omega_0} = \frac{1/\sqrt{LC}}{1/\sqrt{L_0 C}} = \sqrt{\frac{L_0}{L}} = \sqrt{\frac{0.25H}{0.52H}} = 0.693$

(same C in both cases)

so $\boxed{f = 0.693 f_0 = 2.78\text{ GHz}}$

The next two questions involve the following situation:
A copper ring is dropped vertically through a region containing a uniform magnetic field directed into the page. (The field is ZERO outside the shaded region.) The ring enters the region at A and departs at C.



Question value 4 points

(5.1) What is the direction of the induced current in the ring at positions A, B, and C?

- (a) I_A is counter-clockwise, I_B is ~~clockwise~~, I_C is counter-clockwise.
- (b) I_A is clockwise, I_B is ~~counter-clockwise~~, I_C is clockwise.
- (c) I_A is clockwise, I_B is zero, I_C is counter-clockwise.
- (d) I_A is clockwise, I_B is ~~clockwise~~, I_C is clockwise.
- (e) I_A is counter-clockwise, I_B is zero, I_C is clockwise.

Top: flux into page, through loop, is increasing → create flux out of page
→ induce $\boxed{\text{CCW current}}$

middle: flux = constant: $\boxed{\text{no induced current}}$

bottom: flux into page is decreasing → create flux into page
→ induce $\boxed{\text{CW current}}$

Question value 4 points

(5.2) What is the direction of the magnetic force on the ring at positions A, B, and C?

- (a) $\vec{F}_A$ is upward, $\vec{F}_B$ is zero, $\vec{F}_C$ is upward.
- (b) $\vec{F}_A$ is upward, $\vec{F}_B$ is ~~downward~~, $\vec{F}_C$ is upward.
- (c) $\vec{F}_A$ is downward, $\vec{F}_B$ is zero, $\vec{F}_C$ is downward.
- (d) $\vec{F}_A$ is rightward, $\vec{F}_B$ is zero, $\vec{F}_C$ is leftward.
- (e) $\vec{F}_A$ is downward, $\vec{F}_B$ is ~~upward~~, $\vec{F}_C$ is downward.

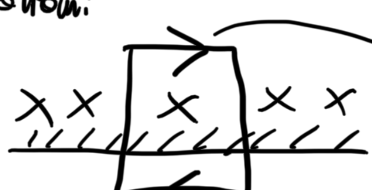
Treat loop as rectangular

At top:

current inside field is $\boxed{\text{to the right}}$
Field is into page ⇒ $\vec{F} = I\vec{L} \times \vec{B} = \boxed{\text{UP}}$

Middle: no current → $\boxed{\text{no magnetic force}}$

At bottom:



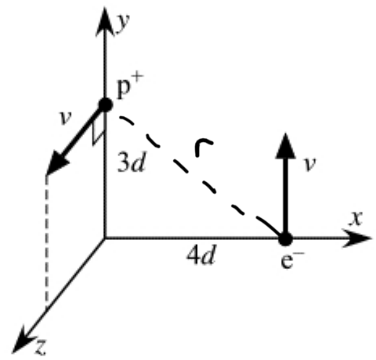
Current inside field is $\boxed{\text{to the right}}$
Field is into page
⇒ $\vec{F} = I\vec{L} \times \vec{B} = \boxed{\text{UP}}$

outside field: no force

The following problem will be hand-graded. <u>Show all supporting work for this problem</u>.

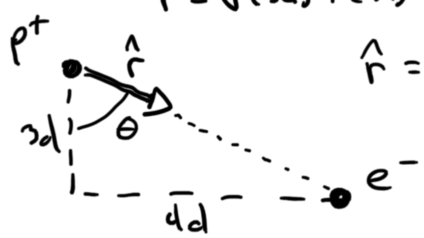
(6) *(20 points)* At a particular moment in time, a proton is located on the positive y-axis at a distance 3*d* from the origin, and is moving with speed v in the positive z-direction. At the same moment, an electron is located on the positive x-axis at a distance 4*d* from the origin, moving in the positive y-direction with the same speed *v*.

Determine the magnetic force on the electron. Be sure to specify magnitude and direction.



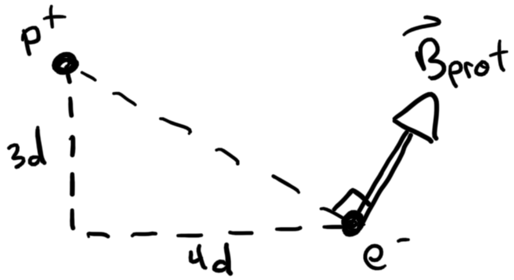
Field due to proton at location of electron

$$r = \sqrt{(3d)^2 + (4d)^2} = 5d$$



$$\hat{r} = +\sin\theta\,\hat{i} - \cos\theta\,\hat{j} = +\frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}$$

so: $$\vec{B}_{prot} = \frac{\mu_0}{4\pi}\,\frac{(+ev\hat{k}) \times \left(+\frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}\right)}{(5d)^2}$$



$$= \frac{\mu_0 ev}{100\pi d^2}\left[\frac{4}{5}(\hat{k}\times\hat{i}) - \frac{3}{5}(\hat{k}\times\hat{j})\right]$$

$$= \frac{\mu_0 ev}{100\pi d^2}\left[\frac{4}{5}\hat{j} + \frac{3}{5}\hat{i}\right]$$

Then: Force on electron is $\vec{F} = q\vec{v}\times\vec{B}$

$$\Rightarrow \vec{F} = (-ev\hat{j}) \times \frac{\mu_0 ev}{100\pi d^2}\left[\frac{3}{5}\hat{i} \times \frac{4}{5}\hat{j}\right]$$

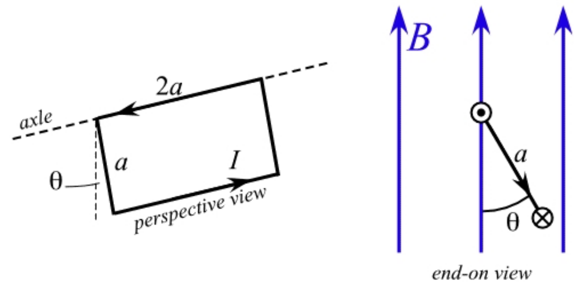
$$= \frac{\mu_0 e^2 v^2}{100\pi d^2}\left[-\frac{3}{5}\underbrace{(\hat{j}\times\hat{i})}_{-\hat{k}} + \frac{4}{5}\xcancel{(\hat{j}\times\hat{j})}^{0}\right]$$

so $$\boxed{\vec{F} = \underbrace{\frac{3\mu_0 e^2 v^2}{500\pi d^2}}_{\text{magnitude}}\ \underbrace{(+\hat{k})}_{\text{direction} = \text{"out of page"}}}$$

The following problem will be hand-graded. Show all supporting work for this problem.

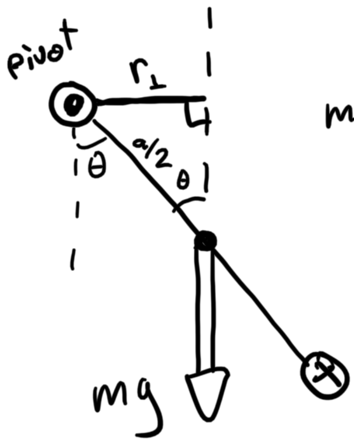
(7) *(20 points)* A rectangular loop of wire of dimensions $a \times 2a$ hangs vertically in Earth's gravitational field, from a horizontal axle along one of the long sides of the loop. The loop has a total mass *m* and carries a steady current *I*. When placed in a vertical magnetic field, the loop is observed to tilt upwards, and hang at a fixed angle θ from true vertical.

Determine the magnitude of the magnetic field.



① Note that when tilted ccw, gravity provides a clockwise torque ⊗

⇒ gravity acts at center of mass = geometric center of loop



moment arm for grav. torque is

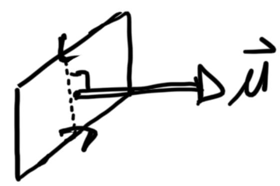
$r_\perp = \frac{a}{2}\sin\theta$

so, if "cw = negative", $\boxed{\vec{\tau}_g = \left(-\frac{a}{2}\sin\theta\right) mg}$

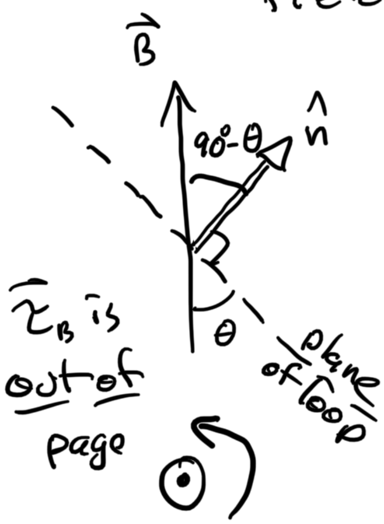
(or $\vec{\tau}_g$ = "into page")

② Loop is a magnetic dipole $\vec{\mu} = I A\hat{n} = I(a)(2a)\hat{n}$

where $\hat{n}$ = RH normal to loop



Torque on dipole in magnetic field is $\vec{\tau}_B = \vec{\mu} \times \vec{B}$



$= |\vec{\mu}||\vec{B}| \sin(\text{angle between vectors})$

$= (2Ia^2)(B)\sin(90° - \theta)$

but $\sin(90 - \theta) = \cos\theta$

⇒ this is a vector out of page, causing ccw rotation

③ Torques must balance

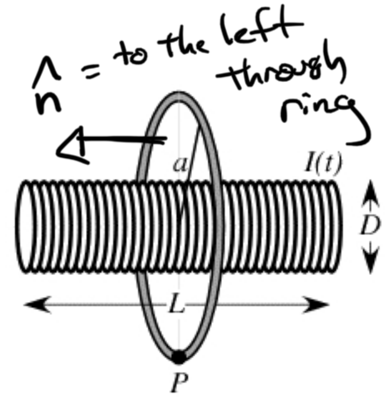
$\Sigma\tau = 0 = (+2Ia^2 B\cos\theta) + \left(-\frac{a}{2} mg \sin\theta\right)$

$$\boxed{B = \frac{mg}{4Ia}\tan\theta}$$

The following problem will be hand-graded. <u>Show all supporting work for this problem</u>.

(08) *(20 points)* A solenoid of diameter D and length L has a total of N windings. A copper ring of radius $a > D/2$ is aligned coaxially with the solenoid. A time-dependent current flows through the solenoid, given by the expression $I(t) = A - Bt^2$, where A and B are positive constants. The solenoid is would such that a positive value of $I(t)$ produces a leftward magnetic field.

Determine the induced electric field (magnitude AND direction) in the copper ring at the labelled point P, at the exact moment the current in the solenoid is zero.



Apply Faradays Law: $\mathcal{E}_{ind} = -\frac{d\Phi}{dt}$

$$\oint \vec{E}_{ind} \cdot d\vec{s} = -\frac{d}{dt}\left[\int \vec{B} \cdot d\vec{A}\right]$$

① Field inside solenoid $B = \mu_0 \frac{N}{L} I$ (and $B \equiv 0$ outside)

② Flux through copper ring

$$\int_{ring} \vec{B} \cdot d\vec{A} = \int_{inside} \vec{B} \cdot d\vec{A} + \cancel{\int_{outside} \vec{B} \cdot d\vec{A}} \rightarrow \text{zero}$$

$$= \left(\mu_0 \frac{N}{L} I\right)\left(\frac{\pi D^2}{4}\right)$$

(choose $\hat{n}$ = to the left, so that flux = pos. when $I(t)$ = pos.)

③ Meanwhile, for loop of radius a

$$\oint \vec{E} \cdot d\vec{s} = E_{ind} \cdot 2\pi a$$

so: $$E_{ind} \cdot 2\pi a = -\frac{\mu_0 N \pi D^2}{4L}\frac{dI}{dt} = -\frac{\mu_0 N \pi D^2}{4L}\left[-2Bt\right]$$

$$E_{ind} = +\frac{\mu_0 N D^2}{4La}(Bt)$$

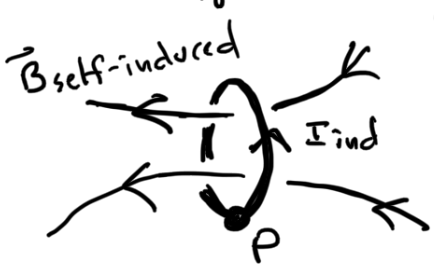
evaluate when $I = A - Bt^2 = 0 \rightarrow t = \sqrt{A/B}$

$$\boxed{E_{ind} = +\frac{\mu_0 N D^2}{4La}\sqrt{AB}}$$

direction: at moment current drops to zero, we are changing from "flux to left" to "flux to right"

$\Rightarrow$ oppose change by trying to make "flux to left"

induced current in ring is "ccw" in perspective view



$$\boxed{\vec{E}_{ind} = \text{out of page, at P}}$$