Solotions

Printed Name

Nine-digit GT ID

signature

Spring 2021 **PHYS 2212 G** **Test 03**

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.

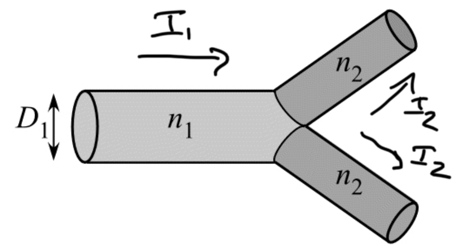
Test Form:

3A

- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on your test, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- A standard formula sheet is provided as the cover page for this test. Please remove it from the test before you submit it to the proctor.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Question value 8 points

(01) A copper wire has n_1 mobile electrons per cubic millimeter and diameter D. At a junction point, the wire is spliced to two identical silver wires having $n_2 = 0.68\, n_1$ mobile electrons per cubic millimeter. For what diameter of the aluminum wires will the drift speed of electrons be the same on both sides of the splice?



(a) $D_2 = 1.17\, D_1$

(b) $D_2 = 0.86\, D_1$

(c) $D_2 = 0.74\, D_1$

(d) $D_2 = 1.21\, D_1$

(e) $D_2 = 0.54\, D_1$

(f) $D_2 = 1.85\, D_1$

$I_{in} = I_{out}$

$I_1 = 2 I_2$

$e n_1 A_1 v_1 = 2\left[e n_2 A_2 v_2\right]$

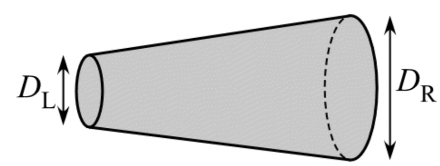
if $v_1 = v_2$, this reduces to $n_1 A_1 = 2 n_2 A_2$

or, $n_1 \dfrac{\pi D_1^2}{4} = 2\, \dfrac{n_2 \pi D_2^2}{4} \Rightarrow D_2^2 = D_1^2 \left(\dfrac{n_1}{2n_2}\right)$

$D_2 = \sqrt{\dfrac{n_1}{2n_2}}\, D_1 = \sqrt{0.735}\, D_1 = \boxed{0.857 D_1}$

Question value 8 points

(02) An aluminum wire has a varying diameter, increasing uniformly from diameter D on the left to diameter $2D$ on the right. A current I_L flows into the wire on the left. The magnitude of the electric field in the wire at that point is E_L. What is the current and the electric field magnitude at the right end of the wire?



(a) $\cancel{I_R = I_L/2}$ and $E_R = E_L/2$

(b) $\cancel{I_R = I_L/4}$ and $E_R = E_L/4$

(c) $I_R = I_L$ and $E_R = E_L/4$

(d) $I_R = I_L$ and $E_R = E_L/2$

(e) $\cancel{I_R = I_L/2}$ and $E_R = E_L/4$

(f) $I_R = I_L$ and $E_R = E_L$

① Same current flows everywhere in wire

$I_R = I_L$

② Current is related to field through current density!

$I = JA = (\sigma E) A$

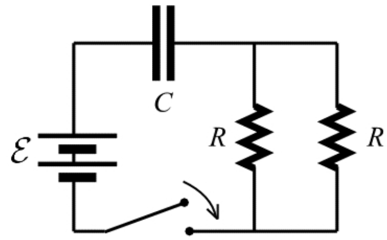
So, $\sigma E_R \dfrac{\pi D_R^2}{4} = \sigma E_L \dfrac{\pi D_L^2}{4}$

$E_R = \left(\dfrac{D_L}{D_R}\right)^2 E_L = \left(\dfrac{1}{2}\right)^2 E_L$

$\boxed{E_R = \frac{1}{4} E_L}$

Question value 8 points

(03) In the circuit below, the capacitor is initially uncharged. After the switch is closed, how much time must elapse for the capacitor to reach 2/3 of its final charge value?



resistors are <u>in parallel</u>

(a) 0.81 RC

(b) 0.33 RC

(c) 1.11 RC

(d) $\boxed{0.55\ RC}$

(e) 0.67 RC

(f) 0.20 RC

Time constant for equivalent circuit is

$$\tau = \frac{R}{2}C = \frac{RC}{2}$$

charge builds up, so write

$$q(t) = Q_f\left[1 - e^{-t/\tau}\right]$$

require $q = \frac{2}{3}Q_f$:

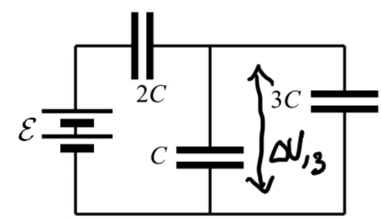
$$\frac{2}{3}Q_f = Q_f\left[1 - e^{-t/\tau}\right]$$

$$\Rightarrow e^{-t/\tau} = \frac{1}{3} \Rightarrow e^{+t/\tau} = 3 \Rightarrow t = \tau \ln 3 = \frac{RC}{2}(1.098)$$

$$\boxed{t = 0.549\,RC}$$

Question value 8 points

(04) Three capacitors are arranged in the network shown below. They are attached to an emf $\mathcal{E}$ and allowed to reach equilibrium. Rank, from greatest to least, the energies stored by each of the three capacitors.



(a) $\cancel{U_3 = U_1} > U_2$

(b) $\boxed{U_2 > U_3 > U_1}$

(c) $U_3 > U_2 > U_1$

(d) $U_2 > \cancel{U_3 = U_1}$

(e) $U_2 = \cancel{U_3 = U_1}$

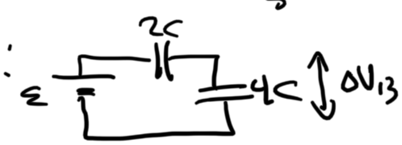
1 C and 3C are <u>in</u> parallel

$\Rightarrow$ same ΔV_{13}

so $U_3 = \frac{1}{2}(3C)\Delta V_{13}^2 = 3\left(\frac{1}{2}C\Delta V_{13}^2\right)$

$U_3 = 3U_1$ so $\boxed{U_3 > U_1}$

2 Equivalent circuit:



<u>In series</u>,

so <u>same</u> Q:

$(2C)\Delta V_2 = Q = (4C)\Delta V_{13} \rightarrow \Delta V_2 = 2\Delta V_{13}$

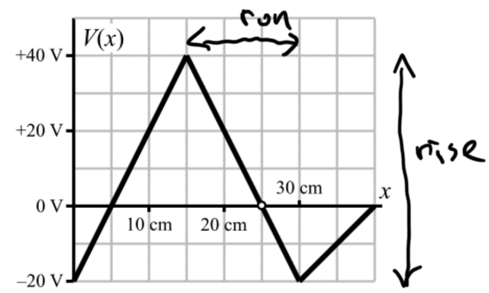
so $U_2 = \frac{1}{2}(2C)(2\Delta V_{13})^2 = 8\left(\frac{1}{2}C\Delta V_{13}^2\right) = 8U_1$

$\Rightarrow (U_2 = 8U_1) > (U_3 = 3U_1)$

so $U_2 > U_3 > U_1$

Question value 4 points

(5.1) In the graph at right, the electric potential is plotted as a function of position along the x-axis.



(a) E = 400 N/C in the positive x-direction

(b) E = 200 N/C in the negative x-direction

(c) E = 200 N/C in the positive x-direction

(d) E = 0 N/C

(e) E = 400 N/C in the negative x-direction

$\vec{E} = -\frac{dV}{dx} = -$(slope of graph, at $x = 25\text{cm}$)

→ linear plot: slope $= \frac{\text{rise}}{\text{run}} = \frac{-60\text{V}}{15\text{cm}} = -\frac{60\text{V}}{0.15\text{m}}$

$= -400\text{ V/m}$

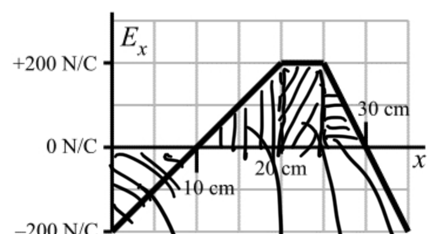
$= -400\text{ N/C}$

so, $\vec{E} = -(-400\text{ N/C}) = +400\text{ N/C}$

$\boxed{\vec{E} = 400\text{ N/C in positive } x\text{-direction}}$

Question value 4 points

(5.2) In the graph at right, the x-component of the electric field is plotted as a function of position along the x-axis. The y- and z-components of the field are identically zero. If the electric potential at the origin is V = +20 volts, what is the electric potential at x = 30 cm?



(a) V = +20 volts

(b) V = –15 volts

(c) V = +35 volts

(d) V = 0 volts

(e) V = +5 volts

(f) V = –20 volts

$\Delta V = -\int_i^f \vec{E}\cdot d\vec{s}$

$= -$[area under curve]

$\underbrace{-2 \quad +2 \quad +2 \quad +1}_{\text{Total area under curve is } +3 \text{ squares}}$

Note: 1 square has area = height × width

$= (100\text{ N/C})(5\text{ cm}) = (100\text{ V/m})(0.05\text{ m}) = 5\text{ V}$

so $\Delta V = -[\text{area under curve}] = -[(5\text{ V/square})(+3\text{ squares})] = -15\text{ V}$

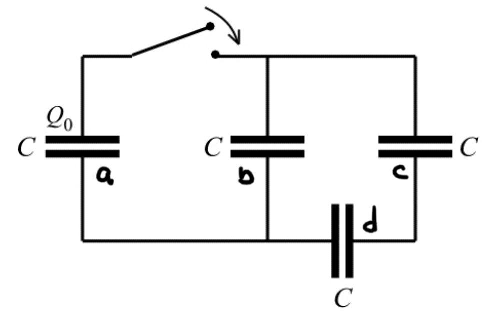
$\Rightarrow V_f = V_i + \Delta V = (+20\text{V}) + (-15\text{V}) = \boxed{+5\text{V}}$

The following problem will be hand-graded. Show all supporting work for this problem.

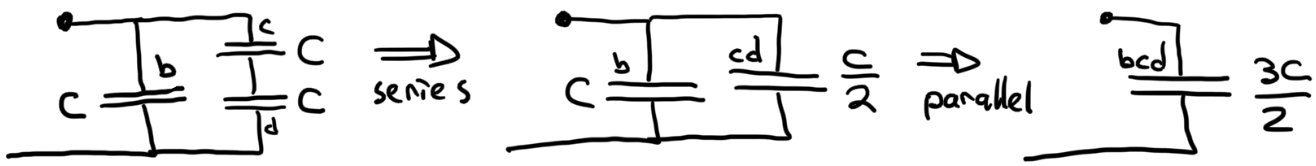
(06) *(20 points)* In the figure at right, capacitor C is initially charged up to store a total charge Q_0. The other capacitors in the network, having the indicated multiples of capacitance C, are initially uncharged. The switch is then closed and the network is allowed to reach equilibrium.

Find the final charge stored on the capacitor at bottom right. Express your answer as a fraction of Q_0.

capacitor "d"

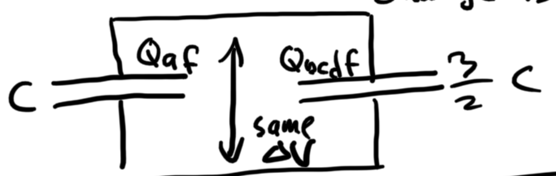


① Simplify right side of network, with switch open



② Close switch: charge flows until equilibrium is reached

charge is conserved — same potential across each side



① $Q_{af} + Q_{bcdf} = Q_0$

② $\Delta V_{af} = \Delta V_{bcdf} \rightarrow \dfrac{Q_{af}}{C} = \dfrac{Q_{bcdf}}{3C/2}$

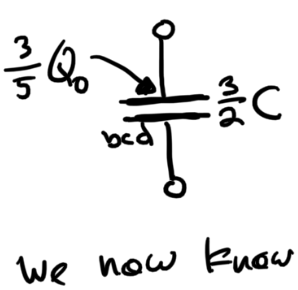
We now have two equations in two unknowns: $Q_{af} = \frac{2}{3} Q_{bcdf}$

$Q_{af} + Q_{bcdf} = Q_0$

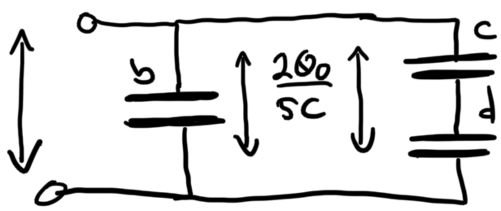
$\left(\frac{2}{3} Q_{bcdf}\right) + Q_{bcdf} = Q_0$

$\frac{5}{3} Q_{bcdf} = Q_0 \Rightarrow \boxed{Q_{bcdf} = \frac{3}{5} Q_0}$

④ Now, reconstruct right side bcd → b+c+d

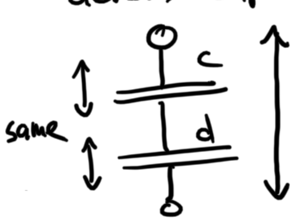


$\Delta V = \dfrac{\frac{3}{5} Q_0}{\frac{3}{2} C} = \dfrac{2}{5} \dfrac{Q_0}{C}$



Parallel: same ΔV as equivalent

We now know the ΔV across caps c and d:



$\dfrac{2Q_0}{5C} = \Delta V_{cd} \Rightarrow$ c and d are:
- identical
- in series

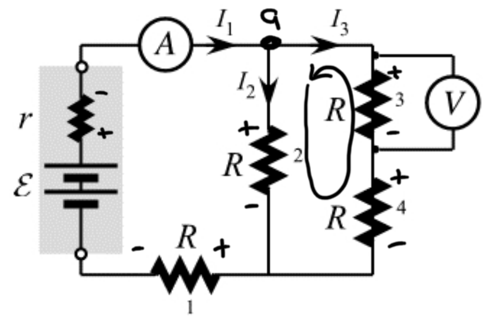
$\Delta V_d = \frac{1}{2} \Delta V_{cd} = \frac{1}{2}\left(\frac{2Q_0}{5C}\right) = \frac{Q_0}{5C}$

so $Q_d = C\Delta V = \boxed{\dfrac{Q_0}{5}}$

The following problem will be hand-graded. Show all supporting work for this problem.

(07) *(20 points)* A real battery having known emf $\mathcal{E}$ and unknown internal resistance r is hooked up to the network of four unknown but identical resistances R shown at right. An ammeter records the current flowing out of the batter as I_p. A voltmeter place across resistor #3 measures a voltage drop $\Delta V = -\mathcal{E}/6$.

Determine the value of the external resistances R and the internal resistance r. In each case, express your answer in terms of $\mathcal{E}$ and I_p.



① junction point a: $\boxed{I_1 = I_2 + I_3}$

② loop rule for right loop (resistors 234): $-I_2 R + I_3 R + I_3 R = 0$

$\boxed{I_2 = 2I_3}$

combining these equations:

$I_1 = 2I_3 + I_3 \rightarrow \boxed{I_3 = \frac{I_1}{3}}$

which also tells us that $\boxed{I_2 = \frac{2}{3} I_1}$

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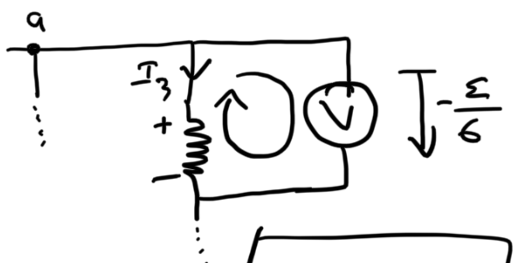
loop rule: resistor 3 and voltmeter



$\Delta V_{\text{voltmeter}} + \Delta V_{R_3} = 0$

$\left(-\frac{\mathcal{E}}{6}\right) + (+I_3 R) = 0$

$\Rightarrow R = \frac{\mathcal{E}}{6I_3}$ but $I_3 = \frac{I_1}{3}$ so $R = \frac{\mathcal{E}}{6(I_1/3)}$ $\boxed{R = \frac{\mathcal{E}}{2I_1}}$

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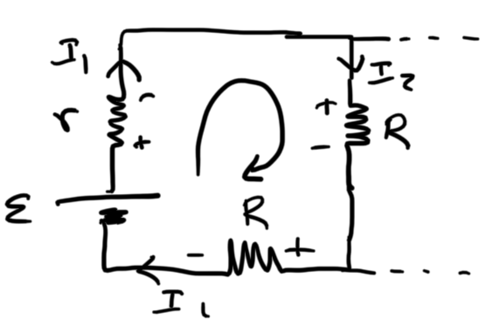
loop rule: battery, resistors 1 and 2



$+\mathcal{E} - I_1 r - I_2 R - I_1 R = 0$

but $I_2 = \frac{2}{3} I_1$ and $R = \frac{\mathcal{E}}{2I_1}$

$\mathcal{E} - I_1 r - \left(\frac{2}{3} I_1\right)\left(\frac{\mathcal{E}}{2I_1}\right) - I_1 \left(\frac{\mathcal{E}}{2I_1}\right) = 0$

$\mathcal{E} - I_1 r - \frac{1}{3}\mathcal{E} - \frac{1}{2}\mathcal{E} = 0$

$\mathcal{E}\left[\frac{6}{6} - \frac{2}{6} - \frac{3}{6}\right] = I_1 r$

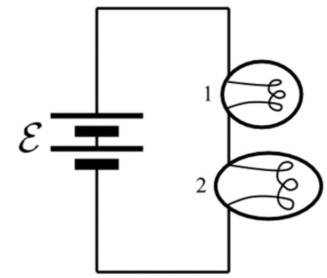
$\frac{\mathcal{E}}{6} = I_1 r$

$\boxed{r = \frac{\mathcal{E}}{6 I_1}}$

The following problem will be hand-graded. Show all supporting work for this problem.

(08) *(20 points)* Two resistors are constructed out of equal volumes of nickel-chromium alloy. The first is extruded to form a cylindrical resistor of length L. The second is extruded to form a cylindrical resistor of length $2L$. Assume both resistors are Ohmic.

Determine the ratio of the power dissipation of the two resistors, P_2/P_1, when they are connected **in series** across an emf $\mathcal{E}$.



① For each wire, volume / length / area are related by:

$V = AL$

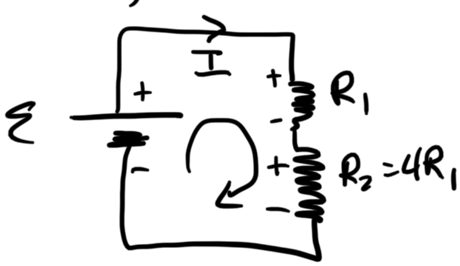
same volume for each: $A_1 L_1 = V = A_2 L_2$

② Find relative resistances: $R_1 = \rho \frac{L_1}{A_1}$

$R_2 = \rho \frac{L_2}{A_2}$ but $L_2 = 2L_1 \rightarrow$ hence, $A_2 = \frac{1}{2} A_1$ – twice the length, half the area

so $R_2 = \rho \frac{(2L_1)}{(A_1/2)} = 4 \rho \frac{L_1}{A_1} \Rightarrow \boxed{R_2 = 4R_1}$

Now, analyze series circuit:



loop rule is $\mathcal{E} - IR_1 - I(4R_1) = 0$

$$\boxed{I = \frac{\mathcal{E}}{5R_1}}$$

Ohm's Law for each resistor:

$\Delta V_1 = IR_1 = \frac{\mathcal{E}}{5}$

$\Delta V_2 = IR_2 = I(4R_1) = \frac{4\mathcal{E}}{5}$

Compute power ratio

$$\frac{P_2}{P_1} = \underbrace{\frac{I \Delta V_2}{I \Delta V_1} = \frac{I \frac{4\mathcal{E}}{5}}{I \frac{\mathcal{E}}{5}}}_{\text{same } I \text{ for both resistors!}} \Rightarrow \boxed{\frac{P_2}{P_1} = 4}$$