Solutions

Printed Name

Nine-digit GT ID

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signature

Spring 2021 **PHYS 2212 G** **Test 02**

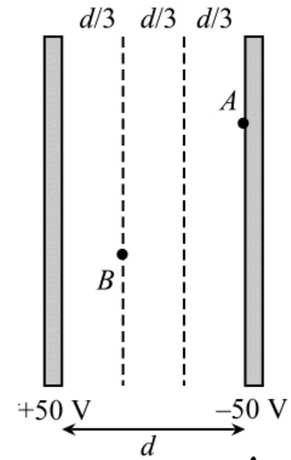
Test Form:

2A

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on your test, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- A standard formula sheet is provided as the cover page for this test. Please remove it from the test before you submit it to the proctor.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Question value 8 points

(01) In the figure at right a capacitor consists of plates separated by a distance d = 0.3 cm. The electric potentials of the positive and negative plates are +50 V and –50 V, respectively. A charge q = +3 μC begins at rest at the negative plate (position A in the figure), and is later observed to pass through position B with a kinetic energy $K_B = 300\ \mu$J. Determine the work done by the electric field, and the work done by external agents, as the charge moves from A to B.



(a) $W_{field} = -200\ \mu J$ and $W_{ext} = +500\ \mu J$

(b) $W_{field} = +300\ \mu J$ and $W_{ext} = 0\ \mu J$

(c) $W_{field} = 0\ \mu J$ and $W_{ext} = +300\ \mu J$

(d) $W_{field} = -200\ \mu J$ and $W_{ext} = -100\ \mu J$

(e) $W_{field} = +200\ \mu J$ and $W_{ext} = +100\ \mu J$

(f) $W_{field} = +300\ \mu J$ and $W_{ext} = -500\ \mu J$

① $W_{field} = -\Delta U_{elec}$

$= -q\Delta V$

$= -(+3\mu C)(+66.7V)$

$\boxed{W_{field} = -200\mu J}$

$\Delta V_{TOT} = +100V$

$\Delta V_{A\to B} = \frac{2}{3}\Delta V_{TOT}$

$= +66.7V$

② $W_{ext} = \Delta E_{system} = \Delta K + \Delta U$

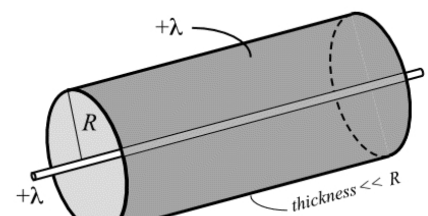
$= (K_f - \cancel{K_i}) + \Delta U_{elec}$

$= (+300\mu J) + (+200\mu J)$

$\boxed{W_{ext} = +500\mu J}$

Question value 8 points

(02) A very thin rod has a uniform linear charge density $+\lambda$. The rod is surrounded by a hollow cylindrical conducting shell of radius R and nonzero thickness $a \ll R$. The conducting shell has a total charge per unit length of $+\lambda$. What are the area densities of charge on the inner and outer surfaces of the shell?



appeared as answer (e) on Gradescope

(a) $\eta_{inner} = -\frac{\lambda}{2\pi R}$ and $\eta_{outer} = +\frac{\lambda}{\pi R}$

(b) $\eta_{inner} = -\frac{\lambda}{4\pi R}$ and $\eta_{outer} = +\frac{\lambda}{4\pi R}$

(c) $\eta_{inner} = +\frac{\lambda}{4\pi R}$ and $\eta_{outer} = +\frac{\lambda}{4\pi R}$

(d) $\eta_{inner} = +\frac{\lambda}{\pi R}$ and $\eta_{outer} = +\frac{\lambda}{\pi R}$

(e) $\eta_{inner} = -\frac{\lambda}{4\pi R}$ and $\eta_{outer} = +\frac{3\lambda}{4\pi R}$

(f) $\eta_{inner} = +\frac{\lambda}{2\pi R}$ and $\eta_{outer} = +2\frac{\lambda}{\pi R}$

①

Exaggerate thickness a

→ GS at radius inside shell has E ≡ 0, so $\Phi_{GS} \equiv 0$

so $Q_{in} \equiv 0$

but $Q_{in} = \underbrace{+\lambda L}_{\text{rod}} + \underbrace{\eta_{inner} \cdot 2\pi RL}_{\text{shell wall}}$

$\Rightarrow \eta_{inner} \cdot 2\pi R = -\lambda$

$\boxed{\eta_{inner} = \frac{-\lambda}{2\pi R}}$

② Shell has charge/length $+\lambda$

→ a length L has total charge

$Q_{shell} = +\lambda L$, split between inner and outer walls

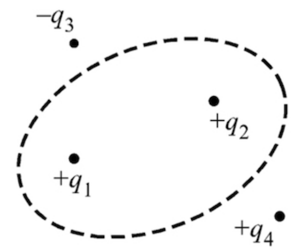
$+\lambda L = \underbrace{\eta_{inner} \cdot 2\pi R}_{=-\lambda} L + \eta_{outer} \cdot 2\pi RL$ →

$\eta_{outer} \cdot 2\pi R = 2\lambda$

$\boxed{\eta_{outer} = \frac{+\lambda}{\pi R}}$

Question value 8 points

(03) In the figure at right, the dashed line represents the cross-section of a three-dimensional Gaussian Surface. The four charges shown have *magnitudes* given by the symbols q_i, and *signs* as indicated explicitly in the figure. Thus, charge 3 is negative, while charges 1, 2, and 4 are positive. Let Φ_i represent the flux through the indicated Gaussian Surface due to charge q_i, only. Which of the following flux comparisons is valid?



(a) $\cancel{\Phi_4 > \Phi_3}$

(b) $\boxed{\Phi_1 + \Phi_2 > \Phi_3 + \Phi_4}$

(c) $\cancel{\Phi_1 + \Phi_2 + \Phi_4 = \Phi_3}$

(d) $\cancel{\Phi_4 > 0}$

(e) $\cancel{\Phi_1 + \Phi_2 - \Phi_4 = \Phi_3}$

(f) $\cancel{\Phi_1 + \Phi_2 = 0}$

$-q_3$ is outside GS

$\to \Phi_3 = 0$

$+q_4$ is outside GS

$\to \Phi_4 = 0$

$\Phi_3 + \Phi_4 = 0$

$+q_1$ and $+q_2$ are inside GS, and both positive

$\to \Phi_1 > 0$ and $\Phi_2 > 0$

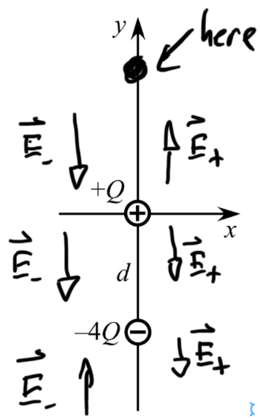
$\to \Phi_1 + \Phi_2 > 0$

only possible alternative is $\Phi_1 + \Phi_2 > \Phi_3 + \Phi_4$

(pos > zero)

Question value 8 points

(04) A point charge $+Q$ is placed at the origin, and a second charge $-4Q$ is placed on the y-axis at $y = -d$. Relative to zero volts at infinity, what is the electric potential at the point on the y-axis where the electric field is zero?



(a) $V = +kQ/2d$

(b) $V = 0$

(c) $V = +3kQ/d$

(d) $\boxed{V = -kQ/d}$

(e) $V = +kQ/d$

• between charges, $\vec{E}_{+Q}$ and $\vec{E}_{-4Q}$ are both downward : $\vec{E}_{net} \neq 0$

• below $-4Q$, $|\vec{E}_{-4Q}| > |\vec{E}_{+Q}|$ – fields can't cancel

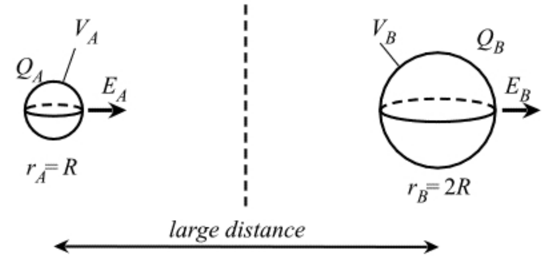
$\Rightarrow$ "$\vec{E}=0$" location must be $\boxed{\text{at some positive } y}$

require $|\vec{E}_+| = |\vec{E}_-| \Rightarrow \frac{kQ}{y^2} = \frac{k(4Q)}{(y+d)^2} \to (y+d)^2 = 4y^2 \to y+d = 2y$ $\boxed{y=d}$

what is $V(y=d)$?

$$V = \frac{+kQ}{d} + \frac{k(-4Q)}{2d} = \frac{+kQ}{d} - 2\frac{kQ}{d} = \boxed{\frac{-kQ}{d}}$$

The next two questions involve the following situation:
Two conducting spheres have radii $r_A = R$ and $r_B = 2R$. They are both given positive charge, but are placed far enough apart that neither sphere effects the other.



Question value 4 points

(5.1) Suppose that the two spheres are charged up to the same (positive) potential, relative to zero volts at infinity. Compare the amount of charge on the two spheres, and the magnitudes of the electric field at the surfaces of the two spheres.

(a) $Q_B = 2Q_A$ and. $E_B = E_A$

(b) $Q_B = Q_A$ and. $E_B = 2E_A$

(c) $Q_B = Q_A$ and. $E_B = \frac{1}{4}E_A$

(d) $Q_B = 2Q_A$ and. $E_B = 2E_A$

(e) $Q_B = Q_A$ and. $E_B = \frac{1}{2}E_A$

(f) $Q_B = 2Q_A$ and. $E_B = \frac{1}{2}E_A$

Require $V_B = V_A \rightarrow \frac{kQ_B}{2R} = \frac{kQ_A}{R}$

$\boxed{Q_B = 2Q_A}$

then $E_A = \frac{kQ_A}{R^2}$

$E_B = \frac{k(Q_B)}{(2R)^2} = \frac{k(2Q_A)}{4R^2} = \frac{2}{4}\frac{kQ_A}{R_2}$

$\boxed{E_B = \frac{1}{2}E_A}$

Question value 4 points

(5.2) Suppose instead that the two spheres are charged up until they have the same electric field magnitude at their surfaces. Compare the amount of charge on the two spheres and the potentials at the surfaces of the two spheres, relative to zero volts at infinity.

(a) $Q_B = 2Q_A$ and. $V_B = V_A$

(b) $Q_B = \frac{1}{2}Q_A$ and. $V_B = 2V_A$

(c) $Q_B = 4Q_A$ and. $V_B = 2V_A$

(d) $Q_B = 4Q_A$ and. $V_B = \frac{1}{2}V_A$

(e) $Q_B = 2Q_A$ and. $V_B = 2V_A$

(f) $Q_B = \frac{1}{2}Q_A$ and. $V = \frac{1}{2}V_A$

Require $E_B = E_A \rightarrow \frac{kQ_B}{(2R)^2} = \frac{kQ_A}{R^2}$

$\Rightarrow \boxed{Q_B = 4Q_A}$

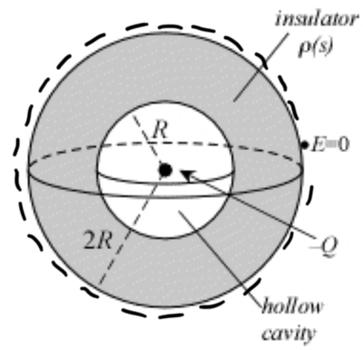
then $V_A = \frac{kQ_A}{R}$

$V_B = \frac{kQ_B}{2R} = \frac{k(4Q_A)}{2R} = \frac{4}{2}\frac{kQ_A}{R}$

$\boxed{V_B = 2V_A}$

The following problem will be hand-graded. *Show all supporting work for this problem.*

(6) *(20 points)* A hollow insulating sphere has inner cavity radius$R$ and outer surface radius $2R$. A point charge $-Q$ is placed at the center of the cavity, and charge is distributed throughout the insulator, given by a volume density $\rho(s) = A/s^2$, where A is a positive constant and s is a dummy radial variable between $s = R$ and $s = 2R$. The electric field is measured to be zero everywhere outside the sphere (i.e. for all radii $r > 2R$).



(i) Use the fact that the field is identically zero outside the sphere, to find an expression for the density $\rho(s)$ that depends only on s, Q, and R. In other words, eliminate the parameter A from the function $\rho(s)$, in favor of the parameters Q and R.

(ii) Use Gauss's Law to find an expression for the electric field (magnitude and direction) inside the insulator, at a distance r in the range $r \in [R, 2R]$. Express you answer entirely in terms of r, R, Q, and ε_0.

(i) Choose GS just barely larger than sphere. $E=0$ outside, so $E=0$ everywhere on GS $\rightarrow \Phi_{GS} = 0 \rightarrow \boxed{Q_{inside} = 0}$

Here, $Q_{inside} = Q_{point} + Q_{insulator}$

$0 = (-Q) + \int \rho \, dV$ where integral is from $s=R$ to $s=2R$ and dV = thin shell at s $= 4\pi s^2 ds$

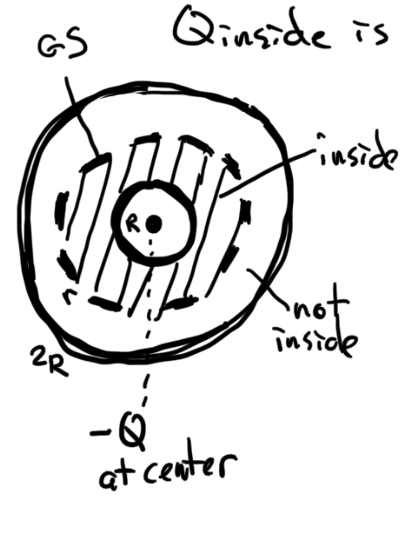
$\Rightarrow \int_R^{2R} \left(\frac{A}{s^2}\right)(4\pi s^2 ds) - Q = 0$

$4\pi A \int_R^{2R} ds = Q \rightarrow 4\pi A R = Q \longrightarrow \boxed{A = \frac{Q}{4\pi R} \text{ and } \rho = \frac{Q}{4\pi R} \cdot \frac{1}{s^2}}$

(ii) To find $E(r)$ for $R < r < 2R$, choose GS at radius r

$\Rightarrow \boxed{\Phi_{GS} = E(r) \cdot 4\pi r^2}$

Q_{inside} is found by summing up charge from $s=R$ to only $s=r$



$Q_{in} = -Q + \int_R^r \rho \, dV = -Q + \int_{s=R}^{s=r} \left[\frac{Q}{4\pi R} \cdot \frac{1}{s^2}\right] 4\pi s^2 ds$

$Q_{in} = Q\left[-1 + \frac{1}{R}\int_R^r ds\right] = Q\left[-1 + \frac{r-R}{R}\right]$

$= Q\left[\frac{-R+r-R}{R}\right] = Q\left[\frac{r-2R}{R}\right]$

so $\Phi = \frac{Q_{in}}{\varepsilon_0}$

$E(r) \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}\left[\frac{r-2R}{R}\right]$

$\boxed{E(r) = \frac{Q}{4\pi\varepsilon_0 R} \frac{r-2R}{r^2}}$

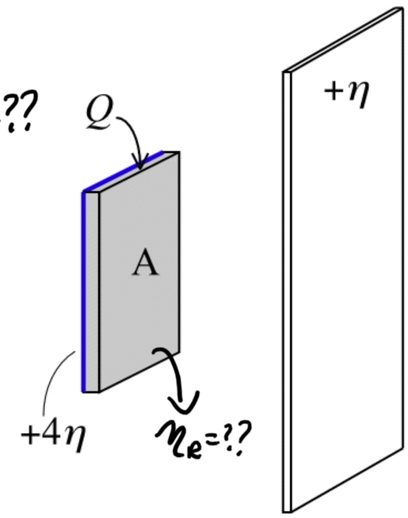
note that $E=0$ when $r=2R$!! That matches what the probem told us!

Form 1A

The following problem will be hand-graded. Show all supporting work for this problem.

(7) *(20 points)* A thin rectangular conducting slab with parallel faces of area A has an unknown total charge Q. The slab is aligned parallel to a thin charged sheet having surface density $+\eta$. After the slab reaches equilibrium, the left face is measured to have a surface density $+4\eta$. density η_R on right face = ??

Determine: (*i*) the total charge Q on the conducting slab, and (*ii*) the electric field (magnitude AND direction) in the region between the slab and the sheet. Express each answer in terms of η, A, and/or ε_0.

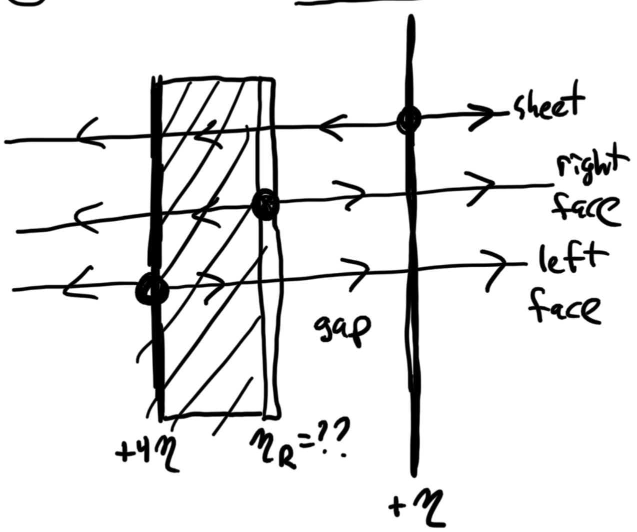


① Total charge on slab must add to equal Q

$$Q = \eta_L A + \eta_R A = (+4\eta)A + \eta_R A$$

$$\boxed{Q = (4\eta + \eta_R)A}$$ η_R = unknown — Not finished!

② Net field inside conductor must be zero



$$\vec{E} = \vec{E}_s + \vec{E}_R + \vec{E}_L = 0 \text{ inside conductor}$$

$$0 = \underbrace{\left(-\frac{\eta}{2\varepsilon_0}\right)}_{\text{sheet}} + \underbrace{\left(-\frac{\eta_R}{2\varepsilon_0}\right)}_{\text{right}} + \underbrace{\left(+\frac{4\eta}{2\varepsilon_0}\right)}_{\text{left}}$$

$$\frac{\eta_R}{2\varepsilon_0} = \frac{4\eta}{2\varepsilon_0} - \frac{\eta}{2\varepsilon_0} = \frac{3\eta}{2\varepsilon_0} \Rightarrow \boxed{\eta_R = 3\eta}$$

Now that we know η_R, we can find Q

$$Q = (4\eta + \eta_R)A = (4\eta + 3\eta)A$$

$$\boxed{Q = 7\eta A}$$

③ Field in the gap

$\vec{E}_R$ and $\vec{E}_L$ to the right

$\vec{E}_{sheet}$ to the left

$$\vec{E}_{gap} = \left(-\frac{\eta}{2\varepsilon_0}\right) + \left(\frac{+3\eta}{2\varepsilon_0}\right) + \left(+\frac{4\eta}{2\varepsilon_0}\right) = \left(+\frac{6\eta}{2\varepsilon_0}\right)$$

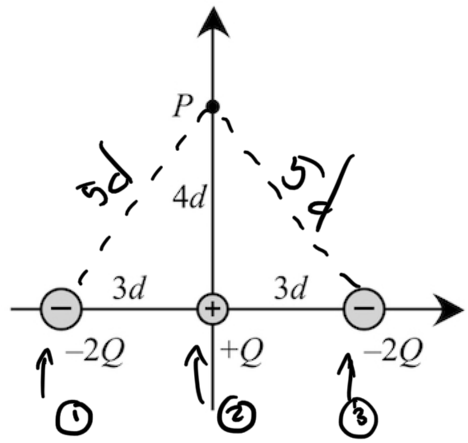
or $\boxed{\vec{E} = \frac{3\eta}{\varepsilon_0} \text{ to the right}}$

The following problem will be hand-graded. *Show all supporting work for this problem.*

(8) *(20 points)* Three charges are arranged on the x-axis as shown above.

(*i*) What is the total potential energy of the charges as shown?

(*ii*) How much work would be required to move charge $+Q$ from the origin to point P on the y-axis? Assume it starts and ends at rest.



(i) $U_{tot} = U_{12} + U_{13} + U_{23}$

$$= \frac{k(-2Q)(+Q)}{3d} + \frac{k(-2Q)(-2Q)}{6d} + \frac{k(+Q)(-2Q)}{3d}$$

$$= -\frac{2}{3}\frac{kQ^2}{d} + \frac{4}{6}\frac{kQ^2}{d} - \frac{2}{3}\frac{kQ^2}{d}$$

$$\boxed{U_{TOT} = -\frac{2}{3}\frac{kQ^2}{d}}$$

Now — move $+Q$ to $y = 4d$ $\quad K_i = K_f = 0$ so $\Delta K = 0$

Also, from above we have $U_i = -\frac{2}{3}\frac{kQ^2}{d}$

$\Rightarrow$ we simply need to compute U_f

$$U_f = \underbrace{U_{12} + U_{13} + U_{23}}_{\text{new values}} = \frac{k(-2Q)(+Q)}{5d} + \frac{k(-2Q)(-2Q)}{6d} + \frac{k(+Q)(-2Q)}{5d}$$

$$= \frac{kQ^2}{d}\left[-\frac{2}{5} + \frac{4}{6} - \frac{2}{5}\right] = \frac{kQ^2}{d}\left[\frac{-12+20-12}{30}\right]$$

$$U_f = -\frac{4}{30}\frac{kQ^2}{d}$$

$$U_i = -\frac{2}{3}\frac{kQ^2}{d} = \frac{-20}{30}\frac{kQ^2}{d}$$

$$\Delta U = \frac{kQ^2}{d}\left[-\frac{4}{30} - \frac{-20}{30}\right] = \frac{kQ^2}{d}\left[+\frac{16}{30}\right] = +\frac{8}{15}\frac{kQ^2}{d}$$

so $W_{ext} = \Delta E_{system} = \Delta K + \Delta U$

$$= 0 + \frac{8}{15}\frac{kQ^2}{d} \Rightarrow \boxed{W_{ext} = +\frac{8}{15}\frac{kQ^2}{d}}$$

positive, as we should expect (we have to pull positive charge away from two negative charges)