Solutions

Printed Name

Nine-digit GT ID

signature

Spring 2021 **PHYS 2212 G** **Test 01**

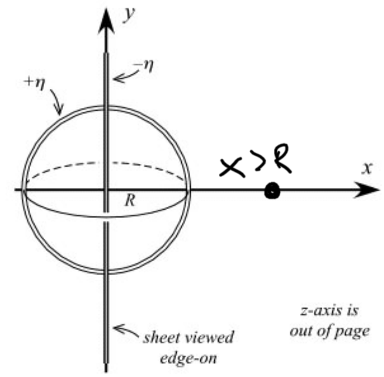
Test Form:

1A

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on your test, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- A standard formula sheet is provided as the cover page for this test. Please remove it from the test before you submit it to the proctor.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Question value 8 points

(01) A hollow insulating spherical shell of radius R is placed at the origin. The shell has a uniform surface charge density $+\eta$. A very large flat sheet with surface charge density $-\eta$ is placed in the yz-plane, bisecting the sphere. At what point (if any) on the positive x-axis will the electric field be exactly zero? (You may assume that the sheet and shell overlap without effecting each other, at their points of contact.)



(a) $x = (2 - \sqrt{2})\,R$
(b) $x = 2\,R$
(c) There is no such place on the positive x-axis.
(d) $x = 4\,R$
(e) $x = (2 + \sqrt{2})$ R
(f) $\boxed{x = \sqrt{2}\,R}$

① Field due to negatively charged sheet: $\vec{E} = \frac{-\eta}{2\varepsilon_0}\hat{\imath}$ for all positive x

②a Total charge on sphere is $Q = \eta A_{surface} = \eta \cdot 4\pi R^2$

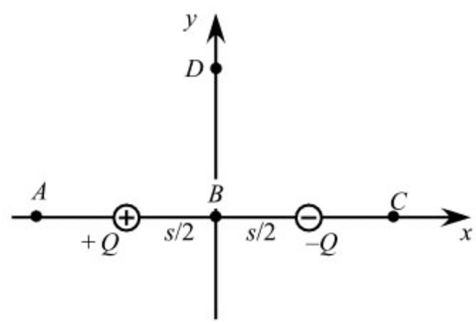
②b Field outside sphere is $\vec{E} = +\frac{\eta \cdot 4\pi R^2}{4\pi\varepsilon_0 x^2}\hat{\imath}$ for all $x > R$

$\vec{E}_{net} = 0$: $\frac{-\cancel{\eta}}{2\cancel{\varepsilon_0}} + \frac{\cancel{\eta}\cancel{4\pi}R^2}{\cancel{4\pi}\cancel{\varepsilon_0}x^2} = 0 \longrightarrow \frac{R^2}{x^2} = \frac{1}{2}$ $\quad x^2 = 2R^2 \Rightarrow \boxed{x = \sqrt{2}R}$

Question value 8 points

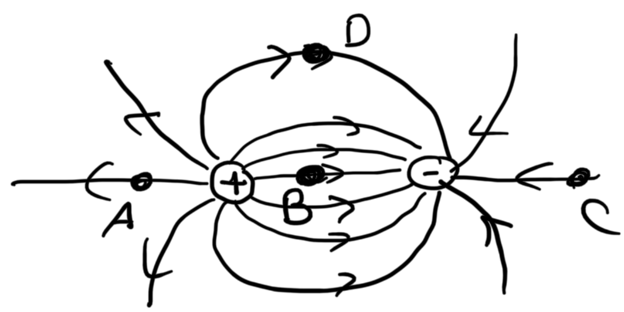
(02) A dipole consists of charges $\pm Q$ located on the x-axis at $\mp s$. Consider an electron that is placed at positions A through D, as shown in the figure at right. What is the direction of the electric field that the electron experiences, at each of the positions?

The sign of the test charge is irrelevant



(a) $\vec{E}_A = -\hat{\imath},\ \vec{E}_B = +\hat{\imath},\ \vec{E}_C = +\hat{\imath},\ \vec{E}_D = +\hat{\jmath}$
(b) $\vec{E}_A = -\hat{\imath},\ \vec{E}_B = 0,\ \vec{E}_C = +\hat{\imath},\ \vec{E}_D = -\hat{\imath}$
(c) $\vec{E}_A = +\hat{\imath},\ \vec{E}_B = -\hat{\imath},\ \vec{E}_C = +\hat{\imath},\ \vec{E}_D = -\hat{\imath}$
(d) $\boxed{\vec{E}_A = -\hat{\imath},\ \vec{E}_B = +\hat{\imath},\ \vec{E}_C = -\hat{\imath},\ \vec{E}_D = +\hat{\imath}}$
(e) $\vec{E}_A = +\hat{\imath},\ \vec{E}_B = 0,\ \vec{E}_C = +\hat{\imath},\ \vec{E}_D = -\hat{\jmath}$

Dipole field pattern:

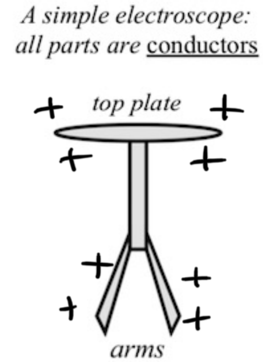


From field pattern, it is clear that
$\vec{E}_A$ and $\vec{E}_C$ are $(-\hat{\imath})$
$\vec{E}_B$ and $\vec{E}_D$ are $(+\hat{\imath})$

This pattern is always the same, no matter what test charge we use.

Question value 8 points

(03) A plastic rod is rubbed with a cloth. If the rod is then touched to a neutral electroscope (see figure), the arms of the electroscope push apart, and remain apart after the rod is removed. What will happen if the cloth that charged the rod is then brought close to the electroscope—without touching it—and is then moved away?



(a) The arms will push further apart when the cloth is nearby, but will return to their original separation after the cloth is removed.

(b) The arms will fall together when the cloth is nearby, but will push apart again after the cloth is removed.

(c) The arms will not change position if the cloth is nearby.

(d) The arms will fall together when the cloth is nearby, and will stay that way after the cloth is removed.

(e) The arms will push further apart when the cloth is nearby, and will stay that way after the cloth is removed.

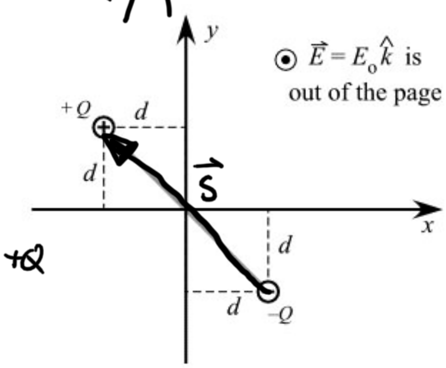
① Scope gains same charge as rod — assume its positive → charge distributes on scope as shown above.

② If rod is positive, cloth is negative:
→ cloth attracts ⊕ on scope to top, leaving none on arms: they fall together

all charge on top, ≃ none on arms

Question value 8 points

(04) An electric dipole lies in the xy plane. It consists of a charge $+Q$ at coordinates $(x, y) = (-d, +d)$ and a charge $-Q$ at coordinates $(x, y) = (+d, -d)$. There is a uniform field present, having magnitude E_0 and pointing in the positive z-direction. What is the torque experienced by the dipole?



(a) $\vec{\tau} = 2\,Qd\,E_0(+\hat{\imath} + \hat{\jmath})$

(b) $\vec{\tau} = 2\,Qd\,E_0(-\hat{\imath} - \hat{\jmath})$

(c) $\vec{\tau} = 2\,Qd\,E_0(+\hat{k})$

(d) $\vec{\tau} = 2\,Qd\,E_0(+\hat{\imath} - \hat{\jmath})$

(e) $\vec{\tau} = 2\,Qd\,E_0(-\hat{\imath} + \hat{\jmath})$

(f) $\vec{\tau} = 2\,Qd\,E_0(-\hat{k})$

Recall $\vec{p} = Q\vec{s}$
where $\vec{s}$ = vector from $-Q$ to $+Q$

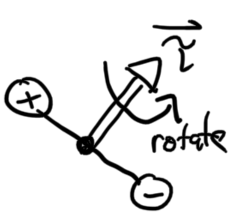
Here, $\vec{s} = (-2d\,\hat{\imath} + 2d\,\hat{\jmath})$

so $\boxed{\vec{p} = -2Qd\,\hat{\imath} + 2Qd\,\hat{\jmath}}$

Torque on dipole is $\vec{\tau} = \vec{p} \times \vec{E} = (-2Qd\,\hat{\imath} + 2Qd\,\hat{\jmath}) \times (E_0\hat{k})$

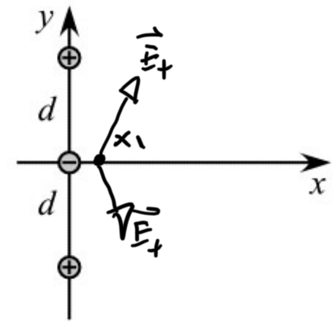
or, $\vec{\tau} = 2QdE_0\left[\underbrace{-\hat{\imath} \times \hat{k}}_{+\hat{\jmath}} + \underbrace{\hat{\jmath} \times \hat{k}}_{+\hat{\imath}}\right]$

so, $\boxed{\vec{\tau} = 2QdE_0(\hat{\imath} + \hat{\jmath})}$ note how this is $\perp$ to $\vec{p}$ and also $\perp$ to $\vec{E}$!



Note how rotation around vector $\vec{\tau}$ will bring ⊕ out "in front" of the page, and push ⊖ back "into" the page — which would tend to align $\vec{p}$ along $\vec{E}$ (as it is supposed to

The next two questions involve the following situation:
Three charges of identical magnitude are arranged along the y-axis as shown at right. Consider the electric field at points on the positive x-axis.



Question value 4 points

(5.1) For values of x that are much smaller than d, what is the approximate expression for the net electric field of the three charges?

(a) $\vec{E} \approx k\frac{Q}{d^2}(+\hat{\imath})$

(b) $\vec{E} \approx k\frac{Q}{x^2+d^2}(-\hat{\imath})$

(c) $\vec{E} \approx k\frac{Q}{x^2}(+\hat{\imath})$

(d) $\boxed{\vec{E} \approx k\frac{Q}{x^2}(-\hat{\imath})}$

(e) $\vec{E} \approx k\frac{Q}{x^2+d^2}(+\hat{\imath})$

(f) $\vec{E} \approx k\frac{Q}{d^2}(-\hat{\imath})$

at x_1 near origin ($\ll d$)
$\vec{E}_+$ vectors are equal, and $\approx$ opposite
$\rightarrow$ they cancel eachother, almost perfectly

so $\boxed{\vec{E}_{net} \approx \vec{E}_- = \frac{kQ}{x^2}(-\hat{\imath})}$

Question value 4 points

(5.2) For values of x that are much larger than d, what is the approximate expression for the net electric field of the three charges?

(a) $\vec{E} \approx k\frac{Q}{x^2}(-\hat{\imath})$

(b) $\boxed{\vec{E} \approx k\frac{Q}{x^2}(+\hat{\imath})}$

(c) $\vec{E} \approx k\frac{Q}{d^2}(+\hat{\imath})$

(d) $\vec{E} \approx 0$

(e) $\vec{E} \approx k\frac{Q}{d^2}(-\hat{\imath})$

Once we are <u>far</u> away from origin, the three charges seem $\approx$ on top of eachother:

$\oplus\ominus\oplus \approx (+-+) \approx (+)$

$\Rightarrow$ Field is roughly that of a single <u>positive</u> charge at the origin

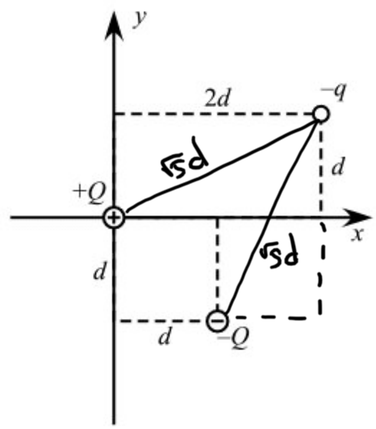
[Equivalently $\vec{E}_{1+}$ and $\vec{E}_{2+}$ are $\approx$ same, and opposite to $\vec{E}_-$]

$\Rightarrow \vec{E}_{net} \approx \vec{E}_{+Q} = \boxed{\frac{kQ}{x^2}(+\hat{\imath})}$

The following problem will be hand-graded. Show all supporting work for this problem.

(6) *(20 points)* A charge $+Q$ is placed at the origin, and a second charge $-Q$ is placed at coordinates $(x, y) = (+d, -d)$, as shown in the figure below. Determine the net force (magnitude and direction) on a charge $-q$ that is placed at position $(x, y) = (+2d, +d)$.

Express the magnitude in terms of k, Q, and d. Express the direction as a numerical angle (to three significant digits) measured relative to the x-axis.



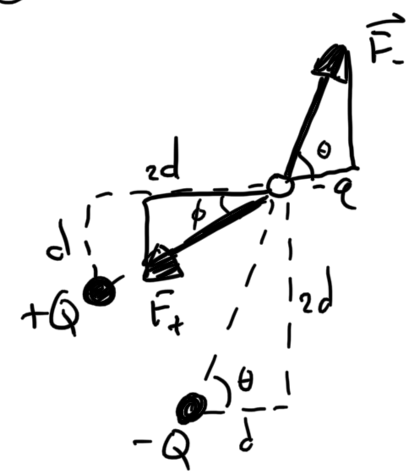
① Both charges $\pm Q$ are the same distance from $-q$:

$$r = \sqrt{(2d)^2 + (d)^2} = \sqrt{5}\,d$$

$\Rightarrow$ The magnitudes of the two forces are identical

$$|\vec{F}_+| = |\vec{F}_-| = \frac{kQq}{r^2} = \boxed{\frac{kQq}{5d^2}}$$

② Charge $-q$ is attracted to $+Q$ and repelled by $-Q$



From positional triangles, we get decomposition of $\vec{F}_+$ and $\vec{F}_-$

$$\vec{F}_- = \left(\frac{kQq}{5d^2}\right)\left(+\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right)$$

$$= \left(\frac{kQq}{5d^2}\right)\left(+\frac{d}{\sqrt{5}d}\,\hat{i} + \frac{2d}{\sqrt{5}d}\,\hat{j}\right) = \boxed{\frac{kQq}{5d^2}\left[\frac{1}{\sqrt{5}}\hat{i} + \frac{2}{\sqrt{5}}\hat{j}\right]}$$

$$\vec{F}_+ = \left(\frac{kQq}{5d^2}\right)\left(-\cos\phi\,\hat{i} - \sin\phi\,\hat{j}\right)$$

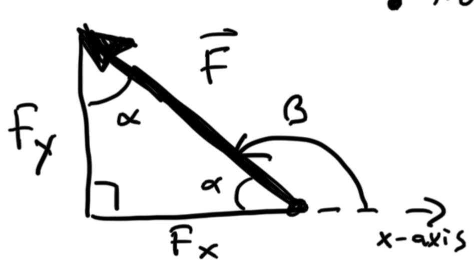
$$= \left(\frac{kQq}{5d^2}\right)\left(-\frac{2d}{\sqrt{5}d}\,\hat{i} - \frac{d}{\sqrt{5}d}\,\hat{j}\right) = \boxed{\frac{kQq}{5d^2}\left[-\frac{2}{\sqrt{5}}\hat{i} - \frac{1}{\sqrt{5}}\hat{j}\right]}$$

③ Perform vector sum!

$$\vec{F}_{net} = \vec{F}_+ + \vec{F}_- = \frac{kQq}{5d^2}\left[-\frac{2}{\sqrt{5}}\hat{i} - \frac{1}{\sqrt{5}}\hat{j} + \frac{1}{\sqrt{5}}\hat{i} + \frac{2}{\sqrt{5}}\hat{j}\right] = \boxed{\frac{kQq}{5d^2}\left[-\frac{1}{\sqrt{5}}\hat{i} + \frac{1}{\sqrt{5}}\hat{j}\right]}$$

④ Draw vector, find magnitude and direction

- Note $|\vec{F}_y| = |\vec{F}_x| = \dfrac{kQq}{5\sqrt{5}\,d^2} \Rightarrow$ This is a 45-45-90 triangle!



$\alpha = 45^\circ$

So:

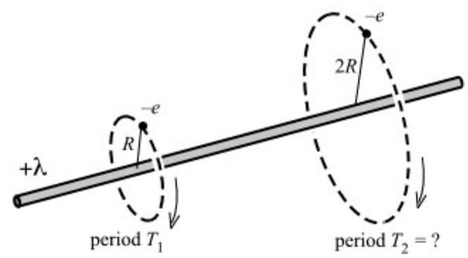
$$|\vec{F}| = \sqrt{2}\left[\frac{kQq}{5\sqrt{5}\,d^2}\right] = \boxed{\frac{\sqrt{2}\,kQq}{5\sqrt{5}\,d^2}}$$

Direction of $\vec{F}$ is $\boxed{\beta = 135^\circ \text{ ccw from x-axis}}$

Form 1A

The following problem will be hand-graded. Show all supporting work for this problem.

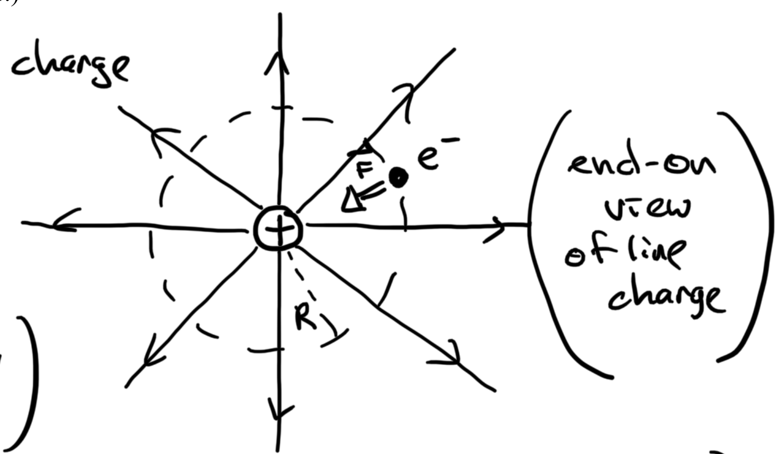
(7) *(20 points)* An electron (mass m, charge $-e$) orbits around a long, uniformly-charged wire (linear charge density $+\lambda$). The orbit is an exactly circular path of radius R, centered on the wire and lying in a plane perpendicular to the wire. The period of the electron's orbit is T_1. If a second electron orbits the wire, in a similar circular path but having a radius $2R$, what will be its period? Express your answer as a numerical multiple of T_1.



(Recall that a particle orbiting in a circle of radius R at speed v experiences a centripetal acceleration of magnitude $a = v^2/R$.)

Field due to a positive line charge

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 R}(+\hat{r})$$



$\Rightarrow$ force on an electron is

$$\vec{F} = (-e)\vec{E} = \frac{-\lambda e}{2\pi\varepsilon_0 R}\hat{r} \quad \left(\text{radially inward}\right)$$

Newton's Law, using centripetal acceleration $\vec{a} = \left(\frac{-v^2}{R}\hat{r}\right)$:

$$\vec{F} = m\vec{a} \rightarrow \frac{-\lambda e}{2\pi\varepsilon_0 R} = m\left(\frac{-v^2}{R}\right) \Rightarrow \boxed{\frac{\lambda e}{2\pi\varepsilon_0} = mv^2}$$

$\boxed{\text{R drops out!}}$

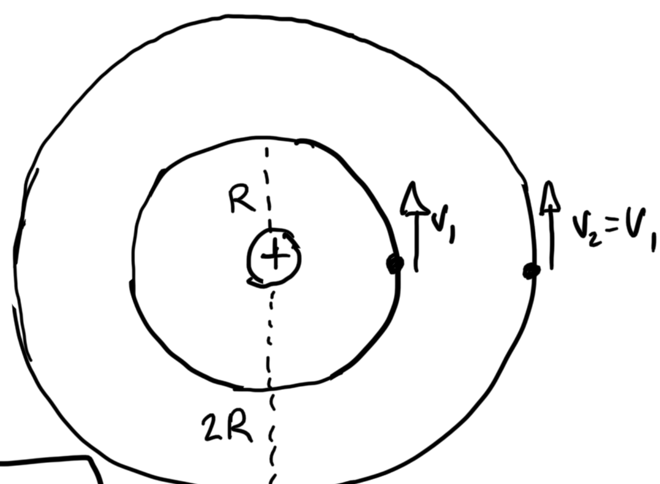
So $V = \sqrt{\dfrac{\lambda e}{2\pi m\varepsilon_0}}$ = constant

$\Rightarrow$ all electrons orbit at the same speed, regardless of orbit radius

$V_1 = V_2$

Clearly, electron #2 must travel twice as far to complete one lap:

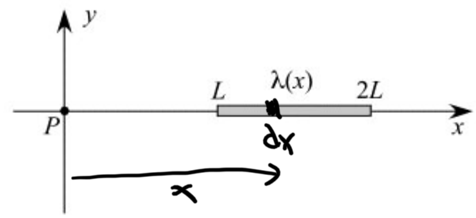
$D_1 = 2\pi R$, $D_2 = 2\pi(2R)$



If they both travel at the same speed, #2 clearly takes twice as long: $\boxed{T_2 = 2T_1}$

The following problem will be hand-graded. Show all supporting work for this problem.

(8) *(20 points)* An insulating rod lies along the x-axis from $x = L$ to $x = 2L$. The rod has a charge density given by $\lambda(x) = Ax^3$, where A is a positive constant. Determine the electric field (magnitude and direction) at the origin. Express your answer in terms of A, L, and ε_0.



Since A is positive for all x, we expect net field to point away from rod:

$$\vec{E} = \sim(-\hat{\imath})$$

charge on segment dx at location x is:

$$\delta Q = \lambda(x)\,dx$$
$$= Ax^3\,dx$$

$\Rightarrow$ Consider one bit of charge, located at position x

The bit of field created by this bit of charge is

$$\delta\vec{E} = \frac{k\,\delta Q}{r^2}\hat{r} \longrightarrow \frac{k\left[Ax^3 dx\right]}{x^2}(-\hat{\imath}) = kAx\,dx\,\hat{\imath}$$

[r = distance from P to bit = x]

[$\hat{r}$ = unit vector at P, away from bit = $-\hat{\imath}$]

So, $$\vec{E}_{net} = \int \delta\vec{E} = \int_{x=L}^{x=2L}\left(-kAx\,dx\,\hat{\imath}\right) = \left[-kA\hat{\imath}\right]\int_L^{2L} x\,dx$$

$$= (-kA\hat{\imath})\left[\frac{x^2}{2}\right]_L^{2L} = (-kA\hat{\imath})\left[\frac{4L^2 - L^2}{2}\right]$$

$$\Rightarrow \boxed{\vec{E}_{net} = -\frac{3kAL^2}{2}\hat{\imath}}$$