Printed Name

Nine-digit GT ID

signature

Fall 2020 **PHYS 2212 G** **Test 04**

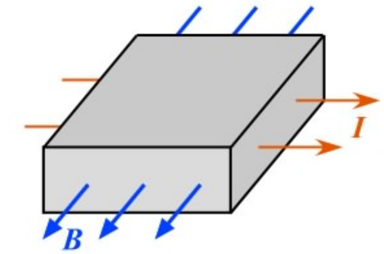
Test Form:

4A

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on your test, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- A standard formula sheet is provided as the cover page for this test. Please remove it from the test before you submit it to the proctor.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Question value 8 points

(01) A rectangular slab of conducting material has a current flowing through it from left to right. The conductor is in a uniform magnetic field, directed from back to front through the slab. What can be said about the Hall Potential across the slab?



(a) There is no Hall Potential across the slab in this situation.

(b) There is a potential difference from front to back, with the front being at higher potential.

(c) There is a potential difference from front to back, with the back being at higher potential.

(d) There is a potential difference from top to bottom, with the top being at higher potential.

(e) There is a potential difference from top to bottom, with the bottom being at higher potential.

(f) There is a potential difference from left to right, with the left side being at higher potential.

(g) There is a potential difference from left to right, with the right side being at higher potential.

$\longrightarrow I$

$\longleftarrow$ electron drift

$(q\vec{v}) = \underline{\text{rightward}}$

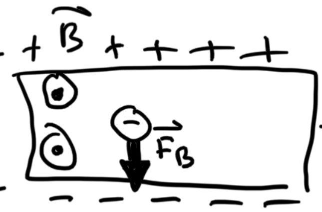
$\odot \longrightarrow (q\vec{v})$

$\vec{B}$ = out of page

$\vec{F} = (q\vec{v}) \times \vec{B}$ = downward

so negative charges accumulate $\underline{\text{at}}$ bottom

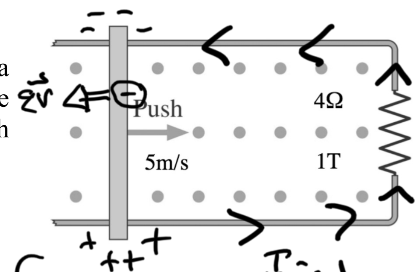
top = high V



bottom = low V

Question value 8 points

(02) In the figure at right, a 40-cm wide slide rail is pushed toward a 4-Ω resistor at a steady speed of 5 m/s. A 1-T magnetic field is directed out of the page. Assume the slide rail and the fixed rails are ideal conductors. What is the induced current through the resistor?



(a) 1.0 A, upward.

(b) 0.5 A , downward.

(c) There is no current at the moment shown.

(d) 1.0 A, downward.

(e) 0.5 A, upward.

① Motional emf

$\mathcal{E}_{ind} = vBL$

$\Rightarrow$ induced current is

$\boxed{I_{ind} = \frac{vBL}{R} = 0.5\,A}$

Direction of current

① Force on mobile electrons

$(q\vec{v}) \times \vec{B}$ = upward

$\Rightarrow$ negative accumulates $\underline{\text{at top}}$

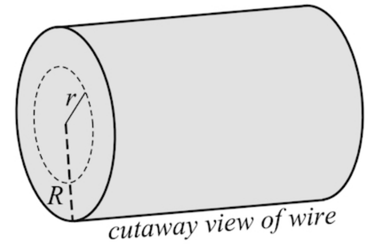
$\Rightarrow$ positive accumulates $\underline{\text{at bottom}}$

② Current flows in rails from pos (high) to neg (low)

$\Rightarrow$ counter-clockwise current $\Rightarrow$ I flows $\underline{\text{upward}}$ through load

Question value 8 points

(03) A conducting wire of radius R has a uniform current density J_0 flowing through its cross-sectional area. What is the magnetic field inside the wire, at a distance r from the central axis of the wire?



(a) $\frac{\mu_0 J_0}{2\pi r}$

(b) $\frac{\mu_0 J_0 R^2}{2 r}$

(c) $\frac{\mu_0 J_0 r}{2}$

(d) $\frac{\mu_0 J_0}{2\pi R}$

(e) $\frac{\mu_0 J_0 r^2}{2 R}$

Ampere's Law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{through}$

$\rightarrow$ for loop of radius $r < R$:

① $\oint \vec{B}\cdot d\vec{s} = B(r)\cdot 2\pi r$

② $I_{Through} = J_0 \cdot A_{loop} = \pi r^2 J_0$

so

$$B(r)\cdot 2\pi r = \mu_0 \pi r^2 J_0$$

$$\boxed{B(r) = \frac{\mu_0 J_0 r}{2}}$$

Question value 8 points

(04) An LC oscillator circuit consists of a capacitor in series with an inductor. At time t_0, the capacitor has full charge Q_{max} and no current flows through the inductor. At some later time t_1, the capacitor has charge $Q_1 = Q_{max}/2$ and a current I_1. If we observe a full cycle of the LC oscillation, what is the maximum current that will flow through the inductor?

(a) $I_{max} = 1.15\, I_1$

(b) $I_{max} = 1.73\, I_1$

(c) $I_{max} = 2.00\, I_1$

(d) $I_{max} = 1.41\, I_1$

(e) $I_{max} = 1.25\, I_1$

Solve as an energy problem:

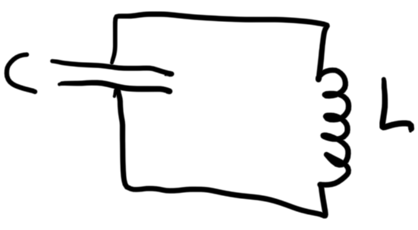
when $Q = max$, $U_c = max = \frac{Q_{max}^2}{2C}$ and $U_L = 0$

when $Q_1 = \frac{Q_{max}}{2}$, $U_c = \frac{(Q_{max}/2)^2}{2C} = \frac{1}{4}\frac{Q_{max}^2}{2C}$

so, at time t_1, $U_c = \frac{1}{4}$ Max

therefore, $U_L = \frac{3}{4}$ Max

Total energy is conserved!

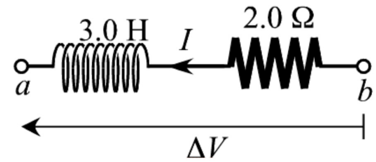


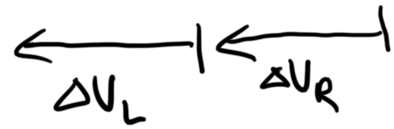
$$\frac{1}{2} L I_1^2 = \frac{3}{4}\left(\frac{1}{2} L I_{max}^2\right)$$

$$I_1^2 = \frac{3}{4} I_{max}^2$$

$$\boxed{I_{max} = \frac{2}{\sqrt{3}} I_1 = 1.15\, I_1}$$

The next two questions involve the following situation:
In the circuit segment at right, a voltmeter measures the potential difference across points a and b to be $\Delta V = V_a - V_b = +8.4$ V, and an ammeter measures the current in the segment to be 1.5 A, flowing right to left.





Question value 4 points

(5.1) At the moment shown, what (if anything) is happening to the current?

(a) The current is decreasing at a rate of 3.8 A/s.
(b) The current is increasing at a rate of 2.8 A/s.
(c) The current is decreasing at a rate of 1.8 A/s.
(d) The current is increasing at a rate of 1.8 A/s.
(e) The current is increasing at a rate of 3.8 A/s.
(f) The current is decreasing at a rate of 2.8 A/s.

Note that $\Delta V = \Delta V_R + \Delta V_L$

where $\Delta V_R = -IR$
$= -(1.5\text{A})(2\Omega)$
$= -3.0\text{V}$

so: $\Delta V = +8.4\text{V} = (-3.0\text{V}) + \Delta V_L$

$\Rightarrow \Delta V_L = +11.4\text{V}$

but Faraday's Law tells us: $\Delta V_L = -L\frac{dI}{dt} \Rightarrow \frac{dI}{dt} = \frac{-\Delta V_L}{L}$

$\boxed{\frac{dI}{dt} = -3.8\text{A/s}}$

Question value 4 points

(5.2) At the moment shown, what is the power output (if positive) or power absorption (if negative) for the inductor coil?

(a) $P_L = 0$ W
(b) $P_L = +8.1$ W
(c) $P_L = -12.6$ W
(d) $P_L = +17$ W
(e) $P_L = +12.6$ W
(f) $P_L = -17$ W
(g) $P_L = -8.1$ W

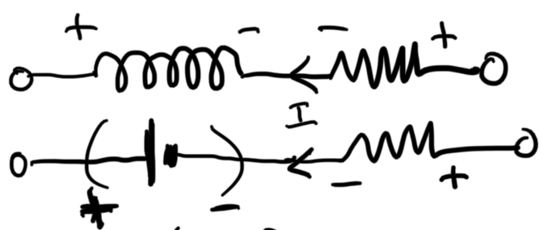
$P_x = I_x \Delta V_x$ for any circuit element X

$\Rightarrow$ for our inductor,

$P_L = I_L \Delta V_L = (1.5\text{A})(+11.4\text{V})$

$\boxed{P_L = +17.1\text{W}}$

Note positive sign:
Inductor is providing energy, trying to sustain the current against a decrease

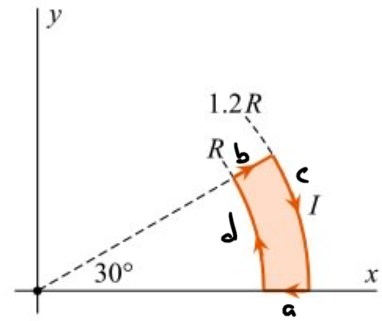


virtual emf opposes current decrease

The following problem will be hand-graded. *Show all supporting work for this problem.*

(6) *(20 points)* A length of wire is bent into a rectangular arc as shown at right. The arc subtends an angle of 30° and extends from radius R to radius $1.2R$. A current I flows clockwise around the loop.

Determine the magnetic field (magnitude and direction) at the origin. Express your answer in terms of I, R, and μ_0.



$\Rightarrow$ A 30° arc is $\frac{1}{12}$ of a circle, or $\frac{\pi}{6}$ radians

Method ① Biot-Savart Law:

- on segments a and b, $d\vec{s} \parallel \hat{r}$, so $d\vec{B} = 0$ $\rightarrow$ only worry about arcs c, d
- on c: $I d\vec{s} \times \hat{r}$ = into page $= -\hat{k}$
- on d: $I d\vec{s} \times \hat{r}$ = out of page $= +\hat{k}$

$$\Rightarrow \vec{B}_c = \int_c \frac{\mu_0}{4\pi} \frac{I d\vec{s} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}(-\hat{k}) \int_c \frac{ds}{(1.2R)^2} = \frac{\mu_0 I}{4\pi (1.2R)^2} \int ds$$

but $\int ds = \frac{1}{12}(\text{circumference}) = \frac{2\pi(1.2R)}{12}$

so $\boxed{\vec{B}_c = \frac{\mu_0 I}{24}\left(\frac{1}{1.2R}\right)(-\hat{k})}$ A similar calculation for $\vec{B}_d$ will give $\boxed{\frac{\mu_0 I}{24}\left(\frac{1}{R}\right)(+\hat{k})}$

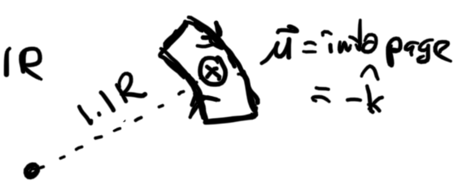
Adding all contributions, we get

$$\vec{B}_{net} = \frac{\mu_0 I}{24}\left[+\frac{1}{R}\hat{k} - \frac{1}{1.2R}\hat{k}\right] = \frac{\mu_0 I}{24}\left[\frac{12}{12R}\hat{k} - \frac{10}{12R}\hat{k}\right]$$

$$= \frac{\mu_0 I}{24}\left(\frac{2}{12R}\hat{k}\right) \stackrel{!}{=} \boxed{\frac{\mu_0 I}{144R}(+\hat{k})}$$

Method ② dipole approximation

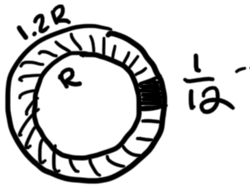
$\rightarrow$ treat loop as a mag. dipole at $r = 1.1R$



$\vec{\mu} = I\vec{A}$ where $|\vec{A}| = \frac{1}{12}$ Area of annulus

$$= \frac{1}{12}\left[\pi(1.2R)^2 - \pi R^2\right]$$

$$= \frac{0.44\pi R^2}{12}$$



Then dipole approx gives $\vec{B} \approx \frac{\mu_0}{4\pi}\frac{(-\vec{\mu})}{r^3} = \frac{\mu_0}{4\pi}\frac{(-I\vec{A})}{(1.1R)^3} = \frac{\mu_0 I \frac{0.44}{12}\pi R^2 \hat{k}}{4\pi(1.1R)^3}$

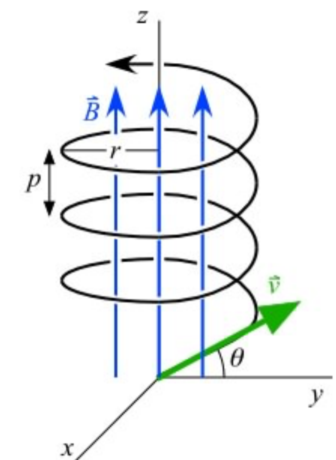
simplification gives $\frac{\mu_0 I}{48R}(+\hat{k}) \cdot \left(\frac{0.44}{(1.1)^3}\right) \approx \boxed{\frac{\mu_0 I}{48R}(0.331)(+\hat{k})}$

$\boxed{\text{that's only a 0.8\% error!}}$

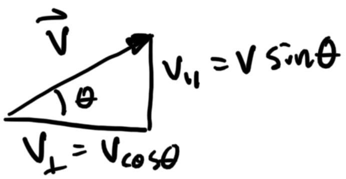
Form 4A

The following problem will be hand-graded. Show all supporting work for this problem.

(7) *(20 points)* A uniform magnetic field of magnitude B lies along the positive z-axis, as shown in the figure below. An electron (mass m, charge $-e$) enters the magnetic field traveling with a speed v directed at an angle θ above the xy-plane, and consequently follows a helical trajectory through the field. The pitch p of the helix is defined to be the vertical distance between two successive laps around the helix (see figure).



Find an expression for the pitch of the helix, in terms of B, m, e, v, and/or θ, as needed. [Hint: start by decomposing the initial velocity vector into horizontal ($v_\perp$) and vertical ($v_\parallel$) components.]



① pitch = motion along $\vec{B}$: $\overrightarrow{\Delta z} = \langle +p \rangle = V_{\parallel}\,\Delta t$
$= V\sin\theta \cdot \Delta t = \boxed{V\sin\theta \cdot T}$

$\Rightarrow$ need to find the time required for one full lap, T

② motion $\perp$ to $\vec{B}$ is uniform circular motion:

$$\sum \vec{F}_{\perp} = m\vec{a}_{\perp} \rightarrow |\vec{F}_B| = m\frac{V_{\perp}^2}{R} \rightarrow eV_{\perp}B = \frac{mV_{\perp}^2}{R}$$

$$\rightarrow \boxed{V_{\perp} = \frac{eBR}{m}}$$

③ we also know, for motion in a circle, $V_{\perp} = \frac{\text{distance}}{\text{time}} = \frac{2\pi R}{T}$

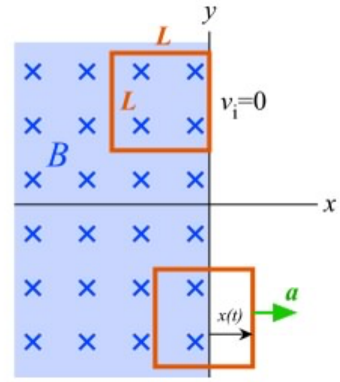
so $\frac{2\pi \cancel{R}}{T} = \frac{eB\cancel{R}}{m} \longrightarrow \boxed{T = \frac{2\pi m}{eB}}$

④ Going back to our expression for pitch:

$$p = V_{\parallel}T = (V\sin\theta)\left(\frac{2\pi m}{eB}\right) = \boxed{2\pi\sin\theta\,\frac{mV}{eB}}$$

The following problem will be hand-graded. *Show all supporting work for this problem.*

(8) *(20 points)* A uniform magnetic field of constant magnitude B fills the $x < 0$ half-plane. The field points in the negative z-direction. A square loop with sides of length L is initially at rest, with its left side at $x = -L$ and its right side at $x = 0$ (top). At time $t = 0$, the loop is pulled to the right with a constant acceleration of magnitude a (bottom).



Determine the magnitude of the induced emf in the loop as a function of time: $\mathcal{E}(t)$, valid for ALL positive values of t. Express your answer in terms of B, L, a, and t. Keep in mind that the loop will fully exit the field at some point! [Hint: start by working out some constant-acceleration kinematics.]

Constant acceleration:

$\vec{a}$ = constant → $\vec{V}(t) = \cancel{\vec{V}_i}^{\,0} + \vec{a}t$

$\boxed{+V(t) = +at}$

→ $\vec{x}(t) = \cancel{\vec{x}_i}^{\,0} + \cancel{\vec{V}_i t}^{\,0} + \frac{1}{2}\vec{a}t^2$ $\quad \boxed{+x(t) = +\frac{1}{2}at^2}$

When will loop completely leave field? when $x(t_f) = +L$

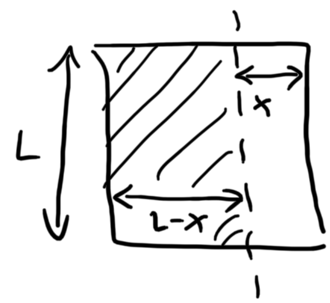
$+L = +\frac{1}{2}at_f^2 \longrightarrow \boxed{t_f = \sqrt{\frac{2L}{a}}}$

Hence, $\mathcal{E}(t) \to 0$ for all $t \geq \sqrt{2L/a}$

For $0 \leq t \leq t_f$, apply Faraday's Law

$\Phi = \int \vec{B} \cdot d\vec{A} = B \cdot A_{in}$ where A_{in} = area of loop that is still inside field

$A_{in} = L(L-x)$



so $\Phi = BL(L-x)$ — into page

$\Rightarrow \mathcal{E}_{ind} = \frac{d\Phi}{dt} = BL\frac{d}{dt}[L-x]$

$= -BL\frac{dx}{dt} = -BLV$

$= -BL(at)$

So $\boxed{\mathcal{E}(t) = \begin{cases} -BLat & 0 \leq t \leq \sqrt{2L/a} \\ 0 & t > \sqrt{2L/a} \end{cases}}$

Problem does not ask about CW-vs-CCW, but $\mathcal{E}_{ind}$ would push current CW

(loop is losing flux into page, so it creates its own flux into page, via a clockwise induced current)