Solutions	
Printed Name	Nine-digit GT ID

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signature

Fall 2020 **PHYS 2212 G** **Test 02**

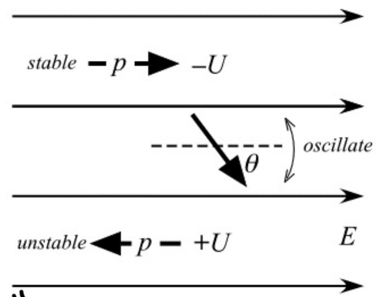
Test Form:

2A

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on your test, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- A standard formula sheet is provided as the cover page for this test. Please remove it from the test before you submit it to the proctor.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Question value 8 points

(01) A dipole is placed in a uniform electric field. When aligned opposite to the field, the dipole is in unstable equilibrium with an energy of +2.00 mJ. When aligned parallel to the field, it is in stable equilibrium with an energy of –2.00 mJ. The dipole is set into oscillation about its stable equilibrium point, with turning points at angles of ±53.1° relative to the equilibrium position. What will be the kinetic energy of the dipole when it rotates through the 0° orientation?



Dipole energy $U = -\vec{p}\cdot\vec{E}$

so parallel : $\boxed{-pE = -2.00\,\text{mJ}}$ = "pE" = 2mJ

(a) 0.60 mJ

(b) 0.80 mJ

(c) 1.00 mJ

(d) 0.40 mJ

(e) 1.20 mJ

initial final

θ

$K_i = 0$ $\quad U_f = -pE$

$U_i = -pE\cos 53.1°$ $\quad K_f = ?$

$= -0.6\,pE$

E

$K_f + U_f = K_i + U_i$

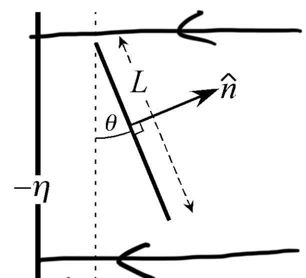
$K_f + (-pE) = 0 + (-0.6pE)$

$K_f = 0.4\,pE$

$= 0.4(2.0\,\text{mJ})$

Question value 8 points

(02) A very large charged surface has a uniform area charge density $-\eta$. A small, square plastic sheet of length L on a side is held near the surface as shown at right. (In the figure, we see an *edge view* of the plastic sheet; it extends a distance L directly <u>into the page</u>.) The sheet is oriented with its surface tilted at angle θ away from being parallel with the larger surface. The normal direction for the plastic sheet is chosen to be "up and to the right", as indicated in the figure. What is the electric flux through the plastic sheet?



$\left(\vec{E} = -\frac{\eta}{2\varepsilon_0}\hat{i}\right)$

(a) $\Phi = 0$, because plastic is an insulating material.

(b) $\Phi = -\frac{\eta L^2}{2\varepsilon_0}\sin\theta$

(c) $\Phi = +\frac{\eta L^2}{2\varepsilon_0}\sin\theta$

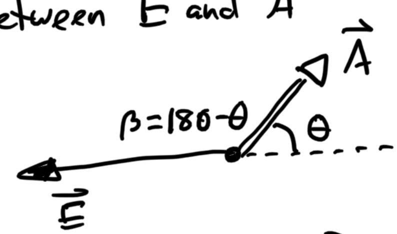
(d) $\Phi = -\frac{\eta L^2}{2\varepsilon_0}\cos\theta$

(e) $\Phi = +\frac{\eta L^2}{2\varepsilon_0}\cos\theta$

$\Phi = \vec{E}\cdot\vec{A}$

$= EA\cos\beta$

where β = angle between $\vec{E}$ and $\vec{A}$

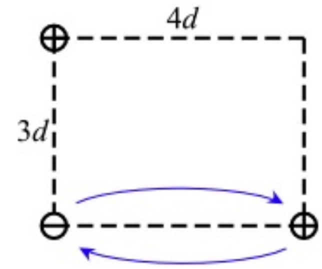


$\Phi = \left(\frac{\eta}{2\varepsilon_0}\right)(L^2)\cos(180-\theta)$

$\boxed{\Phi = -\frac{\eta L^2}{2\varepsilon_0}\cos\theta}$ [using $\cos(180-\theta) = -\cos\theta$]

Question value 8 points

(03) Three charges of identical magnitude Q are at the corners of a rectangle as shown at right. How much external work would be required to exchange the two bottom charges, as indicated by the arrows? Assume the charges begin and end at rest.



(a) $W_{ext} = +\frac{3}{20}\frac{kQ^2}{d}$

(b) $W_{ext} = +\frac{4}{15}\frac{kQ^2}{d}$

(c) $W_{ext} = -\frac{4}{15}\frac{kQ^2}{d}$

(d) $W_{ext} = -\frac{5}{12}\frac{kQ^2}{d}$

(e) $W_{ext} = -\frac{3}{20}\frac{kQ^2}{d}$

(f) $W_{ext} = +\frac{5}{12}\frac{kQ^2}{d}$

$\Delta K = 0$

so $W_{ext} = \Delta E = \cancel{\Delta K} + \Delta U$

$W_{ext} = \Delta U = U_f - U_i$

$$= \left[\frac{-kQ^2}{5d} + \frac{+kQ^2}{3d} + \frac{-kQ^2}{4d}\right] - \left[\frac{-kQ^2}{3d} + \frac{+kQ^2}{5d} + \frac{-kQ^2}{4d}\right]$$

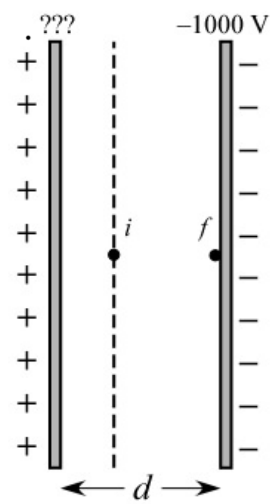
$$W_{ext} = -\frac{kQ^2}{5d} + \frac{kQ^2}{3d} + \frac{kQ^2}{3d} - \frac{kQ^2}{5d}$$

$$= \frac{2kQ^2}{d}\left[\frac{1}{3} - \frac{1}{5}\right] = \frac{2kQ^2}{d}\left[\frac{5-3}{15}\right]$$

$$\boxed{W_{ext} = +\frac{4kQ^2}{15d}}$$

Question value 8 points

(04) A parallel-plate capacitor has plates separated by a distance d = 3.0 cm. When a charge q = +2.0 mC is held 2.0 cm from the negative plate and released, it strikes the negative plate with a kinetic energy K_f = 8.0 J. If the negative plate is assumed to be an potential $V_- = -1000$ volts, what is the potential of the positive plate?



(a) $V_+ = +1000$ volts

(b) $V_+ = -5000$ volts

(c) $V_+ = +5000$ volts

(d) $V_+ = +8000$ volts

(e) $V_+ = -8000$ volts

(f) $V_+ = 0$ volts

Energy is conserved:

$\Delta K + \Delta U = 0$

$\Delta U = -\Delta K = -(+8.0\text{ J})$

hence, the charge fell through

$\Delta V = \frac{\Delta U}{q} = \frac{-8\text{J}}{+2\text{mC}} = -4000\text{V}$

→ that's for $\Delta x = \frac{2}{3}d$

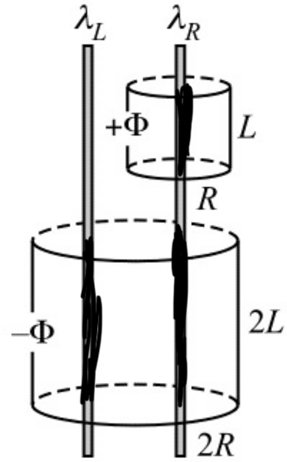
For full capacitor, $\Delta x = d$

$\Rightarrow \Delta V_{\text{pos to neg}} = -6000\text{V}$

or $V_- - V_+ = -6000\text{V}$

$V_+ = V_- + 6000\text{V}$ $\boxed{V_+ = +5000\text{ V}}$

The next two questions involve the following situation:
Two very long charged rods lie parallel to each other, with unknown linear charge densities λ_L and λ_R. If a Gaussian cylinder of radius R and length L encloses *only* the rod on the right, the measured flux through the surface is $+\Phi$. If a larger Gaussian surface of radius 2R and length $2L$ encloses <u>both</u> rods, the total flux through the surface is measured to be $-\Phi$.



Gauss's Law $\Phi_{gs} = \frac{1}{\varepsilon_0} Q_{in}$
or
$Q_{in} = \varepsilon_0 \Phi_{gs}$

Question value 4 points
(5.1) What is the linear charge density of the rod on the right?

(a) $\lambda_R = +\varepsilon_0\Phi/L$
(b) $\lambda_R = +\varepsilon_0\Phi/2\pi R$
(c) $\lambda_R = +\varepsilon_0\Phi/\pi R^2$
(d) $\lambda_R = +\varepsilon_0\Phi/R$
(e) $\lambda_R = +\varepsilon_0\Phi/2\pi RL$

$\Rightarrow$ For small GS, surface encloses a length L of charge density λ_R $\Rightarrow Q_{in} = \lambda_R L$

So, Gauss's Law gives

$$(\lambda_R L) = \varepsilon_0 \Phi \rightarrow \boxed{\lambda_R = \frac{\varepsilon_0 \Phi}{L}}$$

Question value 4 points
(5.2) What is the linear charge density of the rod on the left?

(a) $\lambda_L = -2\varepsilon_0\Phi/4\pi RL$
(b) $\lambda_L = -\varepsilon_0\Phi/4\pi R$
(c) $\lambda_L = -\varepsilon_0\Phi/2R$
(d) $\lambda_L = -3\varepsilon_0\Phi/4\pi R^2$
(e) $\lambda_L = -3\varepsilon_0\Phi/2L$

For <u>large</u> GS, the surface encloses a total length 2L of <u>both</u> line charges

$\rightarrow Q_{in} = (\lambda_R + \lambda_L) 2L$

So, Gauss's Law gives:

$$(\lambda_R + \lambda_L) 2L = -\varepsilon_0 \Phi$$

$$\lambda_R + \lambda_L = -\frac{\varepsilon_0 \Phi}{2L}$$

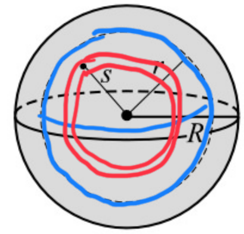
$$\lambda_L = -\frac{\varepsilon_0 \Phi}{2L} - \lambda_R = -\frac{\varepsilon_0 \Phi}{2L} - \left(\frac{\varepsilon_0 \Phi}{L}\right)$$

$$\Rightarrow \boxed{\lambda_L = \frac{-3\varepsilon_0 \Phi}{2L}}$$

The following problem will be hand-graded. Show all supporting work for this problem.

(6) *(20 points)* A solid insulating sphere of radius R has a **non-uniform** charge density placed on it, given by the formula:

$$\rho(s) = \rho_0(1 - s/R)$$



Here, ρ_0 is a *positive* constant, and the parameter s is the distance from the center of the sphere, $0 \le s \le R$.

(i) Determine the electric field magnitude *inside* the sphere, as a function of the distance r from the center of the sphere. Express your answer in terms of $\rho_0, \varepsilon_0, R,$ and r.

(ii) Find the radius r_{max} at which the field magnitude is greatest, and then determine that maximal field value E_{max}. Your answer should be expressed in terms of $\rho_0, \varepsilon_0,$ and R.

① Assume a Gaussian sphere of radius $r < R$ (blue)

→ If $E_{in}(r)$ is the electric field inside the sphere, Then the flux through the sphere is

$$\Phi_{GS} = E_{in}(r) \cdot 4\pi r^2$$

② Find the total charge inside this sphere

⇒ Sum charge by shells (red): $dQ = \rho(s) \cdot 4\pi s^2 ds$

$$\Rightarrow Q_{in} = \int_{s=0}^{s=r} \rho_0\left(1 - \frac{s}{R}\right) 4\pi s^2 ds = 4\pi\rho_0 \int_0^r \left(s^2 - \frac{s^3}{R}\right) ds$$

$$\text{so } Q_{in} = 4\pi\rho_0 \left[\frac{s^3}{3} - \frac{s^4}{4R}\right]_0^r \rightarrow \boxed{Q_{in} = 4\pi\rho_0\left[\frac{r^3}{3} - \frac{r^4}{4R}\right]}$$

③ Apply Gauss's Law: $\Phi_{GS} = Q_{in}/\varepsilon_0$

$$E_{in}(r) \cdot 4\cancel{\pi} r^2 = \frac{\cancel{4\pi}\rho_0}{\varepsilon_0}\left[\frac{r^3}{3} - \frac{r^4}{4R}\right]$$

$$\rightarrow E_{in}(r) = \boxed{\frac{\rho_0}{\varepsilon_0}\left[\frac{r}{3} - \frac{r^2}{4R}\right]}$$

Now: maximize E_{in} by setting $\frac{dE}{dr} = 0$

④ $$0 = \frac{d}{dr}\left[\frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right)\right] = \frac{\rho_0}{\varepsilon_0}\left(\frac{1}{3} - \frac{2r}{4R}\right) = \frac{\rho_0}{\varepsilon_0}\left(\frac{1}{3} - \frac{r}{2R}\right)$$

$$\Rightarrow \boxed{r_{max} = \frac{2}{3}R}$$

⑤ Evaluate E_{in} at r_{max}:

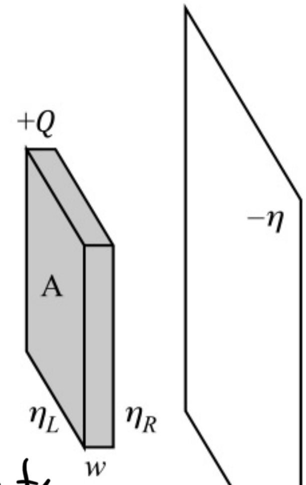
$$E_{max} = \frac{\rho_0}{\varepsilon_0}\left[\frac{1}{3}\left(\frac{2R}{3}\right) - \frac{1}{4R}\left(\frac{4}{9}R^2\right)\right] = \frac{\rho_0}{\varepsilon_0}\left[\frac{2}{9}R - \frac{1}{9}R\right]$$

$$\boxed{E_{max} = \frac{\rho_0 R}{9\varepsilon_0}}$$

The following problem will be hand-graded. Show all supporting work for this problem.

(7) *(20 points)* A conducting slab of width w and area A has a total charge $+Q$ placed upon it. The slab is placed near a large flat sheet having surface charge density $-\eta$. The slab is aligned with its two faces *parallel* to the sheet, as shown at right.

Determine the surface charge density η_L and η_R on the left and right faces of the slab. (You may assume that the edge surfaces of the slab have negligible area.) Express your answer in terms of w, A, Q, and/or η



① Total charge on L/R faces adds to Q:

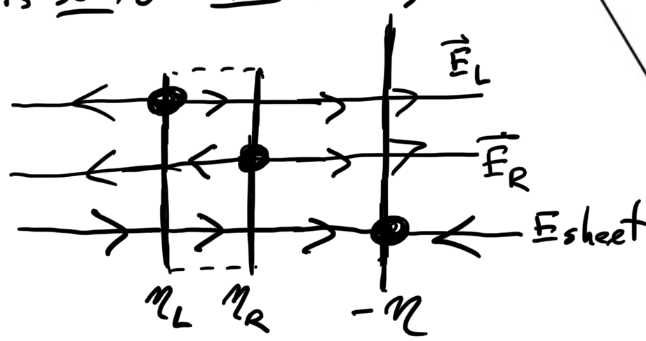
$$\boxed{\eta_L \cdot A + \eta_R \cdot A = Q}$$ two unknowns, η_L η_R

② Net electric field inside conductor must be zero → field is sum of three sheets

at "X" $\vec{E}_L = \langle +\frac{\eta_L}{2\varepsilon_0} \rangle$

$\vec{E}_R = \langle -\frac{\eta_R}{2\varepsilon_0} \rangle$

$\vec{E}_{sheet} = \langle +\frac{\eta}{2\varepsilon_0} \rangle$



$$\Rightarrow \frac{\eta_L}{2\varepsilon_0} - \frac{\eta_R}{2\varepsilon_0} + \frac{\eta}{2\varepsilon_0} = 0 \Rightarrow \boxed{-\eta_L + \eta_R = \eta}$$ two unknowns η_L η_R

combin with $\boxed{\eta_L + \eta_R = Q/A}$ that's two equations in two unknowns

Add: $2\eta_R = \frac{Q}{A} + \eta$

$$\boxed{\eta_R = \frac{1}{2}\left(\frac{Q}{A} + \eta\right)}$$

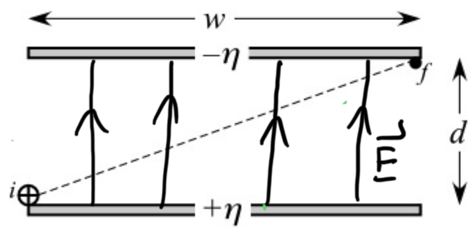
subtract: $-2\eta_L = \eta - Q/A$

$$\boxed{\eta_L = \frac{1}{2}\left(Q/A - \eta\right)}$$

Correction!

The following problem will be hand-graded. *<u>Show all supporting work for this problem</u>.*

(8) *(20 points)* A capacitor has plates with charge density $\pm\eta$ separated by a distance d. The plates have side-to-side width w. A proton (charge $+e$) is initially at at one edge of the positive plate, and is moved to the opposite edge of the negative plate, as shown. It begins and ends at rest.



Find expressions for the following quantities. In each case, express your answer in terms of $\varepsilon_0, \eta, e, w,$ and/or d. Be sure to attach an **explicit** positive or negative sign to each answer!

(i) The potential difference ΔV moved through by the proton.

(ii) The work done on the proton by the electric field, W_{elec}.

(iii) The work done on the proton by an external agent, W_{ext}.

Inside Capacitor:

$$\boxed{\vec{E} = +\frac{\eta}{\varepsilon_0}\hat{j}}$$

(i) In a uniform field: $\Delta V = -\vec{E}\cdot\Delta\vec{s}$

$$\rightarrow \Delta V = -\left(+\frac{\eta}{\varepsilon_0}\hat{j}\right)\cdot\left(w\hat{i}+d\hat{j}\right)$$

$$\boxed{\Delta V = -\frac{\eta d}{\varepsilon_0}}$$

moving from pos plate to neg plate = move to <u>lower</u> potential

(ii) $W_{field} = -\Delta U$

where $\Delta U = q\,\Delta V = (+e)\left(-\frac{\eta d}{\varepsilon_0}\right) = -\frac{e\eta d}{\varepsilon_0}$

so $W_{field} = -\left(-\frac{e\eta d}{\varepsilon_0}\right) \Rightarrow \boxed{W_{field} = +\frac{e\eta d}{\varepsilon_0}}$

positive sign: field is converting PE into KE

(iii) Proton begins and ends <u>at rest</u>: net $\Delta K = 0$

$\Rightarrow$ No KE change, but a PE loss: system <u>loses</u> energy, $E_f < E_i$

then $\Delta E_{sys} = W_{ext}$

$$\rightarrow W_{ext} = \cancel{\Delta K}^{\,0} + \Delta U = 0 + \left(-\frac{e\eta d}{\varepsilon_0}\right)$$

$$\boxed{W_{ext} = -\frac{e\eta d}{\varepsilon_0}}$$