Solution Printed Name	Nine-digit GT ID

signature

Spring 2020 **PHYS 2212 G** **Test 04**

Test Form: **4A**

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on your test, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- A standard formula sheet is provided as the cover page for this test. Please remove it from the test before you submit it to the proctor.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Fill in bubbles for your Multiple Choice answers darkly and neatly.
If you wish to change an answer, draw a clear "X" through the non-answer!

1 ⓐ ⓑ ⓒ ⓓ ⓔ

2 ⓐ ⓑ ⓒ ⓓ ⓔ

3 ⓐ ⓑ ⓒ ⓓ ⓔ

4 ⓐ ⓑ ⓒ ⓓ ⓔ

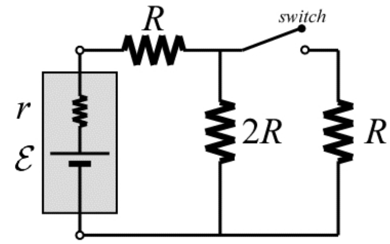
5 ⓐ ⓑ ⓒ ⓓ ⓔ

6 ⓐ ⓑ ⓒ ⓓ ⓔ

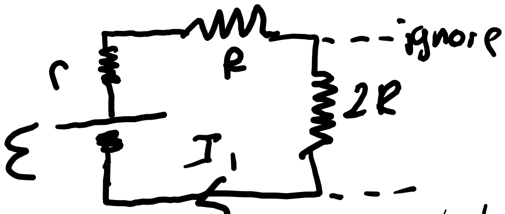
The following problem will be hand-graded. *<u>Show all supporting work for this problem</u>.*

[I] *(20 points)* A real battery having emf $\mathcal{E}$ and unknown internal resistance r is hooked up to the nework of resistors shown at right. When the switch is open, the terminal potential across the emf is measured to be $\frac{6}{7}\mathcal{E}$.

What will be the terminal potential across the emf when the switch is closed? Express your answer as a numerical fraction of $\mathcal{E}$.



(1a) loop rule with switch <u>open</u>



$$+\mathcal{E} - I_1 r - I_1 R - I_1 2R = 0$$

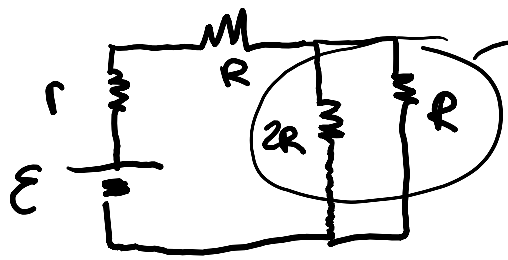
$$\boxed{I_1 = \frac{\mathcal{E}}{3R+r}}$$

(1b) Terminal potential is $V_{Term} \equiv \mathcal{E} - I_1 r$

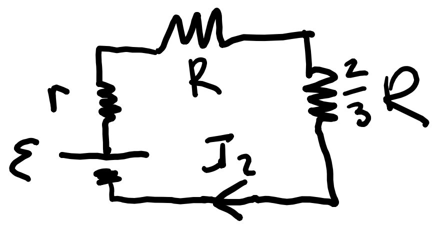
$$\rightarrow \frac{6}{7}\mathcal{E} = \mathcal{E} - \left(\frac{\mathcal{E}}{3R+r}\right) r \rightarrow \frac{6}{7} = \frac{3R+r}{3R+r} - \frac{r}{3R+r} = \frac{3R}{3R+r}$$

$$\rightarrow 6(3R+r) = 7(3R) \rightarrow \boxed{r = R/2}$$

(2a) Use knowledge of r in loop rule with switch <u>closed</u>



$\rightarrow$ in parallel: $R_{eq} = \left(\frac{1}{2R} + \frac{1}{R}\right)^{-1} = \left(\frac{3}{2R}\right)^{-1} = \frac{2}{3}R$



New version of circuit

(2b) Apply loop rule

$$+\mathcal{E} - I_2 r - I_2 R - I_2 \tfrac{2}{3}R = 0$$

$$\mathcal{E} - I_2 R\left(\tfrac{1}{2} + 1 + \tfrac{2}{3}\right) = 0$$

$$\mathcal{E} - I_2 R\left(\tfrac{13}{6}\right) = 0 \qquad \boxed{I_2 = \frac{6\mathcal{E}}{13R}}$$

(2c) $V_{Term} = \mathcal{E} - I_2 r$

$$= \mathcal{E} - \frac{6\mathcal{E}}{13R} \cdot \frac{R}{2} = \mathcal{E} - \frac{3}{13}\mathcal{E} \rightarrow \boxed{V_{Term} = \frac{10}{13}\mathcal{E}}$$

recall: Power $P = I \Delta V$

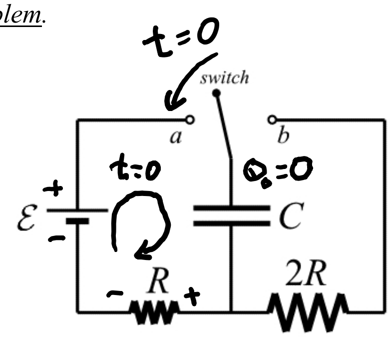
The following problem will be hand-graded. Show all supporting work for this problem.

[II] *(20 points)* In the circuit at right, the capacitor will charge up when the switch is at position a and discharge when the switch is at position b. Suppose that the capacitor is initally uncharged at time $t = 0$ when the switch is moved to position a. The switch is held in that postion until time $t_1 = 2RC$, and is then suddenly switched over to position b at that moment.

(i) What is the power output of the emf immediately before the switch is moved?

(ii) What is the power output of the capacitor immediately after the switch is moved?

In each case, express your answer in terms of $\mathcal{E}$, R, and e (i.e. the natural log of 1).



① Switch at a: charging capacitor with time constant $\tau = RC$

→ current decays: $i(t) = I_0 e^{-t/\tau}$ with $I_0 = \mathcal{E}/R$

② Power provided by emf: $P_\mathcal{E} = I_1 \mathcal{E}$

where $I_1 = i(t_1) = I_0 e^{-t_1/\tau} = \frac{\mathcal{E}}{R} e^{-2} = \frac{\mathcal{E}}{R} \cdot \frac{1}{e^2}$

③ So, $P_\mathcal{E} = \left(\frac{\mathcal{E}}{R} \cdot \frac{1}{e^2}\right) \cdot \mathcal{E} = \boxed{\frac{\mathcal{E}^2}{R} \cdot \frac{1}{e^2}} \approx 0.135\, \mathcal{E}^2/R$

④ at time t_1, capacitor has charge $Q_1 = q(t_1) = Q_f \left[1 - e^{-t_1/\tau}\right]$

using $t_1 = 2RC$ and $Q_f = C\mathcal{E}$, we get:

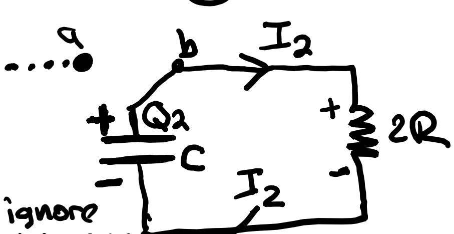
Q_1 = charge before flipping switch

$= C\mathcal{E}\left(1 - e^{-2}\right) = C\mathcal{E}\left(\frac{e^2 - 1}{e^2}\right)$

1 = before
2 = after

charge on cap is conserved when switch is flipped from a to b: $Q_2 = Q_1$

⑤ flip to b: capacitor discharges through 2R



→ loop rule at this moment:

$+\frac{Q_2}{C} - I_2(2R) = 0 \rightarrow \boxed{I_2 = \frac{Q_2}{2RC}}$

not the same current as before!

So, $P_{cap} = I_{cap} \Delta V_{cap} = \frac{Q_2}{2RC} \cdot \frac{Q_2}{C} = \frac{(Q_2/C)^2}{2R}$

$P_{cap} = \frac{\left[\mathcal{E}\left(\frac{e^2-1}{e^2}\right)\right]^2}{2R} = \boxed{\frac{\mathcal{E}^2}{2R}\left(\frac{e^2-1}{e^2}\right)^2} \approx 0.374 \frac{\mathcal{E}^2}{R}$

The following problem will be hand-graded. *Show all supporting work for this problem.*

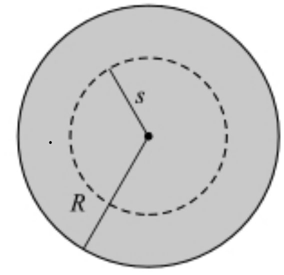
[III] *(20 points)* A long straight wire of radius R has a non-uniform current density throughout its cross-sectional area, given by the expression:

$$J(r) = J_o\left(1 - \frac{r}{R}\right) \quad \text{for} \quad r \le R.$$

where r is the distance from the center of the wire and J_o is a positive constant.

(*i*) Find an expression for the magnetic field inside the wire, at a distance s from the center of the wire (where $s \le R$. Express your answer as a function of s, including the fixed parameters R, J_o, and/or μ_0 as appropriate.

(*ii*) Determine the maximum magnetic field strength that exists anywhere within the wire. Express your answer in terms of R, J_o, and/or μ_0 as appropriate.



end-on view of wire

Ampere's Law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{through}}$

$\rightarrow$ choose circular loop of radius s : $\oint \vec{B}\cdot d\vec{s} \rightarrow B(s)\cdot 2\pi s$

so: find total current through loop of radius s

$$I_{\text{through}} = \int J\, dA \rightarrow \text{use } \underline{\text{area}} = \underline{\text{thin ring}} \begin{bmatrix}\text{radius } r \\ \text{thickness } dr\end{bmatrix}$$

$$= \int_{r=0}^{r=s} \left[J_o\left(1 - \frac{r}{R}\right)\right]\cdot\left[2\pi r\, dr\right]$$

$$= 2\pi J_o \int_0^s \left(r - \frac{r^2}{R}\right) dr = 2\pi J_o\left[\frac{s^2}{2} - \frac{s^3}{3R}\right]$$

Hence, Ampere's Law becomes

$$B(s)\cdot \cancel{2\pi} s = \frac{\mu_0 \cdot \cancel{2\pi} J_o}{6}\left[3s^2 - \frac{2s^3}{R}\right]$$

$$\boxed{B(s) = \frac{\mu_0 J_o}{6}\left[3s - \frac{2s^2}{R}\right]}$$

to find max value, set $\frac{dB}{ds} = 0$ (standard Calc-I technique)

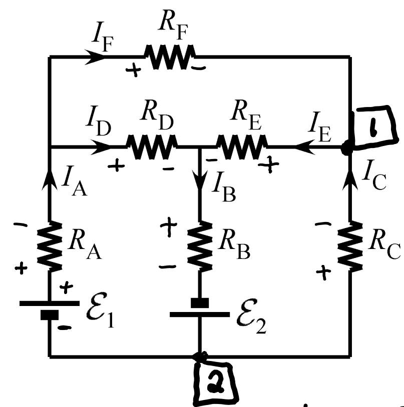
$$\Rightarrow \frac{\mu_0 J_o}{6}\left[3 - \frac{4s}{R}\right] = 0 \rightarrow 3 - \frac{4s}{R} = 0 \rightarrow \boxed{s = \frac{3}{4}R}$$

<u>NOT</u> at surface of wire !!!

so $$B\Big|_{s=\frac{3}{4}R} = \frac{\mu_0 J_o}{6}\left[3\cdot\frac{3R}{4} - 2\frac{\left(\frac{3R}{4}\right)^2}{R}\right]$$

$$= \frac{\mu_0 J_o}{6}\left[\frac{9}{4}R - \frac{9}{8}R\right] = \boxed{\frac{3\mu_0 J_o R}{16}}$$ B_{max}

The next two questions both involve the following situation:
Two ideal emfs and six resistances are arranged in the circuit shown at right. In each branch of the circuit, the assumed direction of current flow is also indicated.



- Assign +/- to each emf, based on its orientation in circuit
- Assign +/- to each resistor, based on direction of current through that resistor

Question value 4 points

(01) Which of the equations below is a valid application of the Loop Rule to the circuit?

(a) $+\mathcal{E}_1 - I_A R_A - I_F R_F - I_C R_C = 0$ (*) one sign error

(b) $-I_D R_D + I_B R_B + \mathcal{E}_2 + \mathcal{E}_1 - I_A R_A = 0$ (*) one sign error

(c) $+\mathcal{E}_2 + I_B R_B + I_D R_D - I_F R_F = 0$

(d) $+I_D R_D + I_E R_E - I_B R_B = 0$

(e) $-I_C R_C - I_F R_F - I_D R_D - I_B R_B + \mathcal{E}_2 = 0$ (*) one sign error

Due to typo, no answers above are correct ~
→ Full credit to answers with only one sign error

Question value 4 points

(02) Which of the equations below is a valid application of the Junction Rule to the circuit?

(a) $\boxed{I_E = I_C + I_F}$ → junction point 1 above: $\Sigma I_{in} = \Sigma I_{out}$ $\boxed{I_F + I_C = I_E}$

(b) $\cancel{I_A + I_D + I_F + I_C = I_B + I_E}$ → there is no such junction!

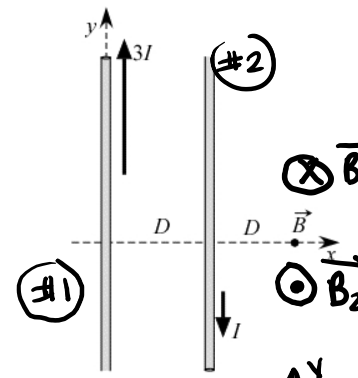
(c) $I_A = I_B + I_C$ → this is junction point 2 but there is a sign error: B flows in, A, C flow out, so $I_B = I_A + I_C$

(d) $\cancel{I_A + I_D = I_C + I_B}$ → there is no such junction!

(e) $\cancel{I_F + I_E = I_D}$ → there is no such junction!

Question value 8 points

(03) Two very long straight wires lie parallel to one another in the xy-plane. One wire lies exactly along the y-axis, carrying a current $3I$ flowing in the positive direction. The other crosses the x-axis at $x = +D$, carrying a current I that flows in the negative direction. What is the magnitude and direction of the magnetic field at $x = +2D$?



(a) $\vec{B} = \frac{\mu_0 I}{4\pi D}\cancel{(+\hat{k})}$

(b) $\boxed{\vec{B} = \frac{\mu_0 I}{4\pi D}(-\hat{k})}$

(c) $\vec{B} = \frac{\mu_0 I}{2\pi D}(+\hat{k})$

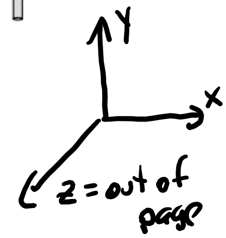
(d) $\vec{B} = \frac{\mu_0 I}{2\pi D}\cancel{(+\hat{k})}$

(e) $\vec{B} = \frac{3\mu_0 I}{2\pi D}(-\hat{k})$

Ampere's Law:

$$B_{wire} = \frac{\mu_0 I}{2\pi r}$$

- Wire 1 — at $x = +2D$,
  $\vec{B}_1$ = into page $= (-\hat{k})$
  $\rightarrow \vec{B}_1 = \frac{\mu_0 (3I)}{2\pi(2D)}(-\hat{k})$



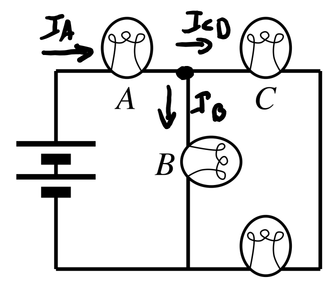
- Wire 2 — at $x = +2D$,
  $\vec{B}_2$ = out of page $= (+\hat{k})$
  $\vec{B}_2 = \frac{\mu_0 I}{2\pi D}(+\hat{k}) \longrightarrow$

$$\vec{B}_{net} = \frac{\mu_0 I}{2\pi D}\left[-\frac{3}{2}\hat{k} + 1\hat{k}\right]$$

$$\boxed{= \frac{\mu_0 I}{4\pi D}(-\hat{k})}$$

Question value 8 points

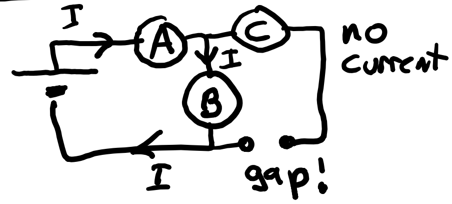
(04) In the diagram at right, all bulbs are identical. What will happen to the brightness of bulbs A and B, if bulb D is removed from its socket?



(a) A and B will both grow dimmer.
(b) A will grow brighter and B will grow dimmer.
(c) Neither bulb will will change brightness
(d) $\boxed{\text{A will grow dimmer and B will grow brighter.}}$
(e) A and B will both grow brighter.

- With D in circuit: All current flows through A, some current flows through B } $\boxed{\text{A is brighter than B}}$ before

- With D removed: no current through C/D branch
  $\rightarrow$ Same current through A and B
  so $\boxed{\text{A and B are equally bright}}$ after



Only answer above that is consistent with these two conditions is: $\boxed{\text{A grows dimmer and B grows brighter}}$

Question value 8 points

(05) A proton (charge $+e$, mass m) and an alpha-particle (charge $+2e$, mass $4m$) are both moving perpendicular to a uniform magnetic field magnitude B, resulting in circular trajectories for both particles in the plane perpendicualr to the field. What are the relative periods of their respective orbits? (Recall that *period* is the time required to complete one lap.)

(a) The periods cannot be compared without knowing the radii of the two orbits.

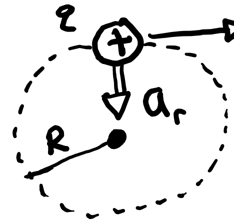
(b) $T_\alpha = T_p/2$

(c) $\boxed{T_\alpha = 2T_p}$

(d) $T_\alpha = T_p$

(e) The periods cannot be compared without knowing the speeds of the two particles.

(Assume $\vec{B}$ out of page)



circular motion: radial accel $a_r = \frac{V^2}{R}$

accel is due to magnetic force (2nd Law)

$F_B = Ma_r$

$qVB = M\frac{V^2}{R} \Rightarrow \boxed{\frac{V}{R} = \frac{qB}{M}}$

→ note speed/radius/period are related:

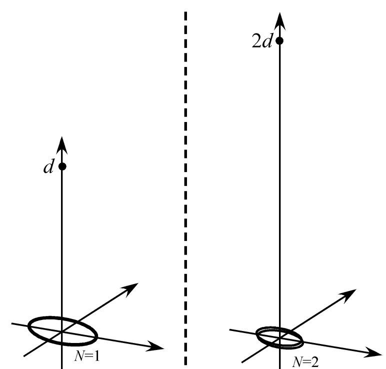
$V = \frac{2\pi R}{T} \rightarrow T = 2\pi\frac{R}{V} = 2\pi\left(\frac{M}{qB}\right)$

so $T_p = 2\pi\left(\frac{m}{eB}\right)$

$T_\alpha = 2\pi\left(\frac{4m}{2eB}\right) = 2\left(2\pi\frac{m}{eB}\right) = \boxed{2T_p}$

Question value 8 points

(06) A wire of length L is formed into a single circular loop. When a current I is driven through the loop, the magnetic field strength at a distance $d = 5L$ along the axis of the loop is B_o. If the <u>same</u> wire is then wound into a 2-loop circular coil, and the <u>same</u> current I is driven through the coil, what will be the magnetic field strength at a distance $2d$ along the axis of the coil? *Hint: in both cases, the distance from the loop is much greater than the size of the loop, so you can use the d >> R limit.*



...not to scale...

(a) $\boxed{B_o/16}$

(b) $B_o/4$

(c) $B_o/2$

(d) $B_o/8$

(e) B_o

On-axis dipole field: $|\vec{B}| \sim \frac{|\vec{\mu}|}{d^3}$

$\Rightarrow$ need to find μ for each loop

- length L = one loop $= 2\pi R_0$

$\rightarrow R_0 = \frac{L}{2\pi}$

$\mu_0 = NIA = NI\pi R_0^2 = 1 \cdot I \cdot \pi \frac{L^2}{4\pi^2} = \frac{IL^2}{4\pi}$

- <u>same</u> length L = <u>two</u> loops $= 2(2\pi R_2)$

$\rightarrow R_2 = \frac{L}{4\pi}$

$\mu_2 = NIA = 2I\pi R_2^2 = 2I\pi\frac{L^2}{16\pi^2} = \frac{IL^2}{8\pi}$

so $\boxed{\mu_2 = \frac{1}{2}\mu_0}$ → half the radius means one-fourth the area, but now there are <u>two</u> loops not one...

③ put it all together:

$B_0 \sim \frac{\mu}{d^3}$

$B_2 \sim \frac{\mu_2}{(2d)^3} = \frac{\frac{1}{2}\mu_0}{2^3 d^3} = \frac{1}{2^4}\frac{\mu_0}{d^3} = \frac{1}{16}\frac{\mu_0}{d^3}$

so $\boxed{B_2 = \frac{B_0}{16}}$