

Solutions

Printed Name

Nine-digit GT ID

signature

Spring 2020

PHYS 2212 G

Test 03

Test Form:

3A

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on your test, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- A standard formula sheet is provided as the cover page for this test. Please remove it from the test before you submit it to the proctor.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

*Fill in bubbles for your Multiple Choice answers darkly and neatly.
If you wish to change an answer, draw a clear "X" through the non-answer!*

1 (a) (b) (c) (d) (e)

2 (a) (b) (c) (d) (e)

3 (a) (b) (c) (d) (e)

4 (a) (b) (c) (d) (e)

5 (a) (b) (c) (d) (e)

6 (a) (b) (c) (d) (e)

The following problem will be hand-graded. Show all supporting work for this problem.

- II (20 points) In a region of space, the electric potential is given by the expression $V(x, y) = Ax^2y - 2Axy^2$, where A is a positively-valued constant. Determine the magnitude and direction of the electric field at position $(x, y) = (+d, -d)$ in the xy -plane. Express the magnitude in terms of A and d . Express the direction as a numerical angle to three-digit precision, measured relative to one of the cardinal axes (i.e. relative to the $+x$, $-x+y$, or $-y$ directions).

Electric field components are found as the gradient of the electric potential:

$$\vec{E}_x = -\frac{\partial V}{\partial x} \text{ (treat } y \text{ as a constant)}$$

$$\vec{E}_x = -[A(2x)y - 2Axy^2] = A[2y^2 - 2xy]$$

$$\vec{E}_y = -\frac{\partial V}{\partial y} \text{ (treat } x \text{ as a constant)}$$

$$\vec{E}_y = -[Ax^2 - 2Ax(2y)] = A[4xy - x^2]$$

Now, evaluate at $x = +d$, $y = -d$

$$\vec{E}_x = A[2(-d)^2 - 2(+d)(-d)] = +4Ad^2$$

$$\vec{E}_y = A[4(+d)(-d) - (+d)^2] = -5Ad^2$$

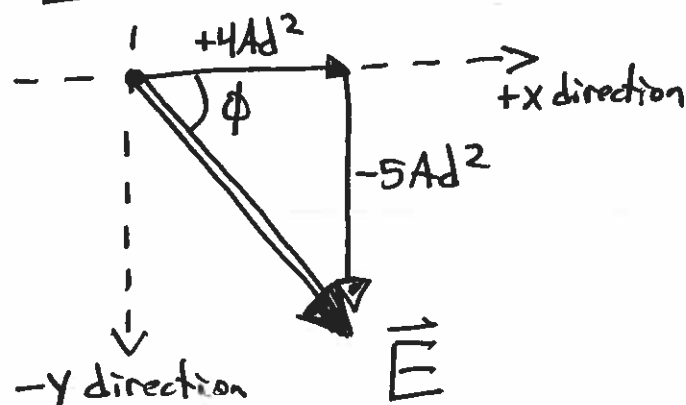
With components known,

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2}$$

$$= Ad^2 \sqrt{(+4)^2 + (-5)^2}$$

$$= Ad^2 \sqrt{16 + 25}$$

$$|\vec{E}| = \sqrt{41} Ad^2$$



direction relative to $+x$ axis is given by

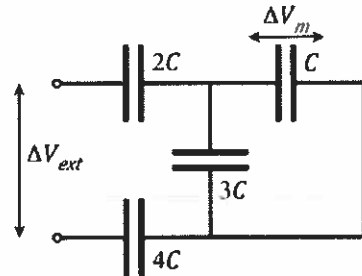
$$\phi = \tan^{-1}\left(\frac{|E_y|}{|E_x|}\right) = \tan^{-1}\left(\frac{5}{4}\right)$$

$$\phi = 51.3^\circ \text{ below positive } x\text{-axis}$$

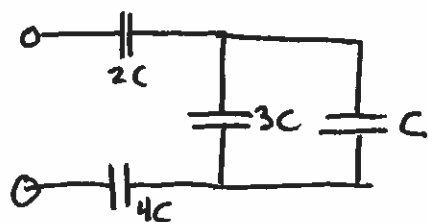
Form 3A

The following problem will be hand-graded. Show all supporting work for this problem.

- [III] (20 points) In the capacitor network at right, C represents an arbitrary value for capacitance. If capacitor C can only sustain a maximum potential difference $\Delta V_m = 10 \text{ V}$, then what maximum external potential ΔV_{ext} can be applied across the network?



Network is equivalent to:

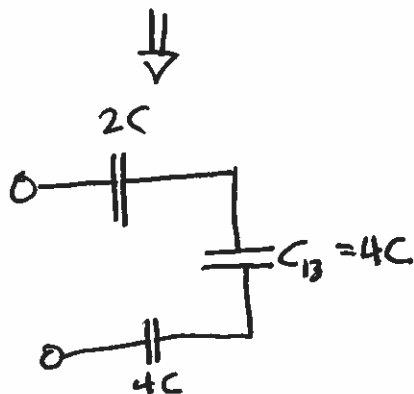


- [1] C and $3C$ are in parallel

- Same ΔV : both are at ΔV_m
- Charges, capacitances add :

$$C_{13} = C_1 + C_3 = 4C$$

$$Q_{13} = C_{13} \Delta V_{13} = (4C)(\Delta V_m)$$



- [2] Remaining capacitors are in series

- Same charge on each

$$Q_{2C} = 4C \Delta V_m$$

$$Q_{13} = 4C \Delta V_m$$

$$Q_{4C} = 4C \Delta V_m$$

- [3] knowing charge on each, we know voltage across each

$$\Delta V_{2C} = \frac{Q_{2C}}{2C} = \boxed{2\Delta V_m}$$

$$\Delta V_{1+3} = \boxed{\Delta V_m} \quad (\text{we already knew this})$$

$$\Delta V_{4C} = \frac{Q_{4C}}{4C} = \boxed{\Delta V_m}$$

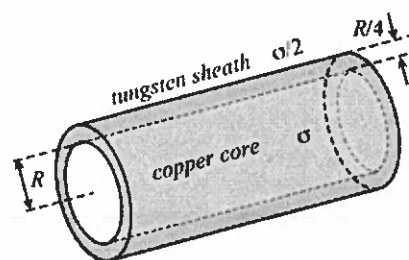
- [4] Total potential across network = sum of series potentials

$$\Delta V_{ext} = \Delta V_{2C} + \Delta V_{1+3} + \Delta V_{4C} = 2\Delta V_m + \Delta V_m + \Delta V_m$$

$$\boxed{\Delta V_{ext} = 4\Delta V_m = 40\text{V}}$$

The following problem will be hand-graded. Show all supporting work for this problem.

- [III] (20 points) A copper wire of radius R and conductivity σ is surrounded by a layer of tungsten (conductivity $\sigma/2$) having thickness $R/4$. (This process, known as *cladding*, is used to prevent corrosion of the inner metal.) A potential difference is applied across the ends of the cladded wire, resulting in some total current I_w in the wire as a whole—some of it flowing through the copper core, and some of it through the tungsten sheath. What *fraction* of the total current flows through each metal? Your answer should be a pair of numerical fractions that add to one (e.g. "core: 8/11 and sheath: 3/11").



Hint: both metals have the same length L and the same potential difference ΔV across them...what other physical quantity is the same in both metals?

Same ΔV and same L implies same E

(assuming internal fields are uniform, $\Delta V = -EL$)

Two Possible solutions:

- [1] Same E , but different σ , different J , different I

$$I_c = J_c A_c = \pi R^2 J_c = \pi R^2 \sigma E$$

$$I_s = J_s A_s = \left[\pi \left(\frac{5}{4}R \right)^2 - \pi R^2 \right] J_s = \pi \left[\frac{25}{16}R^2 - R^2 \right] (\sigma/2) E = \frac{9}{16} \pi R^2 \frac{\sigma}{2} E = \frac{9}{32} \pi R^2 \sigma E$$

$$\Rightarrow \text{comparing, we see } I_s = \frac{9}{32} I_c$$

$$\text{while } I_c = \frac{32}{32} I_c$$

$$\text{so } I_c = \frac{32}{32+9} I_{\text{wire}} = \frac{32}{41} I_{\text{wire}}$$

$$I_s = \frac{9}{32+9} I_{\text{wire}} = \frac{9}{41} I_{\text{wire}}$$

- [2] Same ΔV across each wire

$$I_c R_c = \Delta V = I_s R_s$$

$$I_c \frac{\rho_c L}{A_c} = I_s \frac{\rho_s L}{A_s} \rightarrow \frac{1}{\sigma} \frac{L}{\pi R^2} I_c = \frac{2}{\sigma} \frac{L}{\frac{9}{16} \pi R^2} I_s$$

(using $\rho = \frac{1}{\sigma}$)

$$\downarrow \text{(see area calculations above)}$$

rearranging:

$$I_c = \frac{2 \cdot 16}{9} I_s \quad \text{or} \quad I_s = \frac{9}{32} I_c$$

$$\text{hence, } I_{\text{wire}} = I_s + I_c = \frac{41}{32} I_c$$

$$I_c = \frac{32}{41} I_{\text{wire}}$$

$$I_s = \frac{9}{41} I_{\text{wire}}$$

Question value 8 points

- (01) A parallel-plate capacitor having plates of area A and separation d is connected in series with a second capacitor having the same plate spacing d , but area $2A$. What is the plate area of a single capacitor, also having separation d , that would have a capacitance equal to the two?

(a) $3A$ (b) $(2/3)A$ (c) $(3/2)A$ (d) $(1/2)A$ less than A (e) $4A$

① Before jumping in, recall the rule of thumb:

when in series C_{eq} is less than all individual caps

$$\text{so, if } C_1 = \frac{\epsilon_0 A}{d}, \text{ then } C_{eq} = \frac{\epsilon_0 A_{equiv}}{d}$$

$$\Rightarrow \boxed{\text{we expect } A_{equiv} < A}$$

only two possible answers have $A_{eq} < A$

$$\begin{aligned} \text{② Okay - now do the calc: } \frac{1}{C_{eq}} &= \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{\epsilon_0 A} + \frac{d}{\epsilon_0 2A} \\ &= \frac{2d}{2\epsilon_0 A} + \frac{d}{2\epsilon_0 A} = \frac{3d}{2\epsilon_0 A} \end{aligned}$$

$$\text{hence, } C_{eq} = \frac{2}{3} \epsilon_0 \frac{A}{d} = \frac{\epsilon_0 (\frac{2}{3}A)}{d} \Rightarrow \boxed{A_{equiv} = \frac{2}{3}A}$$

Question value 8 points

- (02) A cylindrical conducting wire lies along the x -axis, extending from $x = 0$ to $x = L$. The wire has an internal electric field that points in the negative x -direction. Describe the potential difference across the wire, the electron current in the wire, and the conventional electric current in the wire.

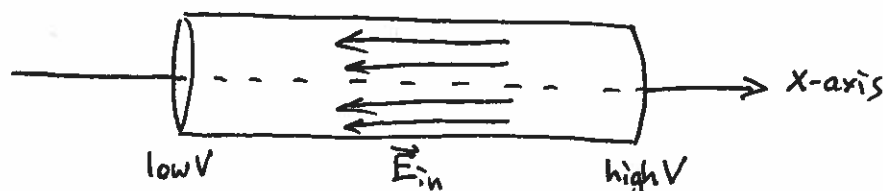
(a) The right side of the wire is at high potential, the electron current is ~~leftward~~, and the conventional current is ~~rightward~~.

(b) The right side of the wire is at high potential, the electron current is rightward, and the conventional current is leftward.

(c) The ~~left~~ side of the wire is at high potential, the electron current is rightward, and the conventional current is ~~rightward~~.

(d) The ~~left~~ side of the wire is at high potential, the electron current is ~~leftward~~, and the conventional current is ~~rightward~~.

(e) The right side of the wire is at high potential, the electron current is rightward, and the conventional current is ~~rightward~~.



① $\vec{E}$ points from high V to low $V \rightarrow$ right side at high potential

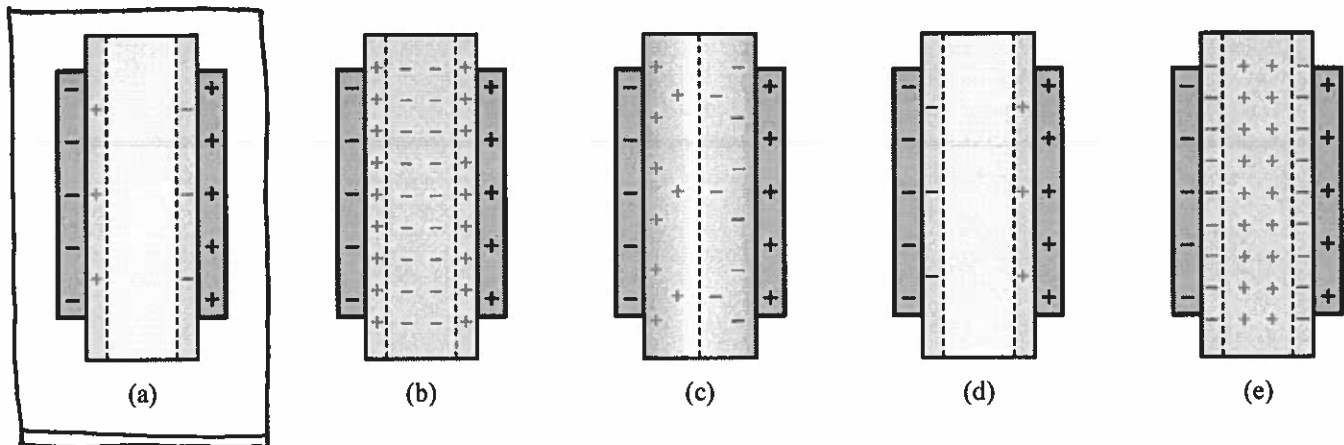
② electron current drifts opposite to $\vec{E} \rightarrow$ i_e is rightward

③ Conventional Electric Current = presumed flow of positive charge

$\rightarrow$ flow is along direction of $\vec{E} \rightarrow$ I is leftward

Question value 8 points

- (03) An insulating dielectric is inserted between the plates of a charged capacitor, and becomes *polarized*. Which of the figures below best characterizes this polarization?



Dielectric polarizes resulting in surface charges, but interior remains neutral
[We demonstrated this in class!]

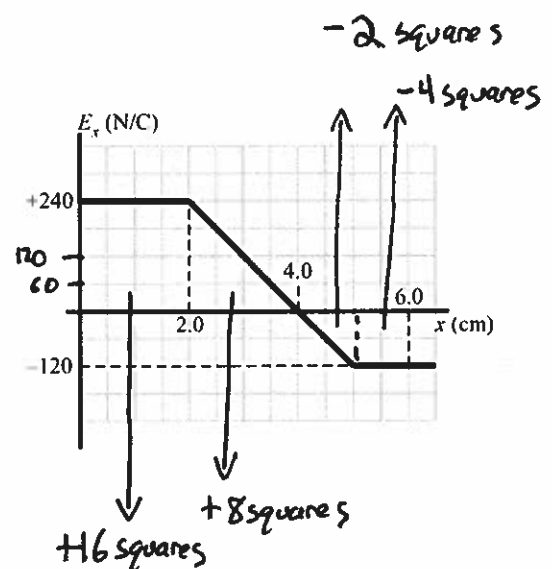
Question value 8 points

- (04) The plot at right displays the x-component of the electric field as a function of position, in the vicinity of the origin. (You may assume that E_y and E_z are both zero in this region.) If the electric potential at the origin is $V = -2.4$ volts, what is the electric potential at $x = 6.0$ cm?

- (a) +3.0 volts
(b) zero volts
(c) -7.8 volts
(d) -2.0 volts
(e) +3.6 volts

$$\Delta V = - \int \vec{E} \cdot d\vec{s}$$

= negative of area under curve



① Looking at graph, total area is

$$+16 + 8 - 2 - 4 = +18 \text{ squares} \rightarrow \Delta V = -(+18)(0.3V)$$

② Each square has height = $60 \text{ N/C} = 60 \text{ V/m}$
width = $0.5 \text{ cm} = 5 \times 10^{-3} \text{ m}$] 1 square = 0.3 V

$$\Delta V = -5.4 \text{ V}$$

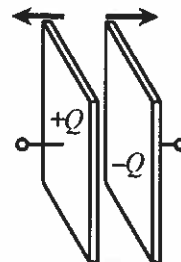
③ Recall that $\Delta V = V_f - V_i = V(6.0 \text{ cm}) - V(0.0 \text{ cm})$
 $-5.4 \text{ V} = V(6.0 \text{ cm}) - (-2.4 \text{ V})$

$$V(6.0 \text{ cm}) = -7.8 \text{ V}$$

Also note: for most of the distance, you are moving in the same direction as $\vec{E}$ so you should expect $V_f < V_i$!

Question value 4 points

- (05) An isolated parallel-plate capacitor has charge Q . The capacitor's plates are initially separated by a distance d . While still isolated, the plates are carefully pulled apart, increasing the separation to $2d$. If the energy initially stored by the capacitor was U_0 , what will be the energy stored after increasing the plate separation?



(a) $U_f = U_0$

(b) $U_f = U_0/4$

(c) $U_f = 2U_0$

(d) $U_f = 4U_0$

(e) $U_f = U_0/2$

① When Q is fixed and C changes:

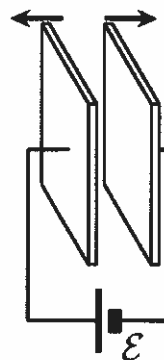
use $U = \frac{Q^2}{2C}$

② if $d \rightarrow 2d$ then $C_0 = \frac{\epsilon_0 A}{d} \rightarrow \frac{\epsilon_0 A}{2d} = \frac{1}{2} \left(\frac{\epsilon_0 A}{d} \right) = \frac{1}{2} C_0$

Hence $U_f = \frac{Q^2}{2C_f} = \frac{Q^2}{2(\frac{1}{2}C_0)} = 2 \left(\frac{Q^2}{2C_0} \right) = \boxed{2U_0}$

Question value 4 points

- (06) A parallel-plate capacitor is charged by attaching it to an emf $\mathcal{E}$. The capacitor's plates are initially separated by a distance d . While still connected to the emf, the plates are carefully pulled apart, increasing the separation to $2d$. If the energy initially stored by the capacitor was U_0 , what will be the energy stored after increasing the plate separation?



(a) $U_f = 4U_0$

(b) $U_f = U_0$

(c) $U_f = U_0/4$

(d) $U_f = U_0/2$

(e) $U_f = 2U_0$

① When $\Delta V = \mathcal{E} = \text{fixed}$, and C changes:

use $U = \frac{1}{2} C \Delta V^2$

② $U_0 = \frac{1}{2} C_0 \mathcal{E}^2$

$U_f = \frac{1}{2} C_f \mathcal{E}^2$ where $C_f = \frac{\epsilon_0 A}{2d} = \frac{1}{2} \frac{\epsilon_0 A}{d} = \frac{1}{2} C_0$

Hence, $U_f = \frac{1}{2} \left(\frac{1}{2} C_0 \right) \mathcal{E}^2 = \frac{1}{2} \left(\frac{1}{2} C_0 \mathcal{E}^2 \right) = \boxed{\frac{1}{2} U_0}$