Solutions

Printed Name

Nine-digit GT ID

signature

Spring 2020 **PHYS 2212 G** **Test 02**

Test Form: **2A**

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on your test, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- A standard formula sheet is provided as the cover page for this test. Please remove it from the test before you submit it to the proctor.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Fill in bubbles for your Multiple Choice answers darkly and neatly.
If you wish to change an answer, draw a clear "X" through the non-answer!

1 ⓐ ⓑ ⓒ ⓓ ⓔ

2 ⓐ ⓑ ⓒ ⓓ ⓔ

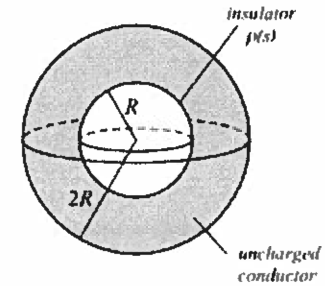
3 ⓐ ⓑ ⓒ ⓓ ⓔ

4 ⓐ ⓑ ⓒ ⓓ ⓔ

5 ⓐ ⓑ ⓒ ⓓ ⓔ

6 ⓐ ⓑ ⓒ ⓓ ⓔ

The following problem will be hand-graded. *Show all supporting work for this problem.*



[I] *(20 points)* A spherical insulator of radius R has a non-uniform but spherically-symmetric charge distributed on it, given by the expression

$$\rho(s) = \rho_0 \frac{s^2}{R^2}$$

(s = "dummy" variable)

where s is the distance from the center of the sphere, and ρ_0 is a positive constant. The insulator is then encased in an $\boxed{\text{uncharged}}$ $\boxed{\text{conducting}}$ spherical shell that extends from radius R to radius $2R$. Important! ditto!

Consider following regions: (*i*) within the inner sphere (i.e. for $r < R$); (*ii*) within the conducting shell (i.e. for $R < r < 2R$); and (*iii*) outside the conducting shell (i.e. for $r > 2R$). In each region find an expression for the magnitude of the electric field as a function of the distance r from the center of the inner sphere, $E(r)$. Express each answer in terms of r, ρ_0, R, and ε_0.

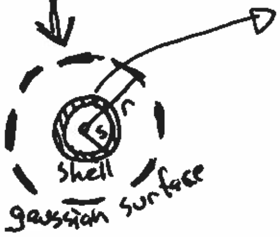
Note that region (ii) is easiest: for $R<r<2R$, you are inside the conductor

$\Rightarrow$ by definition, $\vec{E} \equiv 0$ inside a conductor that is in equilibrium

$$\boxed{E_{ii} \equiv 0}$$

In region (i): choose spherical gaussian surface of radius $r<R$

- Flux through surface is $E_i(r) \cdot 4\pi r^2$
- Charge enclosed by surface is $Q_{in} = \int_{\text{all } s<r} \rho \, dV$
- Volume element dV = thin shells : surface area $4\pi s^2$, thickness ds $\Big]$ $dV = 4\pi s^2 ds$



so $Q_{in} = \int_{s=0}^{s=r} \rho(s) 4\pi s^2 ds = \frac{4\pi\rho_0}{R^2}\int_0^r s^4 ds = \frac{4\pi\rho_0}{R^2} \cdot \frac{r^5}{5}$

Hence, $E_i(r) = \dfrac{\frac{4}{5}\pi\rho_0 \frac{r^5}{R^2}}{4\pi r^2 \varepsilon_0} \rightarrow \boxed{E_i(r) = \frac{\rho_0}{5\varepsilon_0}\frac{r^3}{R^2}}$

In region (iii): again, use gaussian sphere, but now $r>2R$

$E_{iii}(r) \cdot 4\pi r^2 = \frac{1}{\varepsilon_0} Q_{inside}$

but now, $Q_{inside} = Q_{insulator} + \underbrace{Q_{sphere}}_{0 \text{ for uncharged conductor}}$

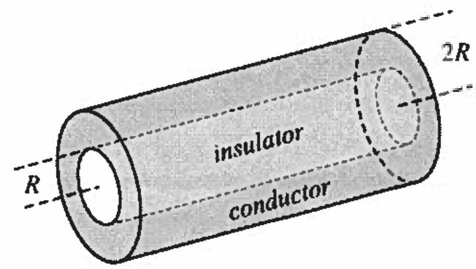
and $Q_{insulator}$ = all charge = $\int_{s=0}^{s=R} \rho \, dV = \frac{4\pi\rho_0}{R^2}\int_0^R s^4 ds = \frac{4}{5}\pi\rho_0 R^3$

all charge is now enclosed!

so $E_{iii}(r) = \dfrac{\frac{4}{5}\pi\rho_0 R^3}{4\pi r^2 \varepsilon_0} \rightarrow \boxed{E_{iii}(r) = \frac{\rho_0}{5\varepsilon_0}\frac{R^3}{r^2}}$

The following problem will be hand-graded. Show all supporting work for this problem.

[II] *(20 points)* A very long insulating rod of radius R has a uniform positive charge density ρ distributed throughout its volume. The insulator is then encased in a conducting cylindrical shell that extends from radius R to radius $2R$. This shell is then given a charge equal to twice the total charge (of the same sign) that was placed on the insulator.



Determine the charge density at all locations in/on the conductor. Keep in mind that there are two surfaces and one volume to consider. Express each answer in terms of ρ and R. two η, one ρ

Hint: start by finding the total charge on a sublength L of the insulator.

① Insulator has cross-sectional area $A_{ins} = \pi R^2$, so a length L of the rod has a volume $V_{ins} = \pi R^2 L$, and hence charge $\boxed{Q_{ins} = \rho V_{ins} = \rho \pi R^2 L}$,

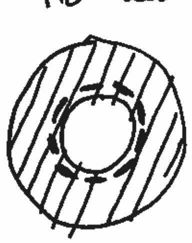
② Since conductor has twice as much charge as rod, $\boxed{Q_{cond} = 2Q_{ins} = 2\rho\pi R^2 L}$

Now — Where does charge reside on conductor in equilibrium?

Answer — NOT on Inside, so $\boxed{\rho \equiv 0 \text{ within conducting sheath}}$

Answer — on inner wall and/or outer wall, as densities η_{in} and η_{out}

To learn about η_{in}, consider gaussian cylinder with radius $R+\varepsilon$ (ε = tiny)



surface is within material of conducting shell (barely)

so $E \equiv 0$ and flux $\Phi \equiv 0$ so $Q_{inside} \equiv 0$

but $Q_{inside} = Q_{insulator} + Q_{inner\ conductor\ wall}$

$$0 = \rho\pi R^2 L + \eta_{inner} \cdot \underbrace{2\pi R L}_{\text{surface area of inner wall of conducting shell}}$$

$\rightarrow \boxed{\eta_{inner} = -\frac{\rho R}{2}}$ neg charge on inner wall

To learn about η_{out}, note that we know total charge and inner charge on conductor

$$Q_{TOT} = Q_{in} + Q_{out} \rightarrow Q_{out} = Q_{TOT} - Q_{in} = 2\rho\pi R^2 L - \left(-\frac{\rho R}{2}\right) \cdot 2\pi R L$$

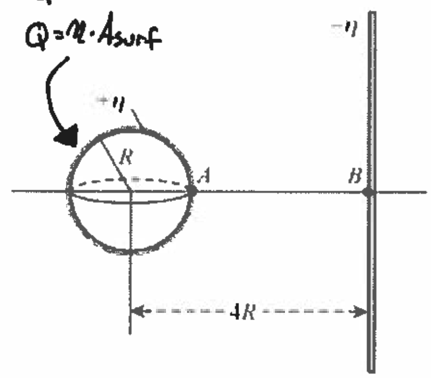
$$\eta_{out} \cdot 2\pi(2R)\cancel{L} = 2\rho\pi R^2 \cancel{L} + \rho\pi R^2 \cancel{L}$$

$$\boxed{\eta_{out} = +\frac{3\rho R}{4}}$$

The following problem will be hand-graded. <u>*Show all supporting work for this problem.*</u>

$Q = \eta \cdot 4\pi R^2$

$Q = \eta \cdot A_{surf}$



|III| *(20 points)* An insulating sphere of radius R has a uniform charge density $+\eta$ on its surface. The sphere is held near an infinite charged sheet having surface density $-\eta$ (yes, the <u>same magnitude</u> η), with the center of the sphere at a distance $4R$ from the charged sheet.

Determine the the electric potential difference between point A on the sphere and point B on the sheet, $\Delta V_{A \to B}$. Express your answer in terms of the parameters R, η, and ε_0. Be sure to include a <u>sign</u> for your answer.

Hint: the principle of Superposition applies here, so think of each object <u>*separately*</u>.

① Potential difference due to sphere:

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$, where $V \to 0$ for $r \to \infty$, and $r =$ <u>distance from center</u>

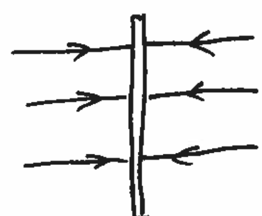
hence, $\Delta V = V_f - V_i = \dfrac{Q}{4\pi\varepsilon_0 r_f} - \dfrac{Q}{4\pi\varepsilon_0 r_i}$ where $Q = +\eta \cdot 4\pi R^2$

$$\Delta V_{sph} = \frac{\eta \cdot 4\pi R^2}{4\pi\varepsilon_0}\left(\frac{1}{r_f} - \frac{1}{r_i}\right) = \frac{\eta R^2}{\varepsilon_0}\left(\frac{1}{4R} - \frac{1}{R}\right) = \frac{\eta R^2}{\varepsilon_0}\left(\frac{-3}{4R}\right)$$

$$\boxed{\Delta V_{sph} = \frac{-3\eta R}{4\varepsilon_0}}$$

moving <u>away</u> from a positively-charged sphere should result in a potential <u>decrease</u>

$-\eta$



② Potential difference due to ∞ charged sheet
= potential difference in a uniform field

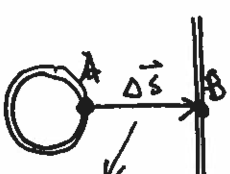
Use $\Delta V = -\vec{E} \cdot \vec{\Delta s}$ in a uniform field

$$= -\left(+\frac{\eta}{2\varepsilon_0}\hat{\imath}\right) \cdot \left(+3R\hat{\imath}\right)$$

$$|\vec{E}| = \frac{\eta}{2\varepsilon_0}$$

$$\boxed{\Delta V_{sheet} = -\frac{3\eta R}{2\varepsilon_0} = \frac{-6\eta R}{4\varepsilon_0}}$$

moving <u>toward</u> a negatively-charged sheet should result in a potential <u>decrease</u>



$\vec{\Delta s} = \langle +3R\hat{\imath} \rangle$

Overall potential difference is the <u>sum</u> of these two terms:

$$\boxed{\Delta V_{A \to B} = \frac{-9\eta R}{4\varepsilon_0}}$$

The next two questions involve the following situation:
A negative source charge is fixed in place, at the position shown. A positive test charge (not shown) is then observed to move from position *i* to position *f*.

Question value 4 points

(01) Describe the work done by the electric field during the displacement, the potential energy change of the system, and the potential difference moved through by the test charge.

(a) The work done by the electric field is ~~**positive**~~; the electric potential energy of the system ~~**decreases**~~; and the test charge moves through a ~~**negative**~~ potential difference.

(b) The work done by the electric field is **negative**; the electric potential energy of the system ~~**decreases**~~; and the test charge moves through a ~~**negative**~~ potential difference.

(c) The work done by the electric field is ~~**positive**~~; the electric potential energy of the system **increases**; and the test charge moves through a **positive** potential difference.

(d) The work done by the electric field is **negative**; the electric potential energy of the system ~~**decreases**~~; and the test charge moves through ~~**zero**~~ potential difference.

(e) The work done by the electric field is **negative**; the electric potential energy of the system **increases**; and the test charge moves through a **positive** potential difference.

① Pos charge displaces away from negative source, but force is toward negative source ∴ $\boxed{\vec{\Delta s} \text{ is opposite to } \vec{F}_{elec}}$ $\boxed{W_{elec} \text{ is negative}}$

② By definition, $\Delta U_{elec} = -W_{elec}$, so if W_{elec} is neg, ΔU is pos. $\rightarrow$ $\boxed{U_e \text{ increases}}$

③ $\Delta V \equiv \frac{\Delta U}{q}$ since q and ΔU are positive, $\boxed{\Delta V = \text{positive}}$

Question value 4 points

(02) Assume that the test charge began at rest at position *i*. What can we say about the external work that may have been done on the system, during the displacement of the test charge?

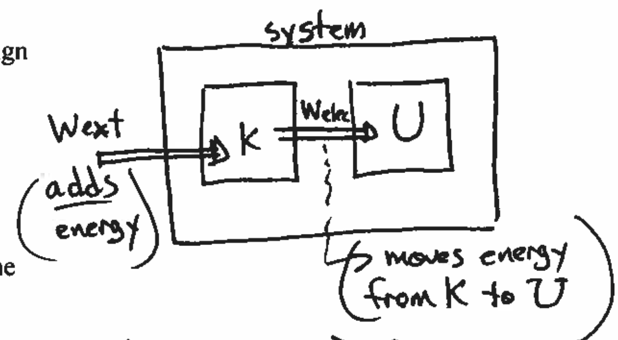
(a) *Non-zero* extrnal work much have been done on the system, but its sign cannot be determined from the information given.

(b) *Negative* external work must have been done on the system.

(c) *Zero* external work must have been done on the system.

(d) *Positive* external work must have been done on the system.

(e) There is not enough information provided to deduce anything about the work that might have been done by an external agent.



① $K_i = 0 \rightarrow K$ cannot decrease below zero, so ΔK must be ≥ 0

② From above, we saw that ΔU must be positive

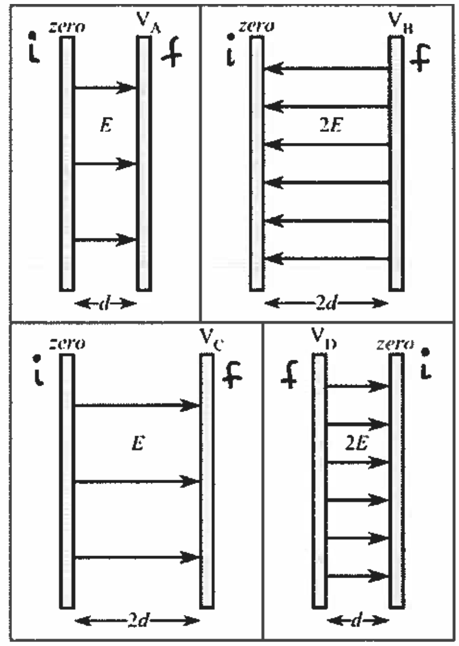
③a If system energy were conserved, $\Delta K + \Delta U = 0$, we would have a contradiction between ① and ②: $E_{system} \neq \text{constant}$

③b Since system energy is not conserved, write $\Delta K + \Delta U = W_{ext}$

ΔK cannot be negative
ΔU is known to be positive
$\boxed{W_{ext} \text{ must be positive}}$

Question value 8 points

(03) In the diagrams at right, four charged capacitors each have one plate specified as being at "zero volts". The field strength and plate separation within each capacitor is indicated. Rank, from highest to lowest, the potentials V_A through V_D at the second plate of each capacitor.



(a) $V_B = V_D > V_A = V_C$

(b) $\boxed{V_B > V_D > V_A > V_C}$

(c) $V_B > V_C = V_D > V_A$

(d) $V_C > V_A > V_D > V_B$

(e) $V_B > V_A > V_D = V_C$

In a uniform field:

$\Delta V = -\vec{E}\cdot\vec{\Delta s}$

$V_f - V_i = -\vec{E}\cdot\vec{\Delta s}$ — $V=0$ at initial plate

so $V_f = -\vec{E}\cdot\vec{\Delta s}$

$A: V_A = -(+E)\cdot(+d) = -Ed$

$B: V_B = -(-2E)\cdot(+2d) = +4Ed$

$C: V_C = -(+E)\cdot(+2d) = -2Ed$

$D: V_D = -(+2E)\cdot(-d) = +2Ed$

• For each, choose sign convention: "to the right" = positive

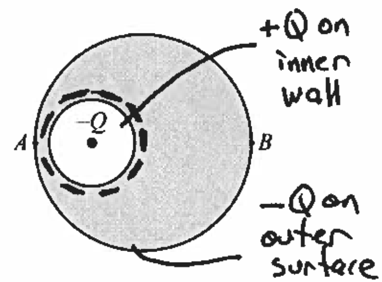
• For each, start at zero plate and displace to non-zero plate

$\Rightarrow$ rank from greatest/most pos. to least/most neg

$$\boxed{V_B > V_D > V_A > V_C}$$

Question value 8 points

(04) An uncharged conducting sphere has an off-center spherical cavity within it. If a point charge $-Q$ is placed at the center of the cavity, what can be said about the distribution of charge on the *outside* surface of the sphere?



(a) There will be a total charge $-Q$, distributed ~~non-uniformly~~ with a higher density at position A (nearest the cavity) and a lower density at position B (furthest from the cavity).

(b) There will be a total charge $-Q$, distributed uniformly around the outer surface of the sphere.

(c) There will be a total charge ~~$+Q$~~, distributed non-uniformly with a lower density at position A (nearest the cavity) and a higher density at position B (furthest from the cavity).

(d) There will be ~~no~~ charge on the outer surface of the conductor, since it is "shielded" from the interior by the conducting material.

(e) There will be a total charge ~~$+Q$~~, distributed uniformly around the outer surface of the sphere.

① Dotted gaussian surface, barely larger than cavity: $E \equiv 0$ on surface (within conductor)
$\rightarrow \Phi_{gs} = 0 \rightarrow Q_{in} = 0 \Rightarrow$ **Charge $+Q$ must be on cavity wall**

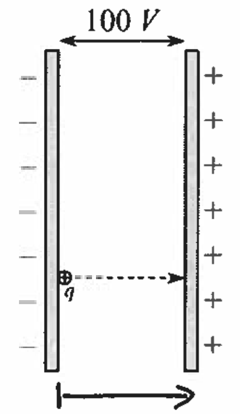
② Since conductor has total charge $= 0$, there must be charge **$-Q$ on outer surface**

③ $E \equiv 0$ inside conductor: outer surface is electrically "shielded" and can't "see inside" $\rightarrow$ it will spread out **uniformly on surface**

Question value 8 points

(05) A capacitor has a potential difference $\Delta V = 100$ volts between ~~its plates.~~ A test charge $q = +1$ C is initially at rest, at the negative plate of the capacitor. An externally applied force moves the charge from the negative plate to the positive plate, in such a way that it arrives at the positive plate with 50 joules of kinetic energy. How much work was done by the applied force during this process?

(ie external work)



(a) + 50 J

(b) − 50 J

(c) 0 J

(d) + 150 J

(e) − 150 J

When external work is involved,

$$\Delta E_{system} = W_{ext}$$

$$\Delta K + \Delta U = W_{ext}$$

$$\Delta K + q\Delta V = W_{ext}$$

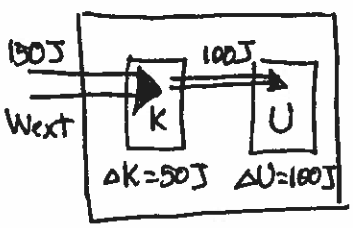
from ⊖ to ⊕

$\Delta V = +100\text{V}$

$= +100 \frac{\text{J}}{\text{C}}$

so: $W_{ext} = (+50\text{J}) + (+1\text{C})(+100\text{V})$

$= 50\text{J} + 100\text{C}\cdot\frac{\text{J}}{\text{C}}$



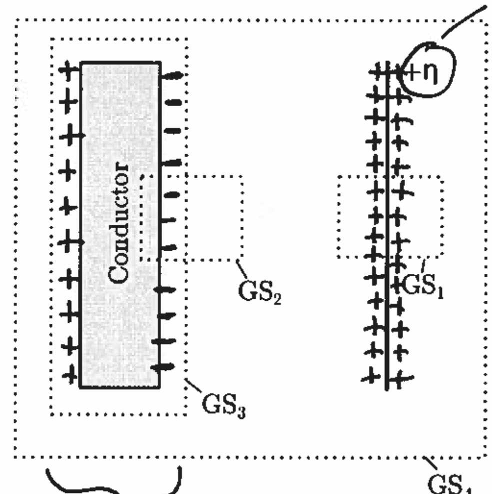
$$\boxed{W_{ext} = +150\text{J}}$$

(Note that any negative answer here is "impossible physics" because it should be clear that both K and U increased)

Question value 8 points

(06) A plane with positive surface charge density $+\eta$ is placed next to an uncharged slab of conducting material, as shown. Rank, from greatest to least, the electric flux Φ_i that passes through each of the Gaussian surfaces (GS_i) shown in the figure.

Positive



(a) $\Phi_4 = \Phi_1 > \Phi_3 = \Phi_2$

(b) $\Phi_2 > \Phi_3 > \Phi_1 > \Phi_4$

(c) $\Phi_4 > \Phi_1 > \Phi_3 > \Phi_2$

(d) $\Phi_3 = \Phi_2 > \Phi_1 > \Phi_4$

(e) $\Phi_4 > \Phi_1 = \Phi_2 > \Phi_3$

① gauss's law:

$$\Phi_{gs} = \frac{1}{\varepsilon_0} Q_{in}$$

so $\boxed{\text{compare enclosed charges!}}$

② Conductor polarizes when placed near charged sheet

Conductor has polarized with ⊖ on right and ⊕ on left

GS 1 contains: small amount of positive charge

GS 2 contains: small amount of negative charge

GS 3 contains: zero net charge → Net charge on conductor is still zero!!

GS 4 contains: large amount of positive charge

large Pos > small pos > zero > small neg

$$\boxed{\Phi_4 > \Phi_1 > \Phi_3 > \Phi_2}$$