Solutions

Printed Name

Nine-digit GT ID

signature

Spring 2020 **PHYS 2212 G** **Test 01**

Test Form:

1A

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on your test, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- A standard formula sheet is provided as the cover page for this test. Please remove it from the test before you submit it to the proctor.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Fill in bubbles for your Multiple Choice answers darkly and neatly.
If you wish to change an answer, draw a clear "X" through the non-answer!

1 ⓐ ⓑ ⓒ ⓓ ⓔ

2 ⓐ ⓑ ⓒ ⓓ ⓔ

3 ⓐ ⓑ ⓒ ⓓ ⓔ

4 ⓐ ⓑ ⓒ ⓓ ⓔ

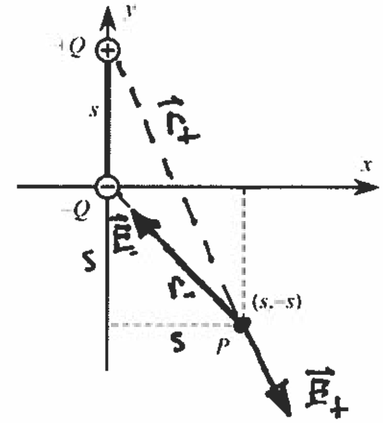
5 ⓐ ⓑ ⓒ ⓓ ⓔ

6 ⓐ ⓑ ⓒ ⓓ ⓔ

The following problem will be hand-graded. *Show all supporting work for this problem.*

[1] *(20 points)* An electric dipole consists of a charge $-Q$ at the origin and a charge $+Q$ located at $y = +s$, as shown at right. What is the direction of the net electric field at point P having coordinates $(x,y) = (+s,-s)$?

Give your answer as a numerical angle, measured relative to one of the cardinal directions ($+x$, $-x$, $+y$, or $-y$), and expressed to three-digit precision.



let $\vec{E}_-$, $\vec{E}_+$ be separate fields due to sources

- For $\vec{E}_-$:

$\hat{r}_-$ = 45° below x-axis

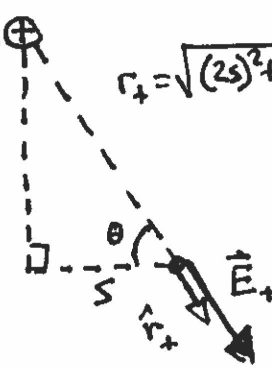
$= +\frac{1}{\sqrt{2}}\hat{\imath} - \frac{1}{\sqrt{2}}\hat{\jmath}$

$r_- = \sqrt{s^2+s^2} = \sqrt{2}s$

so $\vec{E}_- = \frac{k(-Q)}{(\sqrt{2}s)^2}\left(+\frac{1}{\sqrt{2}}\hat{\imath} - \frac{1}{\sqrt{2}}\hat{\jmath}\right)$

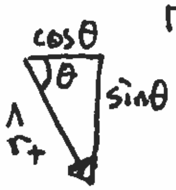
$$\longrightarrow \boxed{\vec{E}_- = \frac{kQ}{s^2}\left[-\frac{1}{2\sqrt{2}}\hat{\imath} + \frac{1}{2\sqrt{2}}\hat{\jmath}\right]}$$

- For $\vec{E}_+$:



$r_+ = \sqrt{(2s)^2+(s)^2} = \sqrt{5}s$

$\hat{r}_+ = \cos\theta\,\hat{\imath} - \sin\theta\,\hat{\jmath}$



$= +\frac{s}{\sqrt{5}s}\hat{\imath} - \frac{2s}{\sqrt{5}s}\hat{\jmath}$

$= +\frac{1}{\sqrt{5}}\hat{\imath} - \frac{2}{\sqrt{5}}\hat{\jmath}$

so $\vec{E}_+ = \frac{k(+Q)}{(\sqrt{5}s)^2}\left[+\frac{1}{\sqrt{5}}\hat{\imath} - \frac{2}{\sqrt{5}}\hat{\jmath}\right]$

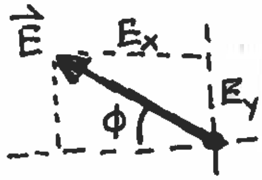
$$\boxed{\vec{E}_+ = \frac{kQ}{s^2}\left[\frac{1}{5\sqrt{5}}\hat{\imath} - \frac{2}{5\sqrt{5}}\hat{\jmath}\right]}$$

Add these to get $\vec{E}_{net}$:

$$\vec{E}_{net} = \frac{kQ}{s^2}\left[-\frac{1}{2\sqrt{2}}\hat{\imath} + \frac{1}{2\sqrt{2}}\hat{\jmath}\right] + \frac{kQ}{s^2}\left[\frac{1}{5\sqrt{5}}\hat{\imath} - \frac{2}{5\sqrt{5}}\hat{\jmath}\right]$$

$$= \frac{kQ}{s^2}\left[-\left(\frac{1}{2\sqrt{2}} - \frac{1}{5\sqrt{5}}\right)\hat{\imath} + \left(\frac{1}{2\sqrt{2}} - \frac{2}{5\sqrt{5}}\right)\hat{\jmath}\right]$$

$$\vec{E}_{net} = \frac{kQ}{s^2}\left[-(0.2641)\hat{\imath} + (+0.1747)\hat{\jmath}\right]$$



direction angle relative to neg x-axis is:

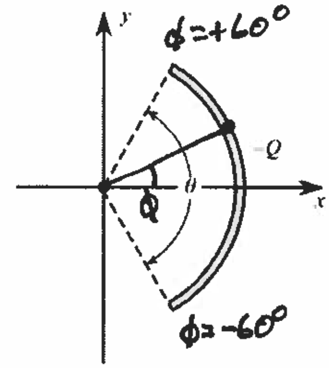
$\phi = \tan^{-1}\left(\frac{|E_y|}{|E_x|}\right)$

$= \tan^{-1}\left(\frac{0.1747}{0.2641}\right)$

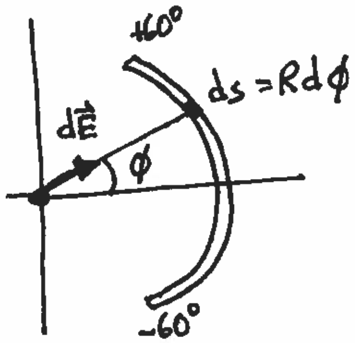
$$\boxed{\phi = 33.5^\circ \text{ above negative x-axis}}$$

The following problem will be hand-graded. *Show all supporting work for this problem.*

[II] *(20 points)* An insulating rod is bent into a circular arc of radius R that subtends a total angle $\theta = 120°$. A **negative** charge of magnitude Q is then distributed uniformly along the arc. What will be the electric field (magnitude and direction) at the center of curvature of the arc? Express your answer in terms of k, Q, and R.



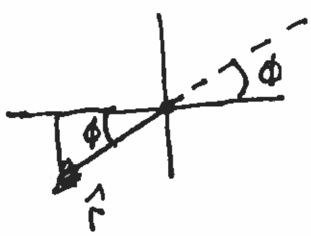
let ϕ = location of one bit of charge, $-60° < \phi < +60°$
[positive ϕ shown in figure below]



charge on segment is
$$\delta Q = \lambda\, ds = \left(\frac{-Q}{L_{TOT}}\right)(R\,d\phi)$$

where L_{TOT} = length of 120° segment = $\frac{1}{3}$ length of 360° segment
$$= \frac{1}{3}(2\pi R) = \frac{2\pi}{3}R$$

so $\boxed{\delta Q = \frac{-3Q}{2\pi}\,d\phi}$



Unit vector at origin, pointing away from δQ, is:
$$\boxed{\hat{r} = -\cos\phi\,\hat{\imath} - \sin\phi\,\hat{\jmath}}$$

But note: charge is symmetric above/below x-axis
$\Rightarrow$ y-components will cancel

so
$$\vec{E} = \int \frac{k\,\delta Q}{r^2}\hat{r} = \int_{-60°}^{+60°} k\left(-\frac{3Q}{2\pi}\right)\frac{d\phi}{R^2}\left[-\cos\phi\,\hat{\imath} - \sin\phi\,\hat{\jmath}\right]$$

$$= +\frac{3kQ}{2\pi R^2}\left[\int \cos\phi\, d\phi\,\hat{\imath} + \int \sin\phi\, d\phi\,\hat{\jmath}\right]$$

$$= +\frac{3kQ}{2\pi R^2}\left[\Big[\sin\phi\Big]_{-60°}^{60°}\hat{\imath} + \underbrace{\cancel{\Big[-\cos\phi\Big]_{-60°}^{60°}}}_{\text{zero}}\hat{\jmath}\right]$$

[see? y-components do cancel]

$$= +\frac{3kQ}{2\pi R^2}\left[\sin 60° - \sin(-60°)\right]\hat{\imath}$$

$$= +\frac{3kQ}{2\pi R^2}\underbrace{\left[\frac{\sqrt{3}}{2} - \left(\frac{-\sqrt{3}}{2}\right)\right]}_{=\sqrt{3}}\hat{\imath}$$

$$\boxed{E_{net} = +\frac{3\sqrt{3}\,kQ}{2\pi R^2}\,\hat{\imath}}$$

ie toward midpoint of negatively-charged arc

The following problem will be hand-graded. *Show all supporting work for this problem.*

[III] *(20 points)* Two parallel charged sheets (seen edge-on) are separated by a distance D. The sheet on the left has a charge dentity $+\eta$, while the sheet on the right has a charge density $+2\eta$. Both sheets have small holes in them, aligned along a common axis. An electron (charge $-e$, mass m) is released from rest at a distance D from the left sheet. It passes through both holes without striking either sheet.

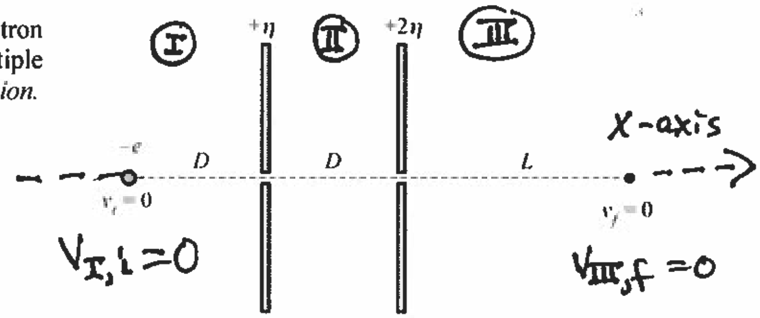
At what distance L beyond the second sheet will the electron come to rest (momentarily)? Express your answer as a multiple of D. *Hint: start by finding the net electric field in each region.*

1D kinematics reminder:

$\overrightarrow{\Delta x} = \vec{v}_i\,\Delta t + 1/2\,\vec{a}\,\Delta t^2$

$\overrightarrow{\Delta v} = \vec{a}\,\Delta t$

$\boxed{v_f^2 = v_i^2 + 2\,\vec{a}\cdot\overrightarrow{\Delta x}}$



Field due to a single positive sheet:

$$\vec{E} = \frac{\eta}{2\varepsilon_0}, \text{ away from sheet}$$

I on far left, both sheets create field to the left

$$\text{so} \quad \vec{E}_I = \left(\frac{-\eta}{2\varepsilon_0} + \frac{-2\eta}{2\varepsilon_0}\right)\hat{i} = \left(\frac{-3\eta}{2\varepsilon_0}\right)\hat{i}$$

II between sheets, η generates rightward field while 2η generates leftward

$$\text{so} \quad \vec{E}_{II} = \left(+\frac{\eta}{2\varepsilon_0} - \frac{2\eta}{2\varepsilon_0}\right)\hat{i} = \left(\frac{-\eta}{2\varepsilon_0}\right)\hat{i}$$

III on far right, both sheets generate rightward field

$$\vec{E}_{III} = \left(+\frac{\eta}{2\varepsilon_0} + \frac{2\eta}{2\varepsilon_0}\right)\hat{i} = \left(\frac{+3\eta}{2\varepsilon_0}\right)\hat{i}$$

Acceleration of electron in each region: $\vec{a} = \frac{\vec{F}}{m} = \frac{-e\vec{E}}{m} = -\left(\frac{e}{m}\right)\vec{E}$

$$\text{so} \quad \underbrace{\vec{a}_I = +\frac{3\eta e}{2m\varepsilon_0}\hat{i} \quad \vec{a}_{II} = +\frac{\eta e}{2m\varepsilon_0}\hat{i}}_{\text{electron speeding up}}, \quad \underbrace{\vec{a}_{III} = -\frac{3\eta e}{2m\varepsilon_0}\hat{i}}_{\text{electron slowing down}}$$

In each region, apply speed equation, noting $V_{I,f} = V_{II,i}$ and $V_{II,f} = V_{III,i}$

(Final speed in one region equals initial speed in next)

$$\text{I:} \quad V_{I,f}^2 = \cancel{V_{I,i}^2}^{\,0} + 2\vec{a}_I \cdot \langle +D \rangle = \frac{3\eta e D}{m\varepsilon_0} = V_{II,i}^2$$

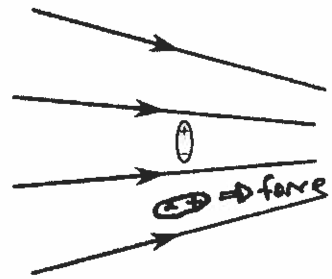
$$\text{II:} \quad V_{II,f}^2 = V_{II,i}^2 + 2\vec{a}_{II} \cdot \langle +D \rangle$$

$$= \frac{3\eta e D}{m\varepsilon_0} + \frac{\eta e D}{m\varepsilon_0} = \frac{4\eta e D}{m\varepsilon_0} = V_{III,i}^2$$

$$\text{III:} \quad V_{III,f}^2 = V_{III,i}^2 + 2\vec{a}_{III} \cdot \langle +L \rangle$$

$$\underbrace{0}_{\text{end at rest}} = \frac{4\eta e D}{m\varepsilon_0} + \left(-\frac{3\eta e L}{m\varepsilon_0}\right) \rightarrow 4D = 3L \rightarrow \boxed{L = \frac{4}{3}D}$$

The next two questions involve the following situation:
An electric dipole is placed as shown in the electric field displayed at right. It it at rest at the moment it is released.



Question value 4 points

(01) What will be the direction of the net force (if any) on the dipole?

(a) The dipole will experience a force directed toward the top of the page.

(b) The dipole will experience a force directed to the right.

(c) The dipole will experience a force directed out of the page.

(d) The dipole will experience zero net force.

(e) The dipole will experience a force directed to the left.

① dipole will almost immediately orient with $\vec{p} \parallel \vec{E}$ (ie to the right)
② In that orientation, dipole will be pulled to right, because ⊕ side of dipole experiences greater force magnitude than ⊖ side

⇒ Dipoles are always pulled toward regions of stronger field magnitude (explains how charged bodies attract neutral bodies)

Question value 4 points

(02) What will be the torque (if any) experienced by the dipole?

(a) The dipole will experience a non-zero torque causing it to rotate around an axis directed out of the page.

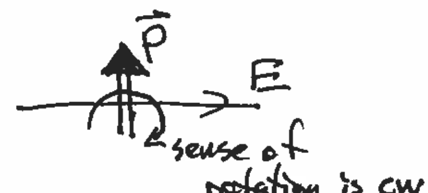
(b) The dipole will experience a non-zero torque causing it to rotate around an axis directed into the page.

(c) The dipole will experience a non-zero torque causing it to rotate around an axis directed to the left.

(d) The dipole will experience zero torque, and will not rotate.

(e) The dipole will experience a non-zero torque causing it to rotate around an axis directed to the right.

Torque on dipole in a field: $\vec{\tau} = \vec{p} \times \vec{E}$

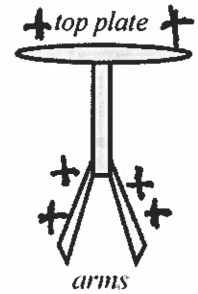


- RHR gives $\vec{\tau}$ directed into page
- note that rotation "around axis into page" results in motion that is clockwise, when viewed from our perspective

Question value 8 points

(03) The figure at right displays a positively-charged electroscope, where the hanging arms push each other apart due to mutual repulsion. Suppose that a positively-charged rod is brought near the top plate (without touching it), then removed, and then a negatively-charged rod is brought near the top plate (also without touching it). How will the hanging leaves of the electroscope respond?

A simple electroscope: all parts are conductors



(a) The arms will push further apart when the positive rod is nearby, and hang closer together when the negative rod is nearby.

(b) The arms will not change their separation when *either* rod is brought nearby.

(c) The arms will hang closer together when *either* rod is brought nearby.

(d) The arms will hang closer together when the positive rod is nearby, and push further apart when the negative rod is nearby.

(e) The arms will push further apart when *either* rod is brought nearby.

- when isolated: some ⊕ on arms, some ⊕ on top plate
- when ⊕ rod is nearby: ⊖ pulled to top plate, extra ⊕ on arms $\boxed{\Rightarrow \text{push farther apart}}$
- when ⊖ rod is nearby: ⊕ pulled to top plate, less ⊕ on arms $\boxed{\Rightarrow \text{hang closer together}}$

Question value 8 points

(04) A solid disk of radius R has a surface charge placed on it *non-uniformly*, with a density function given by:

$$\eta(r) = A\,(r^2 - Rr)$$

Here, A is a positive constant, and r varies between 0 and R. What is the total charge on the disk?

(a) $Q = -\frac{\pi}{6}\,A\,R^4$

(b) $Q = +\frac{\pi}{20}\,A\,R^5$

(c) $Q = +\frac{\pi}{6}\,A\,R^3$

(d) $Q = 0$

(e) $Q = -\frac{\pi}{20}\,A\,R^5$

since $\eta \neq$ constant, we must sum over rings

ring radius $r < R$

ring thickness dr

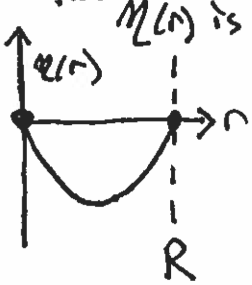
ring area $dA = 2\pi r\,dr$

so $\delta Q = \eta(r)\,dA = A\,(r^2 - Rr)\cdot 2\pi r\,dr$

$$Q_{TOT} = \int_{r=0}^{r=R} 2\pi A\,(r^3 - Rr^2)\,dr = 2\pi A\left[\frac{r^4}{4} - R\frac{r^3}{3}\right]_0^R$$

$$Q_{TOT} = 2\pi A\left[-\frac{R^4}{12}\right] = \boxed{-\frac{\pi A R^4}{6}}$$

note that $\eta(r)$ is negative on range $r \in [0, R]$, so total charge on disk must be negative



Question value 8 points

(05) A 3-g copper penny typically has about $\overbrace{2.6 \times 10^{22}}^{N_0}$ mobile electrons, that are not bound to any particular atom, but instead roam freely throughout the penny. Consider *two* such pennies, separated by a distance $d = 20$ cm. What *fraction* of the mobile electrons in one penny would have to be transferred to the other, in order for the two pennies to feel an attractive force of 450 N (roughly 100 pounds)?

(a) 3.0×10^{-10}

(b) 4.2×10^{-6}

(c) $\boxed{1.1 \times 10^{-8}}$

(d) 6.7×10^{-11}

(e) 1.7×10^{-7}

let N = # pennies transferred

then $Q_1 = +Ne$, $Q_2 = -Ne$

$\Rightarrow$ magnitude of attractive force is $|\vec{F}| = \dfrac{k|Q_1||Q_2|}{d^2}$

$$|\vec{F}| = \frac{kN^2e^2}{d^2}$$

$$N^2 = \frac{Fd^2}{ke^2} \rightarrow N = \sqrt{\frac{Fd^2}{ke^2}} = 2.80 \times 10^{14}$$

fraction of available pennies transferred is: $f = \dfrac{N}{N_0} = \dfrac{2.80 \times 10^{14}}{2.60 \times 10^{22}} \approx \boxed{1.1 \times 10^{-8}}$

Question value 8 points

(06) A test charge $+q$ is use to probe an electric field. When placed at point P, the charge feels a force $\vec{F}$ and sees a field $\vec{E}$. If the charge $+q$ is removed, and a test charge $-2q$ is placed at point P, what will be the force and field?

(a) The force will be $-2\vec{F}$ and the field will be $-2\vec{E}$.

(b) The force will be $-\vec{F}$ and the field will be $2\vec{E}$.

(c) There is no correct answer, because probe charges have to be *positive* for "$\vec{E}$" to have meaning.

(d) The force will be $\vec{F}$ and the field will be $\vec{E}$.

(e) $\boxed{\text{The force will be } -2\vec{F} \text{ and the field will be } \vec{E}.}$

① Field is generated by "something else" (source charge not mentioned in problem)

→ field does not depend on test charge

$\boxed{\vec{E} \text{ does not change}}$ when test charge is replaced

② Given an electric field $\vec{E}$, force on test charge is $q\vec{E}$

$+q$: $\vec{F}_0 = +q\vec{E}$

$-2q$: $\vec{F}_1 = -2q\vec{E} = (-2)(q\vec{E}) = (-2)\vec{F}_0$; $\boxed{\vec{F} \rightarrow -2\vec{F}}$