Solutions

Printed Name

Nine-digit GT ID

signature

Fall 2019 **PHYS 2212 GJ** **Test 04**

Test Form:

4A

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on yourtest, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- The standard formula sheet is on the back of this page, which may be removed from the quiz form if you wish, but it must be submitted.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Fill in bubbles for your Multiple Choice answers darkly and neatly.
If you wish to change an answer, draw a clear "X" through the non-answer!

1 ⓐ ⓑ ⓒ ⓓ ⓔ

2 ⓐ ⓑ ⓒ ⓓ ⓔ

3 ⓐ ⓑ ⓒ ⓓ ⓔ

4 ⓐ ⓑ ⓒ ⓓ ⓔ

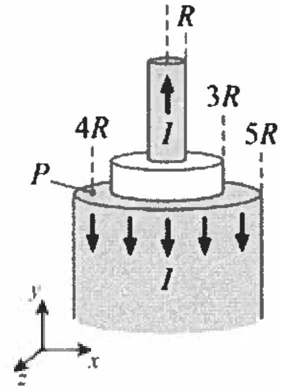
5 ⓐ ⓑ ⓒ ⓓ ⓔ

6 ⓐ ⓑ ⓒ ⓓ ⓔ

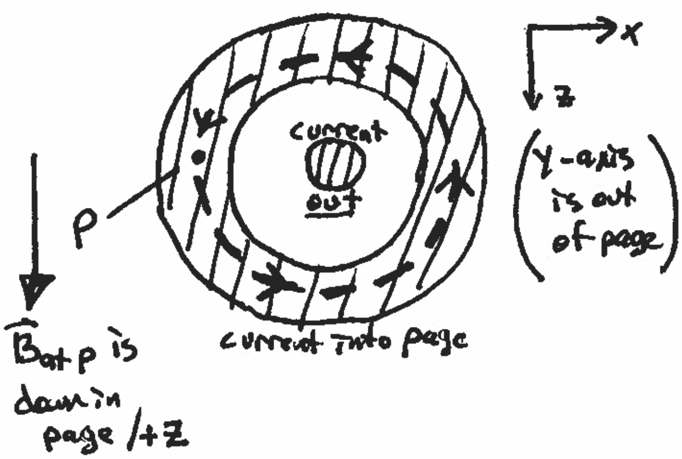
The following problem will be hand-graded. *<u>Show all supporting work for this problem.</u>*

[1] *(20 points)* **Return of the Dreaded Coaxial Cable:** A coaxial cable consists of a solid cylindrical conducting wire of radius R, surrounded by an insulating collar of outer radius $3R$, surrounded in turn by a conducting sheath of outer radius $5R$. A current I flows up the page in the central wire, and an equal return current flows down the page in the sheath. (The return current is distributed <u>uniformly</u> over the cross-sectional area of the sheath.)

Determine magnitude and direction of the magnetic field within the sheath, at the indicated point P, a distance $4R$ from the central axis of the cable. Express the magnitude in terms of μ_0, I, and R. Express the direction as a cartesion unit vector. *(Hint: Ampere's Law will be really handy, here...)*



end-on view of cable:



Choose Amperian loop of radius 4R, passing through point P (dashed curve at left)

- <u>All</u> current I on central wire crosses <u>out</u> of loop
- only part of current I on sheath crosses <u>into</u> loop

→ Choose CCW loop at P

$\boxed{\text{Field } \vec{B} \text{ at P is in } +z \text{ direction}}$

Apply Ampere's Law:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{through}$$

$$|\vec{B}|\cdot 2\pi(4R) = \mu_0\left[+I_{wire} - I_{sheath,\,through\ loop}\right]$$

- $I_{wire} = I$
- $I_{sheath} = J_{sheath}\cdot A_{through\ loop} = \dfrac{I}{A_{sheath}}\cdot A_{loop} = I\,\dfrac{A_{loop}}{A_{sheath}}$

$$\text{so } I_{sheath} = I\,\frac{\pi(4R)^2-\pi(3R)^2}{\pi(5R)^2-\pi(3R)^2} = I\cdot\frac{16-9}{25-9} = \frac{7}{16}I$$

So: <u>Total</u> I passing through our loop is $+I - \frac{7}{16}I = \boxed{+\frac{9}{16}I}$

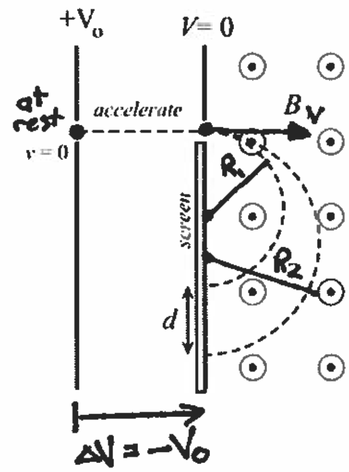
Putting this into Ampere's Law, we get

$$|\vec{B}|\cdot 8\pi R = \mu_0\cdot\frac{9}{16}I \;\longrightarrow\; \boxed{\vec{B} = \frac{9\mu_0 I}{128\pi R}\,\hat{k}}$$

The following problem will be hand-graded. *Show all supporting work for this problem.*

[II] *(20 points)* In a *mass spectrometer*, positively-charged particles are first accelerated from rest through a potential difference $\Delta V = -V_0$, after which they enter a region containing a uniform magnetic field of magnitude B directed perpendicular to their direction of motion, causing them to deflect in a semicircular arc to strike a detector screen. Suppose that first a proton (mass m, charge e) is fired into the device, and then immediately afterward, an alpha-particle (mass $4m$, charge $2e$) is fired in.

What will be the distance d between the proton and alpha-particle impact points on the screen? Express d in terms of the parameters m, e, B, and V_0. (*Hint: start with an energy problem, for the acceleration stage.*)



① Acceleration through potential difference $\Delta V = -V_0$

$\rightarrow$ potential energy change $\Delta U = q\Delta V = -qV_0$

so apply conservation of energy

$$\Delta K + \Delta U = 0 \rightarrow \Delta K = -\Delta U \rightarrow K_f - \cancel{K_i}^{0} = -\Delta U = +qV_0$$

$$\frac{1}{2}mv^2 = qV_0 \rightarrow \boxed{v = \sqrt{\frac{2qV_0}{m}}}$$ Speeds of charges as they enter B-field

② While in field, each particle experiences uniform circular motion

$$\Sigma \vec{F}_{radial} = m\vec{a}_{radial} \rightarrow F_B = m\frac{v^2}{R}$$ where magnetic force has magnitude $|\vec{F}_B| = |q\vec{v}\times\vec{B}| = qvB$

so $$qvB = \frac{mv^2}{R}$$

$\longrightarrow$ solve for R = radius of circular path

$$R = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV_0}{m}}$$ (using speed expression from above)

$$= \sqrt{\frac{2mV_0}{q}}\frac{1}{B}$$

so $$R_{proton} = \frac{1}{B}\sqrt{\frac{2mV_0}{e}}$$ (mass = m, charge = e)

$$R_{alpha} = \frac{1}{B}\sqrt{\frac{2(4m)V_0}{(2e)}}$$ (mass = 4m, charge = 2e)

Both follow half-circles — final separation = differences in diameters

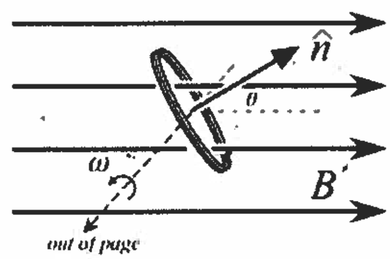
$$d = 2R_{alpha} - 2R_{proton} = \frac{2}{B}\left[\sqrt{\frac{4mV_0}{e}} - \sqrt{\frac{2mV_0}{e}}\right]$$

$$\boxed{d = \frac{2}{B}\sqrt{\frac{mV_0}{e}}\left(2 - \sqrt{2}\right)}$$

The following problem will be hand-graded. *Show all supporting work for this problem.*

[III] *(20 points)* A circular N-turn coil of radius R is placed in a uniform magnetic field of magnitude B. The coil is initially aligned with its normal parallel to the field. The loop is then rotated at constant angular speed ω about an axis perpendicular to the field direction, such that the angle between the coil's normal and the magnetic field is given by $\theta = \omega t$.

Find an expression for the induced emf in the coil, as a function of time.



When loop is oriented with normal $\hat{n}$ making angle θ with $\vec{B}$-field, flux is $\Phi = \int \vec{B}\cdot d\vec{A} \to \vec{B}\cdot\vec{A} = B\pi R^2\cos\theta$

But there are N turns, so N copies of this flux:

$$\Phi_{TOT} = NB\pi R^2\cos\theta = NB\pi R^2\cos(\omega t)$$

(for $t=0$, $\theta=0$, and loop is aligned with $\vec{B}$)

according to Faraday's Law, then:

$$\mathcal{E}_{ind} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left[NB\pi R^2\cos(\omega t)\right]$$

$$= -NB\pi R^2\frac{d}{dt}\left[\cos(\omega t)\right]$$

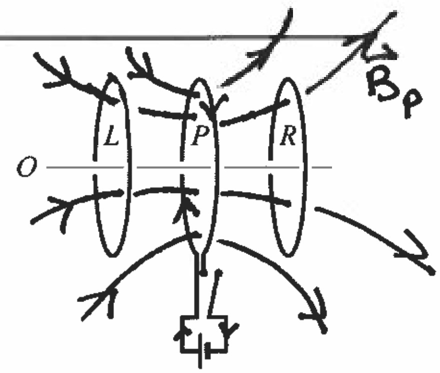
$$= -NB\pi R^2\left[-\omega\sin(\omega t)\right]$$

$$\boxed{\mathcal{E}_{ind} = +\omega NB\pi R^2\sin(\omega t)}$$

oscillates between pos and neg

[When $\mathcal{E}_{ind}$ = positive, induced current is CCW around $\hat{n}$
" " = negative, " " " CW " "]

The next two questionsboth involve the following situation:
Primary loop P is coaxial with nearby loops L (on the left) and R (on the right). Initially, the switch is open and no current flows in the primary. If the switch in the primary circuit closed, current will flow clockwise around loop P, as seen by an observer at position O.



When switch is open, no flux anywhere

Question value 4 points

(01) When the switch is closed, what will be the nature of the induced current in loop R (as seen by an onbserver at O)?

(a) There will be no current in loop R.

(b) There will be a steady, clockwise current in loop R, for as long as the switch remains closed.

(c) There will be a brief, counterclockwise current in loop R, at the moment the switch closes.

(d) There will be a brief, clockwise current in loop R, at the moment the switch closes.

(e) There will be a ~~steady~~, counterclockwise current in loop R, for as long as the switch remains closed.

When switch is closed, rightward flux passes through all loops

Loop R: change from no flux to rightward flux

→ oppose this by self-inducing leftward flux

→ leftward flux would be created by CCW current

→ once current in P stabilizes, $\vec{B}_P \to$ constant

$\Phi_R \to$ constant

$\mathcal{E}_R \to 0$ current dies off

Question value 4 points

(02) When the switch is closed, what will be the nature of the induced current in loop L (as seen by an observer at O)?

(a) There will be a ~~steady~~, counterclockwise current in loop L, for as long as the switch remains closed.

(b) There will be a brief, clockwise current in loop L, at the moment the switch closes.

(c) There will be no current in loop L.

(d) There will be a steady, clockwise current in loop L, for as long as the switch remains closed.

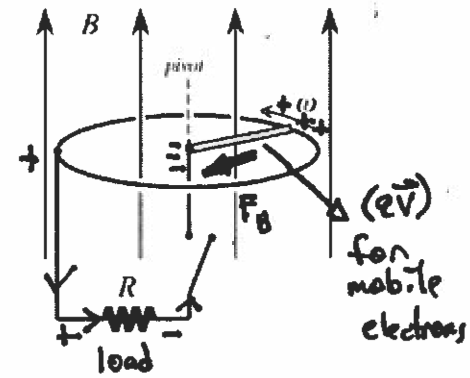
(e) There will be a brief, counterclockwise current in loop L, at the moment the switch closes.

Loop L: change from no flux to rightward flux

→ From that point on, reasoning is the same as for R, above

Question value 8 points

(03) A conducting bar of length L is rotating in a horizontal circle, about a pivot at one end. The other end slides on a frictionless circular conducting rail. The device is placed in a uniform field directed vertically upward. A load resistance is then connected between the axis and the rim as shown. Describe the induced current in the load, after the switch is closed.



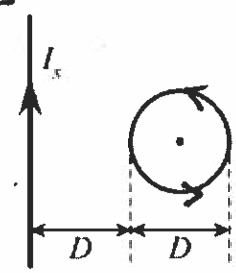
(a) There will be a rightward current that decays away to zero.
(b) There will be no current at all in this situation.
(c) There will be a ~~leftward~~ current that decays away to zero.
(d) There will be a ~~steady~~ rightward current.
(e) There will be a steady leftward current.

As bar rotates ccw (as seen from above), electrons have tangential velocity that is clockwise. $(q\vec{V})$ for electrons is therefore clockwise
Magnetic force on electrons $(q\vec{V})\times\vec{B}$ is toward the pivot
→ pivot accumulates neg charge, rim accumulates pos charge
→ current in resistor flows from pos rim to neg pivot
Current flows rightward through R

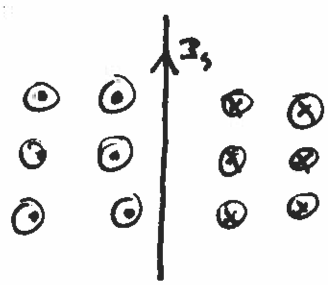
Also: resistor consumes power, I^2R → energy must come from rot. KE of bar
→ **bar slows down, emf and current die off**

Question value 8 points

(04) A long straight wire carrier a current I_S. A 1-turn circular loop of diameter D is placed nearby, with its nearest point a distance D from the wire, as shown. What current I_L in the loop (magnitude and direction) will result in a net magnetic field of zero, at the exact center of the loop?



(a) $I_L = I_S/2$, counter-clockwise
(b) $I_L = I_S/3$, clockwise
(c) $I_L = I_S/3\pi$, clockwise
(d) $I_L = I_S/3\pi$, counter-clockwise
(e) $I_L = I_S/2$, clockwise



$\vec{B}_{wire}$ = into page, at center of circle, with $|\vec{B}_{wire}| = \dfrac{\mu_0 I_s}{2\pi r} = \dfrac{\mu_0 I_s}{2\pi(\frac{3}{2}D)} = \dfrac{\mu_0 I_s}{3\pi D}$

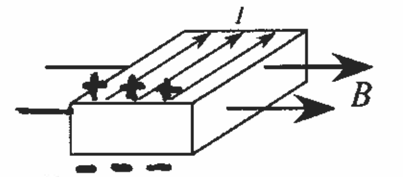
→ require circular loop to generate $\vec{B}_L$ = out of page
→ $\boxed{I_L \text{ must be ccw}}$

Require fields to sum to zero:

$$\vec{B}_{net} = +\frac{\mu_0 I_s}{3\pi D} - \frac{\mu_0 I_L}{2R} = \frac{\mu_0 I_s}{3\pi D} - \frac{\mu_0 I_L}{D} \rightarrow \boxed{I_L = \frac{I_s}{3\pi}}$$

Question value 8 points

(05) A slab of conducting material lies in a rightward-directed magnetic field, and carries a current directed into the page. Which two faces of the slab will develop a Hall voltage across them, and in particular, which of those two faces will be at the <u>higher</u> Hall potential?



(a) The front face will be at a higher Hall potential than the back.

(b) There will be no Hall potential, in this configuration.

(c) The top face will be at a higher Hall potential than the bottom.

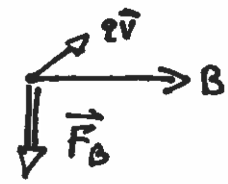
(d) The right face will be at a higher Hall potential than the left.

(e) The bottom face will be at a higher Hall potential than the top.

• electron drift = opposite to I
= out of page

• $q\vec{v}_d = (-e)\vec{v}_d$ = <u>into</u> page

Magnetic force on drifting electrons: $\vec{F}_B = (-e\vec{v}_d)\times\vec{B}$
→ Force is downward

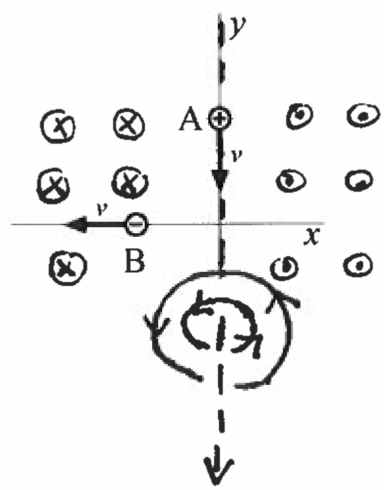


• neg charges accumulate at bottom, <u>positive at top</u>

⇒ $\boxed{\text{Top face is at higher potential}}$

Question value 8 points

(06) Positive point charge A moves with constant velocity along the y-axis. Negative point charge B moves with constant velocity along the x-axis. Both charges are moving with identical speeds v. At the moment shown, what direction is the magnetic force (if any) exerted by A <u>on B</u>?



(a) $\vec{F}_{on\,B}$ is in the <u>negative</u> ~~x-direction~~.

(b) $\vec{F}_{on\,B}$ is zero.

(c) $\vec{F}_{on\,B}$ is in the <u>negative</u> y-direction.

(d) $\boxed{\vec{F}_{on\,B} \text{ is in the positive } y\text{-direction.}}$

(e) $\vec{F}_{on\,B}$ is in the <u>positive</u> ~~x-direction~~.

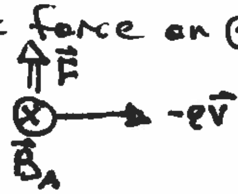
charge B is moving along x-axis — Force on B <u>cannot</u> be along x-axis!

For ⊕ charge moving in $-y$ direction:

Field at ⊖ charge $\vec{B}_A$ is <u>into</u> page

($\vec{B}$ forms circular loops around axis of motion)

Magnetic force on ⊖ charge is $\vec{F} = (-q_B\vec{v})\times\vec{B}_A$



$$\boxed{\vec{F}_{A\,on\,B} = \text{positive } y\text{-direction (up in page)}}$$