Solutions

Printed Name

Nine-digit GT ID

signature

Fall 2019 **PHYS 2212 GJ** **Test 03**

Test Form: **3A**

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on yourtest, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- The standard formula sheet is on the back of this page, which may be removed from the quiz form if you wish, but it must be submitted.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Fill in bubbles for your Multiple Choice answers darkly and neatly.
If you wish to change an answer, draw a clear "X" through the non-answer!

1 ⓐ ⓑ ⓒ ⓓ ⓔ

2 ⓐ ⓑ ⓒ ⓓ ⓔ

3 ⓐ ⓑ ⓒ ⓓ ⓔ

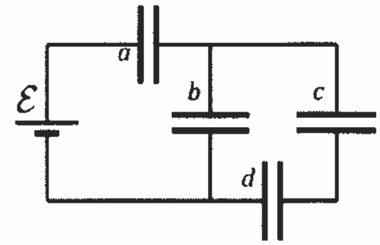
4 ⓐ ⓑ ⓒ ⓓ ⓔ

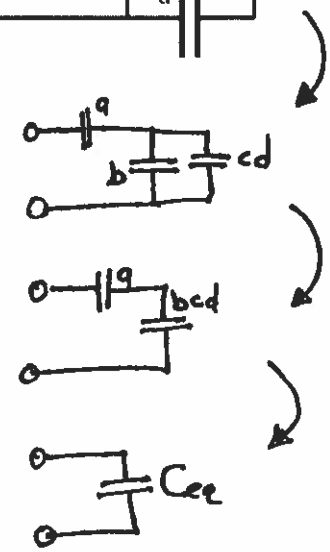
5 ⓐ ⓑ ⓒ ⓓ ⓔ

6 ⓐ ⓑ ⓒ ⓓ ⓔ

The following problem will be hand-graded. *Show all supporting work for this problem.*

[I] *(20 points)* Four identical capacitors C are hooked up to form the network shown at right. Determine the potetnial across capacitor d. Express your answer as a fraction of $\mathcal{E}$.





① Reduce entire network to equivalent:

A] c and d in series

$$\frac{1}{C_{cd}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C} \longrightarrow \boxed{C_{cd} = \frac{C}{2}}$$

B] b and cd in parallel

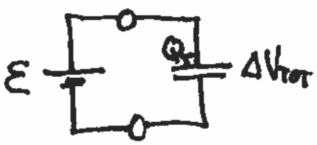
$$C_{bcd} = C + \frac{C}{2} \longrightarrow \boxed{C_{bcd} = \frac{3}{2}C}$$

C] a and bcd in series

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{\frac{3}{2}C} = \frac{3}{3C} + \frac{2}{3C} = \frac{5}{3C}$$

$$\Rightarrow \boxed{C_{eq} = \frac{3}{5}C}$$

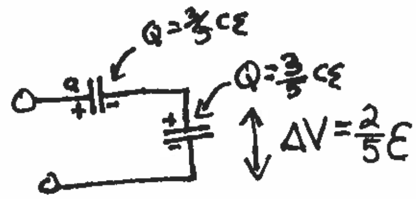
② Analyze equivalent capacitor to find total charge stored



$\Delta V_{TOT} = \mathcal{E}$, so charge stored is

$$Q_{TOT} = C_{eq}\Delta V = \frac{3}{5}C\mathcal{E}$$

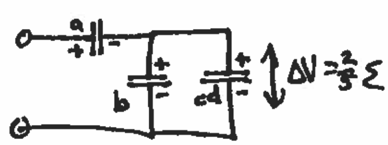
③ Rebuild original circuit



A] a and bcd in series: same stored charge $Q_a = Q_{bcd} = Q_{TOT}$

$\rightarrow$ voltage across bcd is $\Delta V_{bcd} = \dfrac{Q_{bcd}}{C_{bcd}} = \dfrac{3/5\, C\mathcal{E}}{3/2\, C} = \boxed{\frac{2}{5}\mathcal{E}}$

B] b and cd in parallel: same ΔV : $\Delta V_b = \Delta V_{cd} = \Delta V_{bcd} = \frac{2}{5}\mathcal{E}$



$\rightarrow$ charge stored by cd is $Q_{cd} = C_{cd}\Delta V_{cd}$

$$= \frac{C}{2}\cdot\frac{2}{5}\mathcal{E} = \boxed{\frac{1}{5}C\mathcal{E}}$$

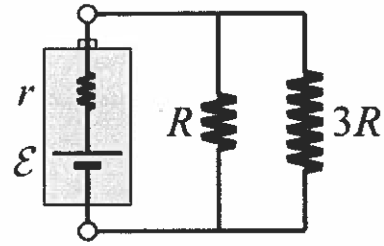
C] c and d in series: same stored charge $Q_c = Q_d = Q_{cd} = \frac{1}{5}C\mathcal{E}$

so, potential across d is

$$\Delta V_d = \frac{Q_d}{C_d} = \frac{\frac{1}{5}C\mathcal{E}}{C} \rightarrow \boxed{\Delta V_d = \frac{\mathcal{E}}{5}}$$

The following problem will be hand-graded. *Show all supporting work for this problem.*

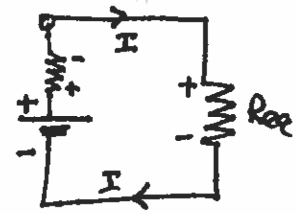
[III] *(20 points)* In the circuit at right, a real battery with unknown internal resistance r is hooked up to the two-resistor network shown. It is found that the net power output of the battery is $P_{out} = \frac{3}{4}\mathcal{E}^2/R$. What is the internal resistance of the battery? Express r as a fraction or multiple of R.



① Reduce resistors to a single equivalent

→ Parallel resistors: $\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{3R} = \frac{3}{3R} + \frac{1}{3R} = \frac{4}{3R}$

$\rightarrow \boxed{R_{eq} = \frac{3}{4}R}$



② Circuit is now a simple loop

→ loop rule gives $+\mathcal{E} - Ir - I R_{eq} = 0$

hence, current is $I = \frac{\mathcal{E}}{r + R_{eq}} = \frac{\mathcal{E}}{r + \frac{3}{4}R}$

③ Note that power out of battery = Power dissipated in R_{eq} (Conservation of Energy)

$(+P_{emf} - P_{\text{internal resistance}}) = \boxed{P_{out} = P_{eq}}$

Power consumed by $R_{eq} = \Delta V_{Eq} I_{eq}$

$= (I R_{eq}) I$ $\qquad P_{eq} = I^2 R_{eq}$

④ Combining expressions:

$P_{eq} = I^2 R_{eq} \longrightarrow \left(\frac{3}{4}\frac{\mathcal{E}^2}{R}\right) = \frac{\mathcal{E}^2}{(r + \frac{3}{4}R)^2} \cdot \frac{3}{4}R$

$\frac{1}{R} = \frac{R}{(r + \frac{3}{4}R)^2}$

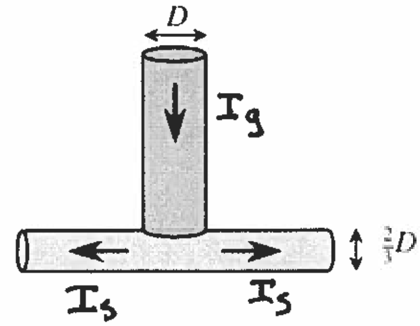
$(r + \frac{3}{4}R)^2 = R^2$

$r + \frac{3}{4}R = R$

$\boxed{r = \frac{1}{4}R}$

The following problem will be hand-graded. *Show all supporting work for this problem.*

[II] *(20 points)* A gold wire having diameter D and conductivity σ carries a current I. The current flows into a T-junction with a silver wire having diameter $\frac{2}{3}D$ and conductivity $\frac{3}{2}\sigma$. Compare the electric field mangitudes in the two wires, by expressing the electric field strength in the silver wire as a numerical multiple or fraction of the electric field strength in the gold wire.



(identical wires means identical currents)

① Junction Rule: $\Sigma I_{in} = \Sigma I_{out}$

$$I_g = I_s + I_s$$

$$\rightarrow \boxed{I_s = \tfrac{1}{2} I_g}$$

② Definition of current density: $J = I/A$ (A = cross-sectional area)

$$\rightarrow I = JA$$

so: $J_s A_s = \frac{1}{2} J_g A_g$ but $A_g = \frac{\pi D^2}{4}$ $A_s = \frac{\pi}{4}\left(\frac{2}{3}D\right)^2 = \frac{\pi D^2}{9}$

$$J_s \cdot \frac{\pi D^2}{9} = \frac{1}{2} J_g \frac{\pi D^2}{4} \rightarrow \boxed{J_s = \frac{9}{8} J_g}$$

③ Microscopic Ohm's Law: $J = \sigma E$

so $\sigma_s E_s = \frac{9}{8} \sigma_g E_g$

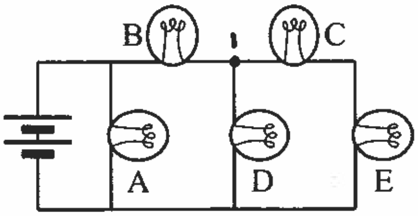
$$\left(\frac{3}{2}\sigma\right) E_s = \frac{9}{8}(\sigma) E_g$$

$$E_s = \frac{2}{3} \cdot \frac{9}{8} E_g$$

$$\boxed{E_s = \frac{3}{4} E_g}$$

Question value 8 points

(01) In the circuit at right, all bulbs are identical. Rank, from greatest to least, the brightness of the five bulbs.



$\boxed{\text{(a)} \quad A > B > D > C = E}$
(b) A = B > C > D = E
(c) A > B = C > D > E
(d) B = C > A = D = E
(e) A > B = D > C = E

① Only A has the full emf across it : $\Delta V_A = \mathcal{E}$ → $\boxed{\text{A } \underline{\text{must}} \text{ be brightest}}$

② At position 1, current splits : I_B must be greater than I_C, I_D, and I_E

→ $\boxed{\text{B is second-brightest}}$

③ C and E together have same potential across them as D : ΔV_D greater than ΔV_C or ΔV_E

→ $\boxed{\text{D is brighter than C or E}}$

$\boxed{A > B > D > C = E}$

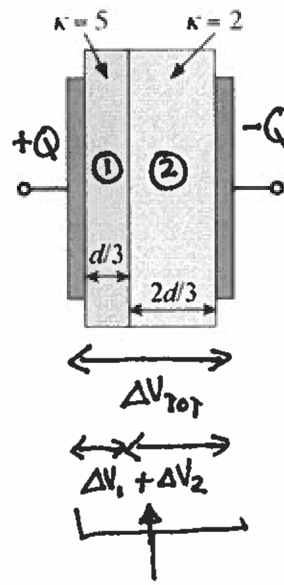
④ C and E are identical, and in series → $\boxed{\text{C and E are equal}}$

(same ΔV) (same I)

Question value 8 points

(02) A capacitor with plate separation d has a vacuum capacitance C_0. A two-layer dielectric is inserted into the capacitor, consisting of a pyrex slab (dielectric constant $\kappa = 5.0$) of thickness $d/3$, and a teflon slab (dielectric constant $\kappa = 2.0$) of thickness $2d/3$. In terms of the vacuum value, what is the capacitance with the dielectric inserted?

Hint: what is the electric field inside each dielectric, when the capacitor is charged?



(a) $2.0\,C_0$
$\boxed{\text{(b)} \quad 2.5\,C_0}$
(c) $3.5\,C_0$
(d) $3.0\,C_0$
(e) $1.5\,C_0$

in vacuum, $E_0 = \dfrac{\eta}{\varepsilon_0} = \dfrac{Q/A}{\varepsilon_0} = \dfrac{Q}{\varepsilon_0 A}$

in dielectric, $E_\kappa = \dfrac{E_0}{\kappa}$ ⇒ reduced field

so, in ①, $\boxed{E_1 = \dfrac{Q}{5\varepsilon_0 A}}$ and in ②, $\boxed{E_2 = \dfrac{Q}{2\varepsilon_0 A}}$

Now, capacitance is found as $C = \dfrac{Q}{\Delta V}$

<u>So</u>: we need to compute expression for <u>total</u> ΔV across capacitor

$\Delta V = \Delta V_1 + \Delta V_2$ but in each dielectric, $\Delta V_i = E_i d_i$

$$\Delta V = \frac{Q}{5\varepsilon_0 A}\cdot\frac{d}{3} + \frac{Q}{2\varepsilon_0 A}\cdot\frac{2d}{3} = \frac{Qd}{\varepsilon_0 A}\left[\frac{1}{15} + \frac{1}{3}\right] = \frac{Qd}{\varepsilon_0 A}\left[\frac{6}{15}\right]$$

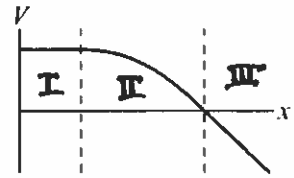
$$= \frac{Qd}{\varepsilon_0 A}\left[\frac{2}{5}\right]$$

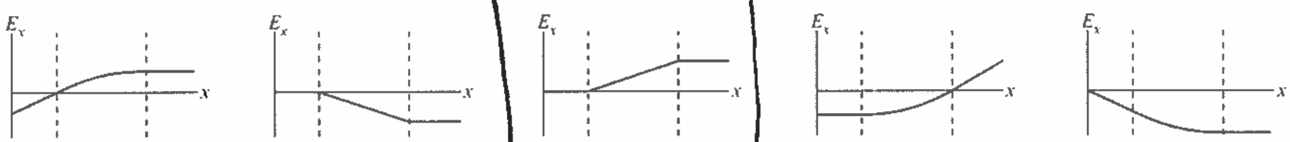
then $C = \dfrac{Q}{\Delta V} = \dfrac{Q}{\frac{Qd}{\varepsilon_0 A}\cdot\frac{2}{5}} = \dfrac{5}{2}\left(\dfrac{\varepsilon_0 A}{d}\right)$ → this is the vacuum capacitance C_0

$$\boxed{C = \frac{5}{2} C_0}$$

Question value 8 points

(03) In a region of space, the electric field depends only on x, with a functional value given by the graph at right. Which of the following plots accurately characterizes the electric field in this region of space?





(a) (b) $\boxed{\text{(c)}}$ (d) (e)

$\vec{E}_x = \langle -\frac{dV}{dx} \rangle$ = negative of slope of $V(x)$

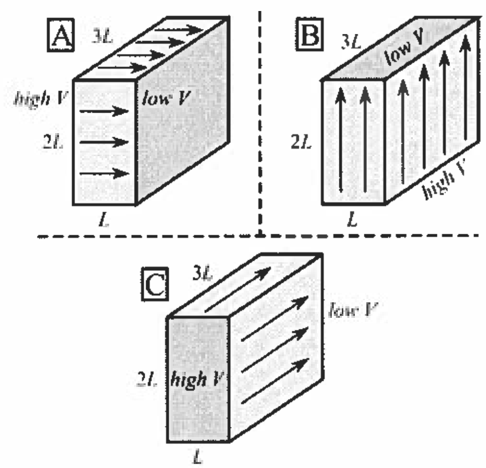
region I : V = constant → slope = 0 → $\boxed{\vec{E}_x = 0}$

region II : slope of V starts zero, becomes negative : $\vec{E}_x$ starts zero, becomes positive

region III : slope of V is constant and negative : $\vec{E}_x$ is constant and positive

Question value 8 points

(04) A slab of material having resistivity ρ has dimensions Lx$2L$x$3L$, as shown at right. The slab can be configured to drive a current left-to-right (diagram A), bottom-to-top (diagram B), or front-to-back (diagram C). Assuming the same potential difference is applied in each case, rank from greatest to least the currents in the three situations.



(a) $I_B > I_A > I_C$
(b) $I_C > I_B > I_A$
(c) $I_A = I_B = I_C$
(d) $I_B > I_C > I_A$
$\boxed{\text{(e) } I_A > I_B > I_C}$

$I = \frac{\Delta V}{R}$ (Ohm's law)

all ΔVs the same

⇓

"rank I, greatest to least" is equivalent to "rank R least to greatest"

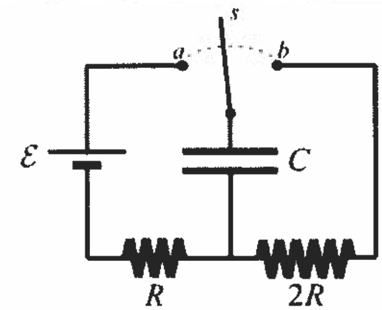
$\boxed{R = \frac{\rho L}{A}}$

so $R_A = \frac{\rho L}{3L \cdot 2L} = \frac{\rho}{6L}$

$R_B = \frac{\rho 2L}{3L \cdot L} = \frac{2\rho}{3L}$

$R_C = \frac{\rho \cdot 3L}{2L \cdot L} = \frac{3\rho}{2L}$

$R_A < R_B < R_C$ so $\boxed{I_A > I_B > I_C}$



The next two questions involve the following situation:

In the circuit at right, capacitor C is initially uncharged with the switch in the neutral position s. At time zero, the switch is moved to position a, charging the capacitor through the left-hand loop. At time $t_1 = 2RC$, the switch is moved to position b, discharging the capacitor through the right-hand loop.

Question value 4 points

(05) What is the charge on the capacitor, at the moment the switch is moved from a to b?

(a) $0.500\, C\mathcal{E}$

(b) $\boxed{0.865\, C\mathcal{E}}$

(c) $0.135\, C\mathcal{E}$

(d) $0.607\, C\mathcal{E}$

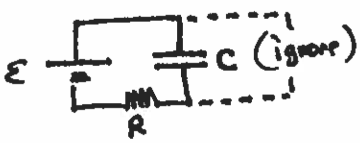
(e) $0.250\, C\mathcal{E}$

Charging circuit: $q(t) = Q_f\left[1 - e^{-t/\tau}\right]$

for $t \to \infty$, $Q \to Q_f$ and $i \to 0$, which means $\Delta V_R \to 0$

$t \to \infty$ loop rule: $+\mathcal{E} - \frac{Q_f}{C} - 0 = 0 \Rightarrow \boxed{Q_f = C\mathcal{E}}$

switch at a:



time constant $\tau = RC$

Now evaluate $q(t)$ at $t = 2RC$

$\rightarrow$ factor $e^{-t/\tau} = e^{-\frac{2RC}{RC}} = e^{-2}$

$q = Q_f\left[1 - e^{-2}\right] = (C\mathcal{E})\left[1 - 0.135\right]$

$\boxed{q = 0.865\, C\mathcal{E}}$

Question value 4 points

(06) Letting Q_s represent the charge on the capacitor when the switch is flipped, what will be the initial current in the discharge circuit, just after the switch is flipped?

(a) $\boxed{I_0 = \frac{Q_S}{2RC}}$

(b) $I_0 = \frac{Q_S}{3R}$

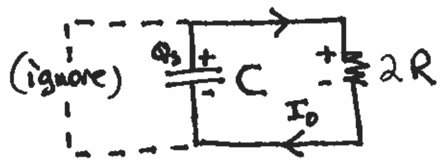
(c) $I_0 = 0$

(d) $I_0 = \frac{Q_S}{3RC}$

(e) $I_0 = \frac{Q_S}{2C}$

Switch at b: charge from above becomes <u>Initial</u> charge Q_S

circuit is a discharge through $2R$



Method ① $t \to 0$ loop rule: $+\frac{Q_S}{C} - I_0(2R) = 0$

$\boxed{I_0 = \frac{Q_S}{2RC}}$

Method ② discharge circuit

$q(t) = Q_S e^{-t/\tau}$ with $\tau = 2RC$

$\hookrightarrow i(t) = -\frac{dQ}{dt} = -\left(\frac{-1}{\tau}\right)Q_S e^{-t/\tau} = \frac{Q_S}{2RC} e^{-t/\tau}$

then $I_0 = i(t=0) = \frac{Q_S}{2RC} e^{0} = \boxed{\frac{Q_S}{2RC}}$