Solutions

Printed Name

Nine-digit GT ID

signature

Fall 2019 **PHYS 2212 GJ** **Test 02**

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.

Test Form:

2A

- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on yourtest, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- The standard formula sheet is on the back of this page, which may be removed from the quiz form if you wish, but it must be submitted.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Fill in bubbles for your Multiple Choice answers darkly and neatly.
If you wish to change an answer, draw a clear "X" through the non-answer!

1 ⓐ ⓑ ⓒ ⓓ ⓔ

2 ⓐ ⓑ ⓒ ⓓ ⓔ

3 ⓐ ⓑ ⓒ ⓓ ⓔ

4 ⓐ ⓑ ⓒ ⓓ ⓔ

5 ⓐ ⓑ ⓒ ⓓ ⓔ

6 ⓐ ⓑ ⓒ ⓓ ⓔ

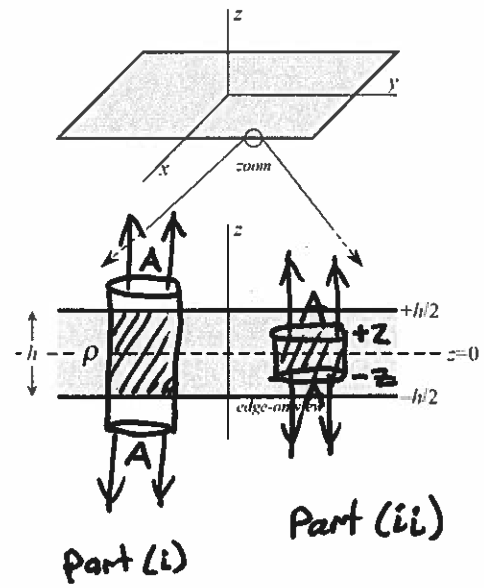
The following problem will be hand-graded. *Show all supporting work for this problem.*

[1] *(20 points)* An insulating sheet of charge having infinite extent lies in the xy-plane. When viewed close-up and edge-on, the sheet appears to be a slab of non-zero thickness h along the z-direction, containing a uniform volume charge density ρ. Assume positive ρ: $\vec{E}$ points away

(i) Determine an expression for the magnitude of the electric field outside the slab, for $z \geq h/2$.

(ii) Determine an expression for the magnitude of the electric field inside the slab, for $z \leq h/2$

In each case, express your answer in terms of ρ, h, z, and ε_0.



(i) choose cylinder with faces of area A, located at $\pm z$, outside sheet (left side)

- Flux through faces are $+EA$ (top) $+EA$ (bottom)
- No flux through side of cylinder: $\vec{E} \perp d\vec{A}$ on sides

$$\Phi_{gs} = 2E(z)A \quad \text{where } E(z) = E_{out}$$

- Charge enclosed is $Q_{in} = \rho V_{in} = \rho\,[A \cdot h]$

$$\text{so} \quad \Phi_{gs} = \frac{Q_{in}}{\varepsilon_0} \longrightarrow 2E(z)\cdot \cancel{A} = \frac{\rho \cancel{A} h}{\varepsilon_0}$$

Area drops out, and is not relevant to final answer

$$\boxed{\underset{\text{(outside)}}{E(z)} = \frac{\rho h}{2\varepsilon_0}} \quad \text{for } |z| \geq \frac{h}{2}$$

does not actually depend on z (uniform field outside sheet)

(ii) Now choose cylinder of height $2z$, centered at $z=0$ (see right side of figure above)

- Flux through cylinder is again $\Phi = 2E_{in}(z)A$
- Charge enclosed is now $Q_{in} = \rho \cdot V_{in} = \rho\,[A \cdot 2z]$

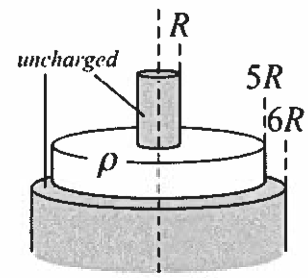
$$\text{so} \quad \Phi_{gs} = \frac{Q_{in}}{\varepsilon_0} \longrightarrow \cancel{2}E_{in}(z)\cdot \cancel{A} = \frac{\rho \cancel{A} \cancel{2} z}{\varepsilon_0}$$

$$\boxed{E_{in}(z) = \frac{\rho z}{\varepsilon_0}}$$

(Note that expression for E_{in} matches E_{out} when $z \to h/2$)

The following problem will be hand-graded. *Show all supporting work for this problem.*

[II] *(20 points)* A coaxial cable consists of a solid cylindrical conducting wire of radius R, surrounded by an insulating collar of outer radius $5R$, surrounded in turn by a conducting sheath of outer radius $6R$. Both conductors are uncharged, but the insulating collar has a uniform charge density ρ throughout its volume.



cutaway view of cable

Determine the surface charge density at each of the following locations:

(i) Outer surface of the inner wire, $\eta(R)$
(ii) Inner surface of the outer sheath, $\eta(5R)$
(iii) Outer surface of the outer sheath, $\eta(6R)$

In each case, express the surface density in terms of ρ and R.

(i) Central wire is uncharged. We also know that there can be no interior charge (wire = conductor in equilibrium) $\Rightarrow$ Hence, there can be no charge on surface, either

$\boxed{\eta(R) \equiv 0}$ on surface of uncharged wire

(ii) Consider gaussian surface = cylinder of radius $5R+\varepsilon$ (ie just outside insulator, and hence, within conducting sheath)

$\Rightarrow$ Sheath = conductor in equilibrium: $E \equiv 0$ everywhere on our GS

so $\Phi_{gs} \equiv 0 \Rightarrow Q_{in} = 0$

but $Q_{in} = Q(\text{on insulator}) + Q(\text{inner wall of sheath})$ (cross-sectional area = annulus)

$= \rho V_{insulator} + \eta A_{sheath} = \rho A_{insulator} \cdot L + \eta \cdot \underbrace{2\pi(5R) \cdot L}_{\text{inner surface area}}$

so $0 = \rho\left[\pi(5R)^2 - \pi(R)^2\right]\cancel{L} + \eta(10\pi R)\cancel{L}$

$\eta \cdot 10\cancel{\pi} R = -\rho \cancel{\pi} 24R^2 \longrightarrow \boxed{\eta(5R) = -\frac{12\rho R}{5}}$

(iii) Total charge on sheath is zero, and charge can only reside on surfaces

$\rightarrow$ for a length L of sheath:

$Q_{TOT} = Q_{inner} + Q_{outer} = \eta(5R) \cdot 2\pi(5R) \cdot L + \eta(6R) \cdot 2\pi(6R) \cdot L$

$\Rightarrow \eta(6R) = -\frac{5}{6}\eta(5R) = -\frac{5}{6}\left(\frac{-12}{5}\rho R\right)$

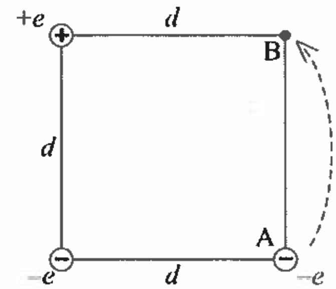
so $\boxed{\eta(6R) = +2\rho R}$

The following problem will be hand-graded. Show all supporting work for this problem.

[III] *(20 points)* A proton and two electrons (all having charge magnitude e) are placed at three corners of a square having sides of length d. An external agent then moves the electron at corner 'A' to corner 'B', while the other two charges are held stationary. The electron that is moved $\boxed{\text{begins and ends at rest.}}$ $K_i = K_f = 0$

(i) Was the work done by the external agent in moving the electron *positive*, *negative*, or *zero*? Justify your answer with a brief explanation, in words.

(ii) Compute an expression for the work done by the external agent. Express your answer symbolically in terms of the electrostatic constant k, e, and d.



(i) electron has moved:

closer to the proton (that means: "falling" toward lower PE)

further from the other electron (that means: "falling away from higher PE)

→ electrons PE has decreased, and hence KE would increase

so: to stop electron at B, we would need to $\boxed{\text{DO NEGATIVE WORK}}$

in order to remove the electrons KE so that it ends at rest

(ii) Energy Principle says: $\Delta E_{system} = W_{external}$

$$\underbrace{\cancel{\Delta K}}_{\text{zero}} + \Delta U = W_{ext}$$

so $W_{ext} = \Delta U = U_f - U_i$ where $U = \sum_{\text{pairs}} \frac{kQ_aQ_b}{r_{ab}}$

$$\Rightarrow U_i = \frac{k(+e)(-e)}{d} + \frac{k(+e)(-e)}{\sqrt{2}d} + \frac{k(-e)(-e)}{d}$$

$$U_f = \frac{k(+e)(-e)}{d} + \frac{k(+e)(-e)}{d} + \frac{k(-e)(-e)}{\sqrt{2}d}$$

$$\text{Thus } W_{ext} = U_f - U_i = \left[\cancel{-\frac{ke^2}{d}} - \frac{ke^2}{d} + \frac{ke^2}{\sqrt{2}d}\right] - \left[\cancel{-\frac{ke^2}{d}} - \frac{ke^2}{\sqrt{2}d} + \frac{ke^2}{d}\right]$$

PE of unmoved charges does not change...

$$W_{ext} = \frac{ke^2}{d}\left[-1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right] = \boxed{-\frac{2ke^2}{d}\left[1 - \frac{1}{\sqrt{2}}\right]}$$

positive, since $\frac{1}{\sqrt{2}} < 1$

See? Overall negative work done by us, to move—and stop—the electron

Question value 8 points

(01) A charged capacitor has plates that are separated by a distance d. The magnitude of the potential difference between the plates is $|\Delta V| = 12\text{V}$. A test charge $q = -500$ nC is placed a distance $d/4$ from the negative plate, and released from rest. How much work is done by the electric field, as the test charge moves all the way to the positive plate?

(a) ~~−4.5 μJ~~

(b) +6.0 μJ

(c) $\boxed{+4.5\ \mu\text{J}}$

(d) ~~Zero work is done by the field, because electric fields are conservative.~~

(e) ~~−6.0 μJ~~

a negative charge is falling from vicinity of neg plate to pos plate

→ losing PE and gaining KE

⇒ $\boxed{\text{Positive work is being done}}$

[this leaves only two viable answers]

→ charge falls 3/4 distance, so it falls through $|\Delta V| = \frac{3}{4}|\Delta V_{TOT}| = \frac{3}{4}(12\text{V}) = \boxed{9\text{V}}$

→ falling from neg plate to pos plate $\Delta V = +9\text{V}$

so $\Delta U = q\Delta V = (-500\text{nC})(+9\text{V}) = -4500\text{nJ} = -4.5\mu\text{J}$

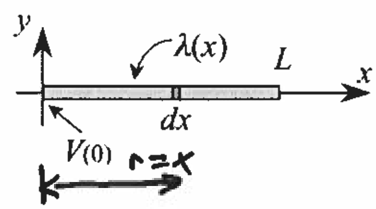
then, since $\Delta U = -W_{field}$, we get $\boxed{W_{field} = -\Delta U = +4.5\mu\text{J}}$

Question value 8 points

(02) A rod of length L lies along the x-axis, extending from $x = 0$ to $x = L$. A total charge Q is distributed non-uniformly along the rod, with a linear density given by the expresssion

$$\lambda(x) = \frac{3Q}{L^3} \cdot x^2 \quad \text{for } 0 \le x \le L.$$

] then $dQ = \lambda(x)dx$



Relative to an assumed value of $V = 0$ at infinity, what is the electric potential at the origin?

(a) $V = \frac{kQ}{L}$

(b) $V = \frac{2kQ}{L}$

(c) $\boxed{V = \frac{3kQ}{2L}}$

(d) $V = \frac{2kQ}{3L}$

(e) $V = \frac{3kQ}{L}$

for subsegment dx: $dV = \frac{k\,dQ}{r}$ where $r = x$

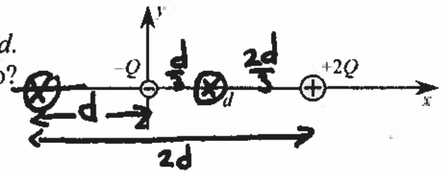
so $V = \int_{x=0}^{x=L} \frac{k(\lambda(x)dx)}{x} = \int_0^L \frac{k\frac{3Q}{L^3}x^2 dx}{x}$

$$V = \frac{3kQ}{L^3}\int_0^L x\,dx = \frac{3kQ}{L^3}\left[\frac{L^2}{2}\right]$$

$$\boxed{V = \frac{3kQ}{2L}}$$

Question value 8 points

(03) A point charge $-Q$ is placed at the origin, and a point charge $+2Q$ is placed at $x = +d$. What is the electric field at a point on the x-axis where the electric potential is zero?



(a) $\vec{E} = -\frac{3kQ}{2d^2}\hat{\imath}$

(b) $\vec{E} = +\frac{3kQ}{2d^2}\hat{\imath}$

(c) $\vec{E} = 0$

(d) $\vec{E} = -\frac{kQ}{2d^2}\hat{\imath}$

(e) $\boxed{\vec{E} = +\frac{kQ}{2d^2}\hat{\imath}}$

Location(s) where $V=0$

$0 = V_+ + V_- = \frac{k(+2Q)}{r_+} + \frac{k(-Q)}{r_-}$ (r_+, r_- = distances from +/- charges)

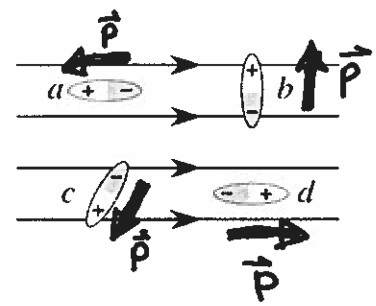
$\frac{2}{r_+} - \frac{1}{r_-} = 0$ $\boxed{r_+ = 2r_-}$ → true for $\boxed{x = -d \text{ and } x = +\frac{d}{3}}$

try $\vec{E}(\frac{d}{3}) = \frac{-kQ}{(d/3)^2}\hat{\imath} + \frac{-2kQ}{(2d/3)^2}\hat{\imath} = -\frac{27}{2}\frac{kQ}{d^2}\hat{\imath}$] not present in answers

try $\vec{E}(-d) = \underbrace{\frac{+kQ}{d^2}\hat{\imath}}_{\text{(toward neg)}} + \underbrace{\frac{-2kQ}{(2d)^2}\hat{\imath}}_{\text{(away from pos)}} = \boxed{+\frac{kQ}{2d^2}\hat{\imath}}$

Question value 8 points

(04) Four identical dipoles are placed in a uniform electric field, with varying orientations as shown at right. Rank in order, from the most positive to most negative, the potential energies U_a to U_d for those four dipoles.



(a) $U_b > U_c > U_d = U_a$

(b) All four dipoles have identically zero potential energy.

(c) $\boxed{U_a > U_c > U_b > U_d}$

(d) $U_d > U_c > U_b > U_a$

(e) $U_a = U_d > U_c > U_b$

dipole in uniform field:

FROM FORMULA SHEET

$U = -\vec{p}\cdot\vec{E}$

where $\vec{p}$ = vector from ⊖ side of dipole to ⊕ side

U = max positive when $\vec{p}$ is opposite to $\vec{E}$ (case a)

U = max negative when $\vec{p}$ is parallel to $\vec{E}$ (case d)

] enough info to find answer

$U_a > U_c > U_b > U_d$

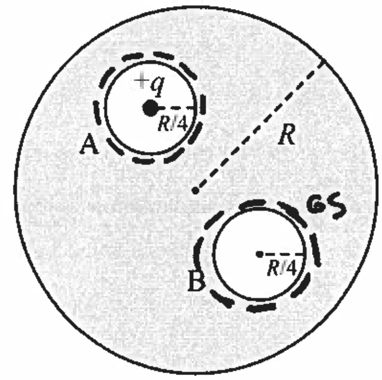
$U = 0$ when $\vec{p} \perp \vec{E}$ (case b)

[Note that only one choice has U_a = max, U_d = min]

U = positive, not max when $\vec{p}$ is "somewhat" opposite to $\vec{E}$ (case c)

The next two questions involve the following situation:

A conducting sphere of radius R has two off-center spherical cavities, both of radius $R/4$, within it. Cavity 'A' has a point charge $+q$ placed at its center. Cavity 'B' is empty.



→ NO charge placed on the conductor itself

Question value 4 points

(05) What is the surface charge density found on the inner wall of Cavity 'B'?

(a) $\boxed{\eta_B = 0}$

(b) $\eta_B = +q/4\pi R^2$

(c) $\eta_B = -4q/\pi R^2$

(d) $\eta_B = -q/4\pi R^2$

(e) $\eta_B = +4q/\pi R^2$

choose GS = sphere that contains cavity B

→ surface is within conductor material, so $\vec{E} \equiv 0$

hence, $\Phi_{gs} \equiv 0$ so that $\boxed{Q_{in} \text{ must} \equiv 0}$

We conclude: There is no charge on the wall of cavity B

Question value 4 points

(06) What is the surface charge density found on the outer surface of the sphere?

(a) $\eta_{out} = -4q/\pi R^2$

(b) $\eta_{out} = 0$

(c) $\boxed{\eta_{out} = +q/4\pi R^2}$

(d) $\eta_{out} = -q/4\pi R^2$

(e) $\eta_{out} = +4q/\pi R^2$

choose GS = sphere that contains cavity A

→ again $\vec{E} = 0$ on surface, so $\Phi_{gs} \equiv 0$

And hence, $Q_{in} = 0$

BUT NOW: $Q_{in} = (+q)_{center} + Q_{wall\,A}$

→ We conclude $Q_{wall\,A} = -q$

Now: total charge on conductor is zero

so $Q_{TOT} = 0 = Q_{outer} + Q_{wall\,A} + Q_{wall\,B}$

$0 = Q_{outer} + (-q) + 0$ (from prior problem)

$Q_{outer} = +q \rightarrow \boxed{\eta_{outer} = \dfrac{+q}{4\pi R^2}}$