Solutions

Printed Name

Nine-digit GT ID

signature

Fall 2019 **PHYS 2212 GJ** **Test 01**

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.

Test Form:

1A

- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on yourtest, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- The standard formula sheet is on the back of this page, which may be removed from the quiz form if you wish, but it must be submitted.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Fill in bubbles for your Multiple Choice answers darkly and neatly.
If you wish to change an answer, draw a clear "X" through the non-answer!

1 ⓐ ⓑ ⓒ ⓓ ⓔ

2 ⓐ ⓑ ⓒ ⓓ ⓔ

3 ⓐ ⓑ ⓒ ⓓ ⓔ

4 ⓐ ⓑ ⓒ ⓓ ⓔ

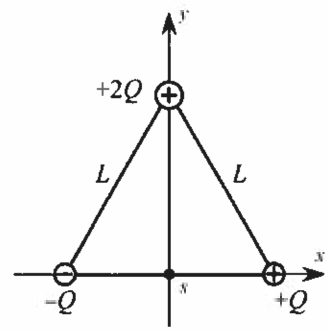
5 ⓐ ⓑ ⓒ ⓓ ⓔ

6 ⓐ ⓑ ⓒ ⓓ ⓔ

The following problem will be hand-graded. *Show all supporting work for this problem.*

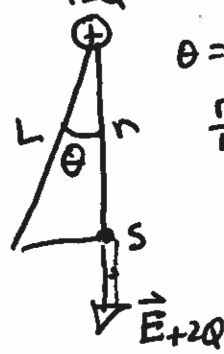
[1] *(20 points)* Three charges, $+Q$, $-Q$, and $+2Q$ are arranged at the corners of an equilateral triangle having sides of length L, as shown at right. Determine the electric field at the origin (point s in the figure).

Express the magnitude of $\vec{E}$ in terms of the parameters k, Q, and L. Express the direction of $\vec{E}$ as a numerical angle relative to one of the coordinate axes. Give the angle to three-digit precision.



$\sin 60° = \sqrt{3}/2 = \cos 30°$
$\cos 60° = 1/2 = \sin 30°$

① Field due to +2Q is straight down] $-\hat{j}$
② Field due to +Q is leftward
③ Field due to −Q is also leftward] $-\hat{i}$



$\theta = 30°$

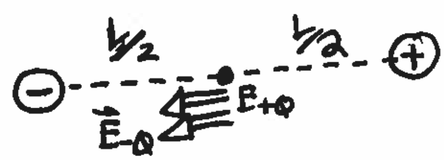
$\frac{r}{L} = \cos 30° = \frac{\sqrt{3}}{2}$

$r = \frac{\sqrt{3}}{2} L$

$r^2 = \frac{3}{4} L^2$

① Net y-component of field at s:

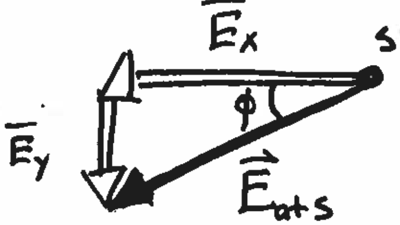
$$\vec{E}_y = \vec{E}_{+2Q} = \frac{k(2Q)}{\frac{3}{4}L^2}(-\hat{j}) = \boxed{\frac{8kQ}{3L^2}(-\hat{j})}$$



②+③ Net x-component of field at s:

$$\vec{E}_x = \vec{E}_{+Q} + \vec{E}_{-Q}$$
$$= \frac{kQ}{(L/2)^2}(-\hat{i}) + \frac{kQ}{(L/2)^2}(-\hat{i}) = \boxed{\frac{8kQ}{L^2}(-\hat{i})}$$

Now, consider overall $\vec{E}$-field vector



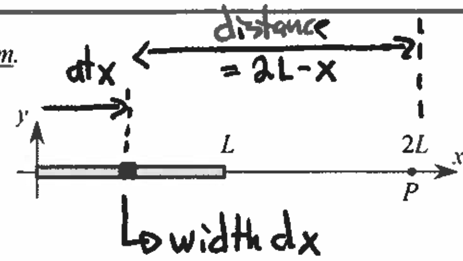
$$|\vec{E}| = \sqrt{E_x^2 + E_y^2} = \frac{8kQ}{L^2}\sqrt{1^2 + (1/3)^2}$$

$$\boxed{|\vec{E}| = \frac{8kQ}{L^2}\sqrt{10/9}}$$

direction is given by angle ϕ

$$\tan\phi = \frac{|\vec{E}_y|}{|\vec{E}_x|} = \frac{8kQ/3L^2}{8kQ/L^2} = \frac{1}{3} \Rightarrow \boxed{\phi = 18.4° \text{ below negative x-axis}}$$

The following problem will be hand-graded. *Show all supporting work for this problem.*



[II] *(20 points)* An insulating rod lies along the x axis, extending from $x = 0$ to $x = L$. A non-uniform distribution of charge is placed on the rod, given by the linear density function:

$$\lambda(x) = \frac{A}{(2L-x)^2} \quad \text{for} \quad 0 \le x \le L$$

Here, A is a positive constant—and consequently, λ is positively-valued for all x in the given range.

(*i*) Determine the total charge Q on the rod. Express your answer in terms of the parameters A and L.

(*ii*) Find an expression for the electric field on the positive x-axis at a distance $2L$ from the origin (point P in the figure). Express your answer in terms of k, A, and L—and remember that $\vec{E}$ is a vector, and must have a direction specified as well as a magnitude!

For a segment of width dx located at position x: "pointlike" charge dQ is:

$$dQ = \lambda(x)\,dx = \frac{A}{(2L-x)^2}\,dx$$

(i) Total charge is thus $Q = \int dQ = \int_{x=0}^{x=L} \frac{A}{(2L-x)^2}\,dx$

use: $\int u^n\,du = \frac{u^{n+1}}{n+1} + C$

let $u = (2L-x)$, $du = -dx$ $\rightarrow$ $Q = -A\int_0^L (2L-x)^{-2}(-dx)$

$$= -A\left[\frac{(2L-x)^{-1}}{-1}\right]_0^L = -A\left[\frac{-1}{L} - \frac{-1}{2L}\right]$$

$$\boxed{Q = \frac{A}{2L}}$$

(ii) Field due to dQ at point P: $d\vec{E} = \frac{k\,dQ}{r^2}\hat{r}$

$$= \frac{k\left[\frac{A}{(2L-x)^2}dx\right]}{(2L-x)^2}(+\hat{\imath})$$

So, $\vec{E}_{Net} = \int d\vec{E}$

$$= \int_{x=0}^{x=L} \frac{kA\,dx}{(2L-x)^4}\hat{\imath} = -kA\hat{\imath}\int_0^L \frac{-dx}{(2L-x)^4} = -kA\hat{\imath}\int_0^L (2L-x)^{-4}(-dx)$$

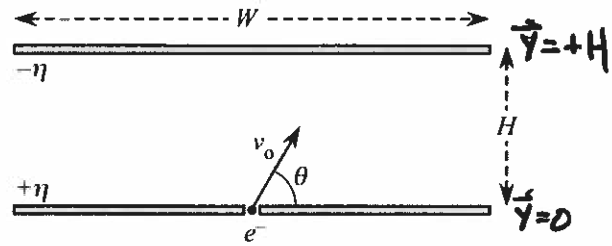
$$= -kA\hat{\imath}\left[\frac{(2L-x)^{-3}}{-3}\right]_0^L$$

$$= -kA\hat{\imath}\left[\frac{-1}{3L^3} - \frac{-1}{3(2L)^3}\right] = +\frac{kA\hat{\imath}}{L^3}\left[\frac{1}{3} - \frac{1}{24}\right]$$

$$\boxed{\vec{E} = +\frac{7kA}{24L^3}\hat{\imath}}$$

The following problem will be hand-graded. Show all supporting work for this problem.

[III] *(20 points)* An electron (mass m, charge $-e$) is fired into a charged parallel-plate capacitor, through a small hole in one plate. The capacitor plates are separated by a height H, and have width W. The hole is in the very middle of the positive plate, and the electron enters moving at an angle θ above the horizontal. If the charge densities on the two plates are $\pm\eta$, what maximum initial speed can the electron have without hitting the negative plate?



Express your answer sybolically in terms of the paramaters provided here, plust the permittivity constant ε_0.

① Field between plates of capacitor $\vec{E}_{net} = \vec{E}_{+\eta} + \vec{E}_{-\eta}$

$$= \left(+\frac{\eta}{2\varepsilon_0}\hat{j}\right) + \left(\frac{+\eta}{2\varepsilon_0}\hat{j}\right) = +\frac{\eta}{\varepsilon_0}\hat{j}$$

② Force on electron within capacitor:

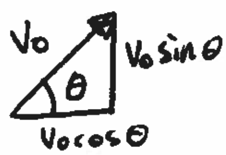
$$\vec{F} = q\vec{E} = (-e)\left(+\frac{\eta}{\varepsilon_0}\hat{j}\right) = -\frac{e\eta}{\varepsilon_0}\hat{j}$$

③ acceleration of electron, from 2nd Law:

$$\vec{a} = \vec{F}/m = \left(-\frac{e\eta}{m\varepsilon_0}\hat{j}\right) = \left(-a_y\hat{j}\right)$$

"a_y" represents magnitude of accel!

Now, apply kinematics: inside capacitor, at moment of entry:



$\vec{V}_{ix} = +V_0\cos\theta$ = constant (no horizontal accel)

→ actually irrelevant to question about vertical motion

$\vec{V}_{iy} = \langle +V_0\sin\theta\rangle$ not constant because $a_y \neq 0$

"barely reach top plate" require $\vec{V}_{yf} = 0$ at moment $\vec{y} = +H$

↳ $\vec{V}_{yf} = \vec{V}_{yi} + \vec{a}\,\Delta t \rightarrow 0 = +V_0\sin\theta + (-a_y)\Delta t$

downward-directed vector!

$$\boxed{\Delta t = V_0\sin\theta / a_y}$$

then displacement equation gives:

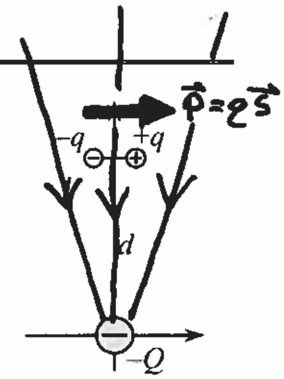
$$\vec{y}_f = \vec{y}_i + \vec{V}_{iy}\Delta t + \tfrac{1}{2}\vec{a}\,\Delta t^2$$

$$+H = 0 + V_0\sin\theta\left(\frac{V_0\sin\theta}{a_y}\right) + \tfrac{1}{2}(-a_y)\frac{V_0^2\sin^2\theta}{a_y^2} = \frac{V_0^2\sin^2\theta}{2a_y}$$

$$\text{so } V_0 = \sqrt{\frac{2a_yH}{\sin^2\theta}} = \boxed{\sqrt{\frac{2e\eta H}{m\varepsilon_0\sin^2\theta}}}$$

Question value 8 points

(01) A point charge $-Q$ is fixed at the origin. A small electric dipole (size s, charges $\pm q$) is placed on the positive y-axis as shown at right. When the dipole is released, what interaction(s) will it experience?



- (a) It will experience a torque directed into the page, followed by a force directed straight down.
- (b) It will experience a torque directed to the right.
- (c) It will experience a force directed to the left.
- (d) It will experience a torque directed out of the page, followed by a force directed straight down.
- (e) It will experience a torque directed to the left, followed by a force directed to the right.

① Field $\vec{E}$ at dipole is downward

Dipole moment $\vec{p}=q\vec{s}$ is to the right

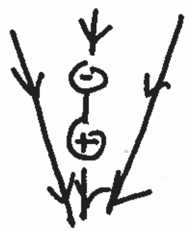
• Torque is $\vec{\tau}=\vec{p}\times\vec{E}$ $=\boxed{\text{into page}}$ or $\boxed{\text{clockwise}}$

② Once dipole has rotated cw:

+q feels stronger field

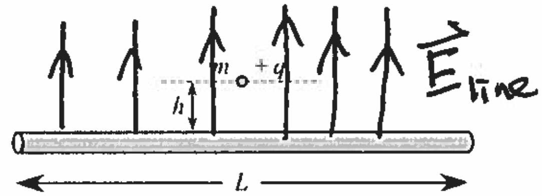
−q feels weaker field

⇒ dipole feels $\boxed{\text{net force down}}$ (attracted to −Q)



Question value 8 points

(02) A conducting wire of length $L = 24$ cm is laid out horizontally. A small insulating bead of mass $m = 0.33$ grams is charged to $q = +11$ nC and placed at a height $h = 1.0$ cm above the midpoint of the wire. How much charge would have to be placed on the wire in order to suspend the bead in midair, balancing its weight electrostatically? (Hint: charged placed on a conducting wire will distribute itself uniformly along the wire.)



- (a) $Q = +160$ nC
- (b) $Q = +3.2$ nC
- (c) $Q = +1.2$ μC
- (d) $Q = +17$ nC
- (e) $Q = +39$ nC

charge Q uniformly distributed on rod of length L

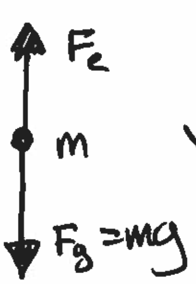
① linear charge density is $\lambda = Q/L$

② Field nearby ≈ "infinite line charge field"

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 h}\text{, upward}$$

③ Electrostatic force on charge is

$$\vec{F}_e = \frac{\lambda q}{2\pi\varepsilon_0 h}\text{, upward}$$



④ Now require charge to be in equilibrium:

$$\langle +F_e\rangle + \langle -mg\rangle = 0 \rightarrow \frac{\lambda q}{2\pi\varepsilon_0 h} = mg = \frac{(Q/L)q}{2\pi\varepsilon_0 h}$$

$$\boxed{Q = \frac{2\pi\varepsilon_0 mghL}{q}} = 3.9\times10^{-8}\,C = 39.2\,nC$$

Question value 8 points

(03) Three insulating rods—rubber R, plastic P, and glass G—are rubbed with three separate burlap rags. It is found afterwards that R attracts P, P attracts G, and G attracts R. We conclude from this that

(a) all three rods must have the same kind of charge.

(b) at least two of the rods must be *uncharged.*

(c) only one of the rods can be charged.

(d) exactly one of the rods must be *uncharged.*

(e) all three rods must have different kinds of charge.

① No two objects repel ⇒ none of the three have the same charge
so: at least one body is neutral

② Two neutral bodies would not interact at all
so: there must be less than two neutral bodies

Conclusion: there is exactly $N=1$ uncharged (neutral) body

Question value 8 points

(04) A spherical rubber balloon has a radius R. A total charge Q is then uniformly distributed over the balloon's entire surface. The balloon is then carefully inflated from radius R to radius $1.5\,R$, without losing any charge in the process. The balloon is then sloppily deflated to radius $0.5\,R$, losing *half* its total charge in the process. Rank, from greatest to least, the surface charge densities on the balloon: initial (η_o); when inflated (η_{inf}); and when deflated (η_{def}).

(a) $\eta_o > \eta_{inf} > \eta_{def}$

(b) $\eta_{inf} > \eta_{def} > \eta_o$

(c) $\eta_{def} > \eta_o = \eta_{inf}$

(d) $\eta_{def} > \eta_o > \eta_{inf}$

(e) $\eta_{inf} = \eta_o > \eta_{def}$

$$\eta = \frac{\text{charge}}{\text{surface area}} = \frac{Q}{4\pi r^2} \text{ for spheres}$$

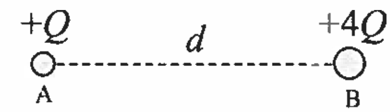
so $$\eta_o = \frac{Q}{4\pi R^2}$$

$$\eta_{inf} = \frac{Q}{4\pi(3R/2)^2} = \frac{Q}{9\pi R^2}$$

$$\eta_{def} = \frac{\tfrac{1}{2}Q}{4\pi(R/2)^2} = \frac{2Q}{4\pi R^2}$$

so $$\boxed{\eta_{def} > \eta_o > \eta_{inf}}$$

The next two questions involve the following situation:
Two charged spheres, $+Q$ and $+4Q$, are separated by a fixed distance d. The size of both spheres is much smaller than their separation distance.



Question value 4 points
(05) Compare the magnitude of the coulomb force acting on A to the magnitude of the coulomb force acting on B.

(a) $|\vec{F}_{on\,A}| = 4|\vec{F}_{on\,B}|$
(b) $|\vec{F}_{on\,A}| = |\vec{F}_{on\,B}|/2$
(c) $|\vec{F}_{on\,A}| = |\vec{F}_{on\,B}|$
(d) $|\vec{F}_{on\,A}| = 2|\vec{F}_{on\,B}|$
(e) $|\vec{F}_{on\,A}| = |\vec{F}_{on\,B}|/4$

basic third law stuff : $|\vec{F}_{A\,on\,B}| = |\vec{F}_{B\,on\,A}|$
→ still holds true for electrical force

$$\boxed{|\vec{F}_{on\,A}| = |\vec{F}_{on\,B}|}$$

Question value 4 points
(06) Compare the magnitude of the electric field acting on A to the magnitude of the electric field acting on B.

(a) $|\vec{E}_{on\,A}| = 4|\vec{E}_{on\,B}|$
(b) $|\vec{E}_{on\,A}| = |\vec{E}_{on\,B}|/2$
(c) $|\vec{E}_{on\,A}| = |\vec{E}_{on\,B}|/4$
(d) $|\vec{E}_{on\,A}| = |\vec{E}_{on\,B}|$
(e) $|\vec{E}_{on\,A}| = 2|\vec{E}_{on\,B}|$

$$\vec{E}_{on\,A} = \vec{E}_{at\,A,\,by\,B} = \frac{k(+4Q)}{d^2}(-\hat{i})$$

$$\vec{E}_{on\,B} = \vec{E}_{at\,B,\,by\,A} = \frac{k(+Q)}{d^2}(+\hat{i})$$

so $\boxed{|\vec{E}_{on\,A}| = 4\,|\vec{E}_{on\,B}|}$