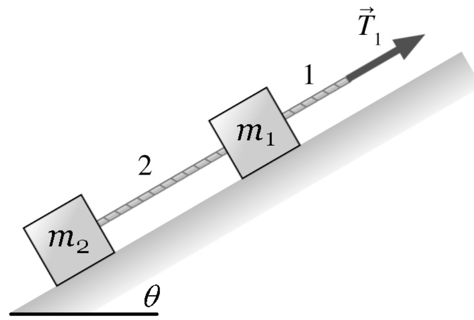
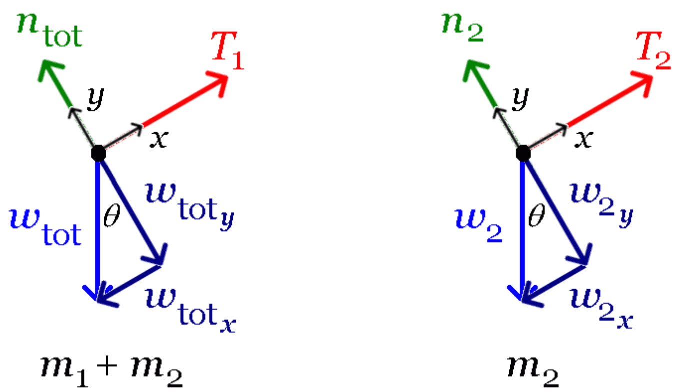
I. (16 points) Two blocks of mass m_1 and m_2 are tied together and pulled so that it travels up a frictionless incline that makes an angle θ with the horizontal, as illustrated. The force pulling them is a tension of magnitude T_1, and their resulting acceleration is not zero. What is the tension magnitude T_2 in rope 2, which ties the blocks together, in terms of other parameters defined in the problem, and physical or mathematical constants? (*On Earth.*)



There is more than one approach to solving this Newton's Second Law problem. The easiest is probably to look first at the two blocks m_1 and m_2 along with rope 2 as a single object being pulled up the incline by rope 1. A Free-Body Diagram has been sketched for this object. A coordinate system has been chosen so the acceleration on one axis is zero, and the weight force has been resolved into components. Applying Newton's Second Law to the x direction:



$$\sum F_x = T_1 - w_{\text{tot}_x} = m_{\text{tot}} a_x$$

So

$$T_1 - (m_1 + m_2)\, g \sin\theta = (m_1 + m_2)\, a_x \qquad \Rightarrow \qquad a_x = \frac{T_1 - (m_1 + m_2)\, g \sin\theta}{m_1 + m_2}$$

Looking next at just the block with mass m_2, which is pulled up the ramp by rope 2, a Free-Body Diagram has been sketched for this object. The same coordinate system is used, so the acceleration of this block will be represented by the same symbol as the acceleration of the two blocks together. The weight force has been resolved into components. Again, applying Newton's Second Law to the x direction:

$$\sum F_x = T_2 - w_{2x} = m_2 a_x \qquad \Rightarrow \qquad T_2 - m_2 g \sin\theta = m_2 a_x$$

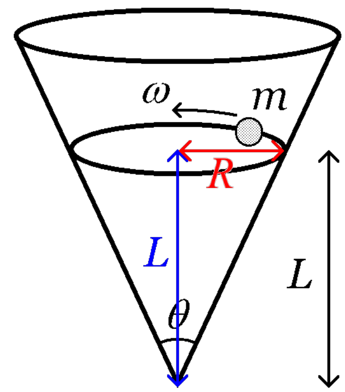
Substituting the expression for the acceleration and solving for T_2:

$$T_2 - m_2 g \sin\theta = m_2 \left(\frac{T_1 - (m_1 + m_2)\, g \sin\theta}{m_1 + m_2} \right) \qquad \Rightarrow \qquad T_2 = m_2 \left(\frac{T_1 - (m_1 + m_2)\, g \sin\theta}{m_1 + m_2} \right) + m_2 g \sin\theta$$

That is sufficient to answer the question. A bit more algebra, though, yields an interesting result:

$$\begin{aligned} T_2 &= m_2 \left(\frac{T_1 - (m_1 + m_2)\, g \sin\theta}{m_1 + m_2} \right) + \left(\frac{m_1 + m_2}{m_1 + m_2} \right) m_2 g \sin\theta \\ &= m_2 \left(\frac{T_1 - (m_1 + m_2)\, g \sin\theta + (m_1 + m_2)\, g \sin\theta}{m_1 + m_2} \right) = \left(\frac{m_2}{m_1 + m_2} \right) T_1 \end{aligned}$$

1. (6 points) A ball of mass m is placed at a height L in the frictionless hollow cone shown. It is put into uniform circular motion. What is the radius R of its path? (*On Earth.*)

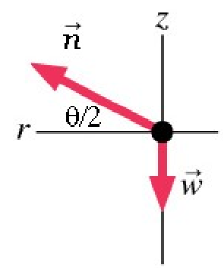


. . . .

From the diagram,

$$\tan(\theta/2) = R/L \qquad \Rightarrow \qquad R = L\tan(\theta/2)$$

II. (16 points) In the problem above, what angular speed ω is required to keep the ball at the height L? Express your answer in terms of R, other parameters defined in the problem and figure, and physical or mathematical constants.



. . . .

Use Newton's Second Law. Choose one axis horizontal toward the center, in the direction of the known acceleration. Then the vertical, or z, direction, will yield an expression for the normal force.

$$\sum F_z = n_z - w = ma_z = 0 \qquad \Rightarrow \qquad n\sin\theta/2 = mg \qquad \Rightarrow \qquad n = \frac{mg}{\sin(\theta/2)}$$

In the radial, or r, direction, the acceleration is a centripetal acceleration.

$$\sum F_r = n_r = ma_r = m\frac{v^2}{R} \qquad \Rightarrow \qquad n\cos(\theta/2) = mR\omega^2$$

Substituting the expression for n,

$$n\cos(\theta/2) = mR\omega^2 \quad \Rightarrow \quad \left(\frac{mg}{\sin(\theta/2)}\right)\cos(\theta/2) = \frac{mg}{\tan(\theta/2)} = mR\omega^2$$

Solving for ω,

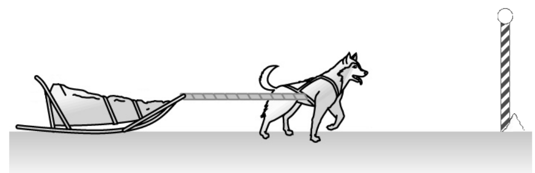
$$\omega = \sqrt{\frac{g}{R\tan(\theta/2)}}$$

2. (6 points) It is the end of the dog sled race, and a dog approaches the finish line over level ground pulling a 55 kg sled. At 5.0 m from the finish line, the sled is traveling at 6.0 m/s. The coefficient of kinetic friction between the sled and the ground is 0.18 in these last 5.0 m. The tiring dog exerts a force, $\vec{F}$, on the sled whose magnitude decreases as the finish line is approached, according to

$$\left|\vec{F}(x)\right| = \left(32\,\mathrm{N/m^{1/2}}\right)\sqrt{x}$$

where x is the distance from the sled to the finish line. Is the work done by the dog on the sled the force exerted by the dog times the distance of 5.0 m? *(On Earth.)*

. . . .



Work is only "force times distance" when the force is constant and parallel to the displacement. In this case, the force depends on position, so

No, work is not "force times distance" in this case.

III. (16 points) In the problem above, what is the speed of the sled as it crosses the finish line?

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Let the sled be the system, and use the Energy Principle:

$$W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{th}}$$

With this choice of system, there are no potential energy changes ($\Delta U = 0$). The weight force and the normal force each do no work on the system, as they are always perpendicular to the displacement. The force exerted by the dog is an external force parallel to the displacement, and the force of friction changes the thermal energy.

$$\int_0^d F(x)\cos 0^\circ\, dx = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) + f_k d$$

where d is the distance traveled, 5.0 m. Note that the normal force is equal in magnitude to the weight in this situation, so the frictional force $f_k = \mu_k n = \mu_k mg$. Representing $32\,\mathrm{N/m^{1/2}}$ as a constant C,

$$\int_0^d C\sqrt{x}\, dx = C\frac{x^{3/2}}{3/2}\bigg|_0^d = \frac{2C}{3}\,d^{3/2} = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) + \mu_k mgd$$

Solving for the final speed

$$\begin{aligned} v_f &= \sqrt{v_i^2 + \frac{4C}{3m}\,d^{3/2} - 2\mu_k gd} \\ &= \sqrt{(6.0\,\mathrm{m/s})^2 + \frac{4\cdot 32\,\mathrm{N/m^{1/2}}}{3\,(55\,\mathrm{kg})}\,(5.0\,\mathrm{m})^{3/2} - 2\,(0.18)\,(9.8\,\mathrm{m/s^2})\,(5.0\,\mathrm{m})} = 5.2\,\mathrm{m/s} \end{aligned}$$

3. (8 points) Two cars, A and B, on level ground have the same mass and their engines provide the same power. Each increases its speed by the same amount Δv. Car A, however, increases its speed from rest, while car B is already traveling at speed v_0 before its speed increases. Which car, if either, requires less time to complete its speed change?

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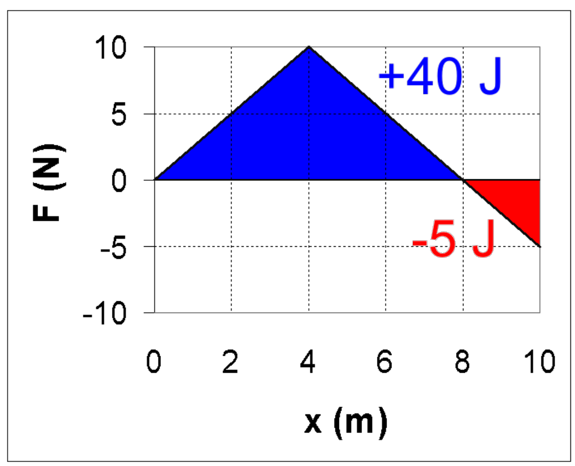
Since the kinetic energy of an object depends on the square of its speed, the same change in speed results in a greater kinetic energy change for car B. Since the engines of the two cars provide the same power (*i.e.*, the same energy is provided per unit time), and car A has a smaller kinetic energy change,

Car A requires less time than car B to change its speed by Δv.

4. (8 points) A 2.0 kg particle moving on the x axis is subject to the force shown on the graph. If the particle's velocity is +5.0 m/s as it passes through the origin, what is its kinetic energy when it reaches +10 m?



. . . .

Since $W = \int \vec{F} \cdot d\vec{s}$, the work done is the area under the Force *vs.* Position graph, or +35 J. Work is also the change in kinetic energy. Initially, the kinetic energy is $\frac{1}{2}mv_i^2 = \frac{1}{2}(2.0\,\text{kg})(+5.0\,\text{m/s})^2 = +25\,\text{J}$. So, with an initial kinetic energy of +25 J and a change of +35 J, the resulting kinetic energy must be

+60 J

5. (8 points) I slid the book a distance d to the left across the level table at constant speed. In terms of magnitudes, there is an applied force F_h from my hand, a gravitational force mg from the Earth, a normal force n from the table, and a kinetic friction force f_k from the table. What description(s) of the energy transformations is/are valid? Just as you do with "Choose all that apply" questions in class, express your answer as number, with your choices in numeric order. (*On Earth.*)

1 For a system consisting of just the book, my hand does external work $F_h d$.

2 For a system consisting of just the book, my hand does external work $F_h d$, and friction does external work $-f_k d$.

3 For a system consisting of just the book, my hand does external work $F_h d$, and friction increases the thermal energy by $f_k d$.

4 For a system consisting of the book and the table, my hand does external work $F_h d$.

5 For a system consisting of the book and the table, my hand does external work $F_h d$, and friction does external work $-f_k d$.

6 For a system consisting of the book and the table, my hand does external work $F_h d$, and friction increases the thermal energy by $f_k d$.

. . . .

Thermal energy increases in both the book and the table. The only choice consistent with this is

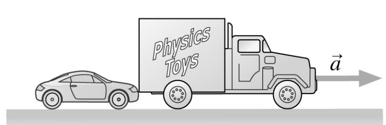
6 For a system consisting of the book and the table, my hand does external work $F_h d$, and friction increases the thermal energy by $f_k d$.

6. (8 points) A small car is pushing a larger truck with a dead battery, causing it to speed up to the right, as illustrated. The mass of the truck is greater than that of the car. Which of the following statements is true? (*On Earth.*)

. . . .

The force of the car on the truck and the force of the truck on the car constitute an action/reaction force pair.

The car exerts the same magnitude force on the truck as the truck exerts on the car.



7. (8 points) Rank the gravitational acceleration at the surface of the four illustrated planets, from greatest to least. Planet i has mass M and radius R.

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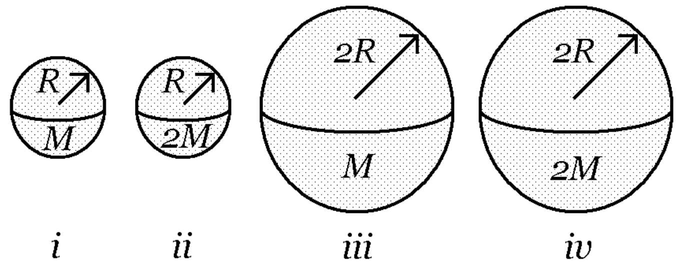

Weight is the force of gravity.

$$F_G = G\frac{Mm}{r^2} = mg \quad \Rightarrow \quad g = G\frac{M}{r^2}$$

For planet i this is GM/R^2, for planet ii this is $2GM/R^2$, for iii it's $\frac{1}{4}GM/R^2$, and for planet iv it is $\frac{1}{2}GM/R^2$. So

$$ii > i > iv > iii$$