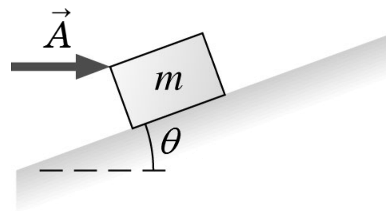
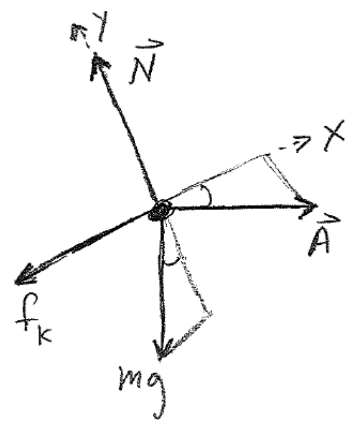
I. (16 points) A constant horizontal applied force $\vec{A}$ moves a block with mass m up an incline that makes an angle θ with the horizontal, as shown. The coefficient of static friction between the block and the incline is μ_s and that of kinetic friction is μ_k. What is the magnitude of the acceleration of the block up the incline? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants. (*On Earth.*)



Use Newton's Second Law. Sketch a Free Body Diagram. Choose a coordinate system. I'll choose the x axis in the direction of the known acceleration up the plane, and the y axis perpendicular to that. With that choice of coordinate system, the normal force $\vec{N}$ and the kinetic friction force $\vec{f}_k$ lie along axes, while the applied force $\vec{A}$ and the gravitational force $m\vec{g}$ must be resolved into components. Write out Newton's Second Law (the sum of the forces is ...) for each axis. I'll show signs explicitly, so symbols represent magnitudes. Note that since the block is moving, the frictional force is kinetic.



$$\sum F_y = N - A_y - mg_y = ma_y = 0 \qquad \Rightarrow \qquad N = A\sin\theta + mg\cos\theta$$

and

$$\sum F_x = A_x - mg_x - f_k = ma_x \qquad \Rightarrow \qquad a_x = \frac{A\cos\theta - mg\sin\theta - \mu_k N}{m}$$

Substitute the expression for N.

$$a_x = \frac{A\cos\theta - mg\sin\theta - \mu_k\left(A\sin\theta + mg\cos\theta\right)}{m}$$

II. (16 points) A child is riding a merry-go-round (a horizontal circular ride found in an amusement park). The merry-go-round is rotating with a constant speed ω_0. The operator then applies the brakes, giving the merry-go-round a constant angular acceleration so it comes to a stop in time Δt. Through what angle does it turn from the time the operator applies the brakes to the time it comes to a complete stop? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants. (*On Earth.*)

. . . .

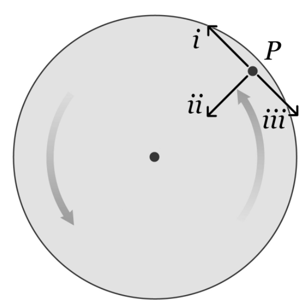
Use constant angular acceleration kinematics.

$$\theta = \theta_0 + \omega_0\,\Delta t + \tfrac{1}{2}\alpha\,(\Delta t)^2 \qquad \text{and} \qquad \omega = \omega_0 + \alpha\,\Delta t$$

The merry-go-round comes to a stop, so $\omega = 0$. Therefore, $\alpha = -\omega_0/\Delta t$.

$$\theta - \theta_0 = \omega_0\,\Delta t + \tfrac{1}{2}\left(\frac{-\omega_0}{\Delta t}\right)(\Delta t)^2 = \tfrac{1}{2}\omega_0\,\Delta t$$

1. (6 points) The figure illustrates a top-view of the merry-go-round, as it rotates counterclockwise with a child sitting at point P. What is the direction of the child's acceleration as the merry-go-round slows down?

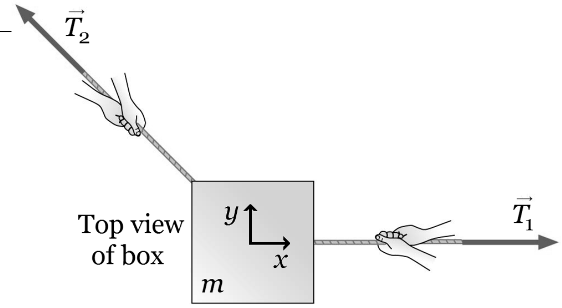


. . . .

Since the merry-go-round is slowing down as it turns counterclockwise, the child's tangential acceleration is in direction *iii*. The child is still, however, moving in a circle, so there is a centripetal acceleration in direction *ii*. The child's total acceleration is the vector sum of these,

Somewhere between directions *ii* and *iii*.

III. (16 points) A block with mass $m = 5.0\,\text{kg}$ is at rest on a frictionless horizontal surface, as shown in a top view (*i.e.*, the surface is in the plane of the page). Two ropes are attached to it. One is pulled is pulled with tension $\vec{T}_1 = 440\,\text{N}$ in the $+x$ direction. The other is pulled with tension $\vec{T}_2 = 240\,\text{N}$ at an angle $\theta = 120°$ from the $+x$ axis. What is the magnitude of the block's acceleration? (*On Earth.*)



Use Newton's Second Law.

$$\vec{F}_{\text{net}} = \vec{T}_1 + \vec{T}_2 = m\vec{a}$$

In the x direction,

$$F_{\text{net}_x} = T_{1x} + T_{2x} = T_1 + T_2\cos\theta = 440\,\text{N} + 240\,\text{N}\cos(120°) = 320\,\text{N}$$

and in the y direction

$$F_{\text{net}_y} = T_{1y} + T_{2y} = 0 + T_2\sin\theta = 0\,\text{N} + 240\,\text{N}\sin(120°) = 208\,\text{N}$$

The magnitude can be found with the Pythagorean Theorem:

$$F_{\text{net}} = \sqrt{F^2_{\text{net}_x} + F^2_{\text{net}_y}} = \sqrt{(320\,\text{N})^2 + (208\,\text{N})^2} = 380\,\text{N}$$

Since m is a scalar,

$$\vec{a} = \frac{1}{m}\vec{F}_{\text{net}} \qquad \Rightarrow \qquad a = \frac{F_{\text{net}}}{m} = \frac{380\,\text{N}}{5.0\,\text{kg}} = 76\,\text{m/s}^2$$

2. (6 points) In the problem above, what if the block were not at rest, but was moving in the $+y$ direction at a speed of $35\,\text{m/s}$ at the instant those same two tensions were applied? How would the block's acceleration compare to the situation when it was initially at rest?

In a reference frame moving in the $+y$ direction at a speed of $35\,\text{m/s}$, the block *is* at rest. Such a frame is inertial, and accelerations must be the same in all inertial frames.

The acceleration of the initially moving block would have the same magnitude and direction as the initially stationary one.

3. (8 points) A ball rolls inside a hose that has been shaped as a semicircle and laid flat on a table, as shown in the figure. At point P the ball leaves the hose. Which of the arrows best reflects the ball's trajectory after leaving the hose? Assume there is no friction. (*On Earth.*)

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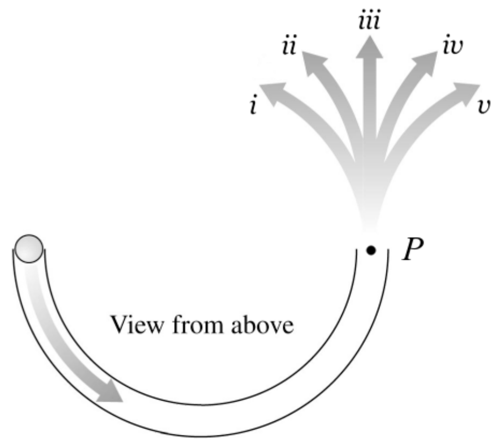

Once the ball leaves the hose, no net force acts on it. Its velocity will be constant, both magnitude and direction. That is trajectory

iii

4. (8 points) A skydiver is falling at terminal speed v, and then opens their parachute. Soon, they reach a new terminal speed v'. How does the magnitude of the skydiver's acceleration change during the time from just after their parachute is opened, to just before they reach their new terminal speed? (*On Earth, do NOT neglect drag!*)

. . . .

The skydiver is moving rapidly at the instant they open their parachute. The drag force upward (which is proportional to the square of the speed) is much greater than the gravitational force downward, so they have a large upward acceleration. However, once they reach their new terminal speed, their acceleration must be zero. Changing from a large acceleration to zero acceleration means

The magnitude of their acceleration **decreases**.

5. (8 points) A bullet is shot through two cardboard disks attached a distance D apart to a shaft turning with a rotational period (the time to make one revolution) T, as shown. Each disk has a hole, and the holes lie at the same radial distance from the shaft. If the two holes are 180° apart, what speed must the bullet have if it is to pass through both holes within a single revolution of the shaft?

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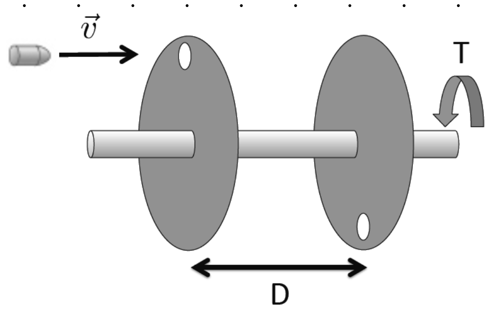


The disks have a constant angular acceleration of zero.

$$\theta = \theta_0 + \omega_0\,\Delta t + \tfrac{1}{2}\alpha\,(\Delta t)^2 \qquad \Rightarrow \qquad \Delta\theta = \omega_0\,\Delta t$$

The bullet has a constant linear acceleration of zero.

$$s = s_0 + v_0\,\Delta t + \tfrac{1}{2}a\,(\Delta t)^2 \qquad \Rightarrow \qquad \Delta s = v_0\,\Delta t$$

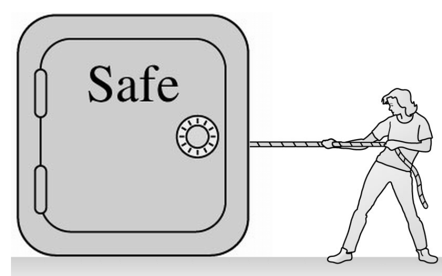
The time for the disks to rotate half a revolution ($\Delta\theta = \pi$) must be the same as the time for the bullet to travel a distance $\Delta s = D$.

$$\Delta t = \frac{\Delta\theta}{\omega_0} = \frac{\Delta s}{v_0} \qquad \Rightarrow \qquad v_0 = \omega_0\frac{\Delta s}{\Delta\theta} = \omega_0\frac{D}{\pi}$$

The angular speed ω_0 is the angular displacement of one revolution (2π) divided by the time for one revolution, T.

$$v_0 = \omega_0\frac{D}{\pi} = \left(\frac{2\pi}{T}\right)\frac{D}{\pi} = \frac{2D}{T}$$

6. (8 points) A 6000 N safe is at rest on level ground. The coefficient of kinetic friction between the safe and the ground is 0.33, and the coefficient of static friction between the safe and the ground is 0.50. A student ties a rope to the safe, and pulls with a horizontal force of 1000 N, but the safe doesn't move. What is the magnitude of the friction force on the safe? (*On Earth.*)

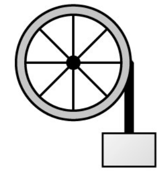


. . . .

The safe is in equilibrium, so the net force on it is zero. The force f_s exerted by friction must balance tension T in the rope. In terms of magnitudes,

$$f_s = T = 1000\,\text{N}$$

7. (8 points) The wheel in the figure has a diameter of 3.0 m. It is wrapped with a cord, which is tied to a block with mass 4.0 kg. The block is released from rest. At the instant the wheel has turned though an angle of 540°, how far has the block fallen? (*On Earth.*)



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The block will have fallen a distance equal to the arc length through which the rim of the wheel has turned.

$$s = r\theta = \frac{d}{2}\,\theta = \frac{3.0\,\mathrm{m}}{2}\left(540^\circ \times \frac{\pi\ \text{radians}}{180^\circ}\right) = 14\,\mathrm{m}$$