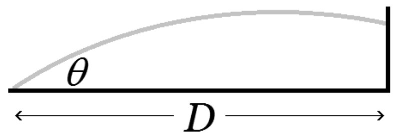

I. (16 points) A spring-loaded toy cannon launches its ball at an angle $\theta = 35°$ above the horizontal. The ball reaches its maximum height at a time $t_{\max} = 0.48\,\mathrm{s}$ after launch. At what time after launch does the ball hit the wall a distance $D = 3.7\,\mathrm{m}$ from the cannon? (*On Earth.*)



This is a projectile motion problem, which is a special case of constant-acceleration kinematics, in which the vertical acceleration is solely due to gravity, and the horizontal acceleration is zero.

Choose a coordinate system. I'll put the origin at the launch point, with x horizontal to the right, and y vertical upward. The vertical component of the ball's velocity is zero at time $\Delta t = t_{\max}$.

$$v_y = v_{0y} + a_y\,\Delta t \qquad \Rightarrow \qquad 0 = v_0 \sin\theta - g t_{\max}$$

$$v_0 = \frac{g t_{\max}}{\sin\theta} = \frac{\left(9.81\,\mathrm{m/s^2}\right)(0.48\,\mathrm{s})}{\sin(35°)} = 8.21\,\mathrm{m/s}$$

Remember that the horizontal component of the acceleration is zero.

$$x = x_0 + v_{0x}\,\Delta t + \tfrac{1}{2} a_x\,(\Delta t)^2 \qquad \Rightarrow \qquad D = 0 + v_0 \cos\theta\,\Delta t$$

$$\Delta t = \frac{D}{v_0 \cos\theta} = \frac{(3.7\,\mathrm{m})}{(8.21\,\mathrm{m/s})\cos(35°)} = 0.55\,\mathrm{s}$$

1. (6 points) An object moves in one dimension with a velocity $\vec{v}$ that depends on time t according to

$$\vec{v} = \left(At^2 - Bt - C\right)\hat{\imath}$$

where $A = 9.0\,\mathrm{m/s^3}$, $B = 6.0\,\mathrm{m/s^2}$, and $C = 4.0\,\mathrm{m/s}$. Which constant-acceleration kinematic equation from your text can be used to find the acceleration of the object?

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Acceleration is related to velocity

$$a = \frac{dv}{dt} = (2At - B)\,\hat{\imath}$$

Since the acceleration depends on time, it is not constant, and

No constant-acceleration kinematic equation from the text can be used.

II. (16 points) In the problem above, if the object is at position $\vec{x} = +5.0\hat{\imath}\,\mathrm{m}$ at time $t = 0\,\mathrm{s}$, what is its position at time $t = 2.0\,\mathrm{s}$?

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Since this motion is in one dimension, signs can indicate direction, and the vector notation can be dropped. Then

$$v = \frac{dx}{dt} \qquad \Rightarrow \qquad x = \int v\,dt = \int \left(At^2 - Bt - C\right)dt = \frac{A}{3}t^3 - \frac{B}{2}t^2 - Ct + K$$

where K is an integration constant. The object is at position $x = +5.0\,\mathrm{m}$ at time $t = 0\,\mathrm{s}$, so

$$+5.0\,\mathrm{m} = \frac{A}{3}(0\,\mathrm{s})^3 - \frac{B}{2}(0\,\mathrm{s})^2 - C\,(0\,\mathrm{s}) + K \qquad \Rightarrow \qquad K = +5.0\,\mathrm{m}$$

At time $t = 2.0\,\mathrm{s}$, then,

$$\begin{aligned} x &= \frac{A}{3}(2.0\,\mathrm{s})^3 - \frac{B}{2}(2.0\,\mathrm{s})^2 - C\,(2.0\,\mathrm{s}) + K \\ &= \frac{9.0\,\mathrm{m/s^3}}{3}(2.0\,\mathrm{s})^3 - \frac{6.0\,\mathrm{m/s^2}}{2}(2.0\,\mathrm{s})^2 - 4.0\,\mathrm{m/s}\,(2.0\,\mathrm{s}) + 5.0\,\mathrm{m} \\ &= 24\,\mathrm{m} - 12\,\mathrm{m} - 8.0\,\mathrm{m} + 5.0\,\mathrm{m} = +9\,\mathrm{m} \text{ or } +9\hat{\imath}\,\mathrm{m} \end{aligned}$$

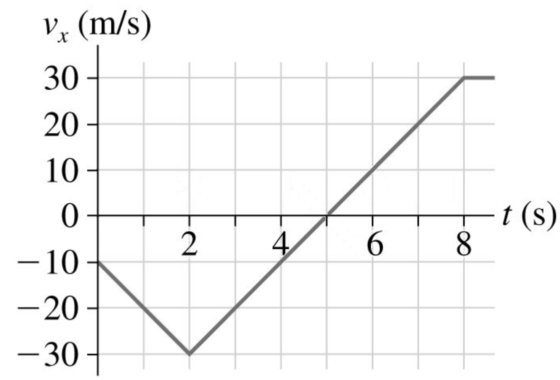
III. (16 points) A particle moves along the x axis, starting at position $\vec{x} = +10\,\mathrm{m}$ at time $t = 0\,\mathrm{s}$. Its velocity $\vec{v}$ depends on time, as shown. What is the position of the particle at time $t = 8\,\mathrm{s}$?

The displacement of the object can be related to its velocity as

$$v = \frac{dx}{dt} \qquad \Rightarrow \qquad \Delta x = \int v\,dt$$

Graphically, then, the displacement is the area under the velocity curve.

Note that the negative area between $t = 5\,\mathrm{s}$ and $t = 8\,\mathrm{s}$, and the positive are between $t = 2\,\mathrm{s}$ and $t = 5\,\mathrm{s}$, exactly cancel. Therefore, the displacement will be the area under the velocity curve from $t = 0\,\mathrm{s}$ to $t = 2\,\mathrm{s}$.



Each box on the graph represents $10\,\mathrm{m}$. There are three full boxes, and two half boxes, all on the negative side of the v axis, between $t = 0\,\mathrm{s}$ to $t = 2\,\mathrm{s}$, representing a displacement of -40 m.

$$\Delta x = x_f - x_i \qquad \Rightarrow \qquad x_f = x_i + \Delta x = 10\,\mathrm{m} + (-40\,\mathrm{m}) = -30\,\mathrm{m}$$

2. (6 points) If it can be determined in the problem above, at what time in the range $t = 0\,\mathrm{s}$ to $t = 8\,\mathrm{s}$ does the particle achieve its maximum distance from the origin?

The particle moves in the negative direction from time $t = 0\,\mathrm{s}$ to $t = 5\,\mathrm{s}$, when it stops to turn around to come back. Therefore, $t = 5\,\mathrm{s}$ is the time at which it is the greatest negative distance from its starting point. Since there are more "negative boxes" between time $t = 0\,\mathrm{s}$ to $t = 8\,\mathrm{s}$, the particle doesn't even return to its starting point by time $t = 8\,\mathrm{s}$. Only one "negative box" was required to bring the particle to the origin. Therefore, the particle has its maximum distance from the origin

At time $t = 5\,\mathrm{s}$.

3. (8 points) Mary needs to row her boat across a 100 m wide river that flows in the $-y$ direction at 1 m/s. Mary can row a a speed of 2 m/s. Assume that Mary wants to land directly across the river (i.e., in the $+x$ direction) from the point at which she started. What will be her speed with respect to the shore?

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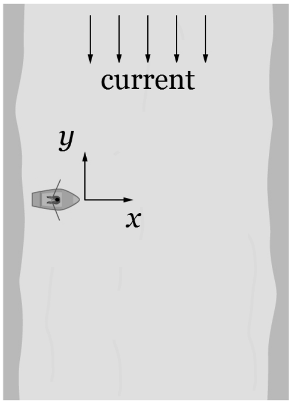
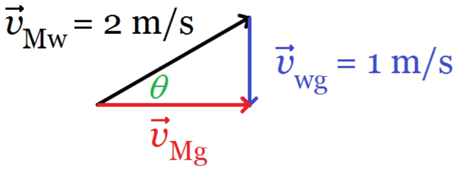


The velocity $\vec{v}_{\mathrm{Mg}}$ of Mary with respect to the ground is

$$\vec{v}_{\mathrm{Mg}} = \vec{v}_{\mathrm{Mw}} + \vec{v}_{\mathrm{wg}}$$

where $\vec{v}_{\mathrm{Mw}}$ is the velocity of Mary with respect to the water, and $\vec{v}_{\mathrm{wg}}$ is the velocity of the water with respect to the ground. Representing this graphically, one can see that her speed is

Less than 2 m/s.



4. (8 points) A cannon is aimed 35° above the horizontal, and fires a shell which lands on a plain a height h above the cannon. The cannon is then aimed at 55° above the horizontal, and fires an identical shell at the same speed, which also lands on that same plain. Which shell, if either, travels a greater horizontal distance to the point at which it lands? Assume that, unlike the figure, the size of the cannon is small compared to the height h. Remember that both shells *do* land on the plain! (*On Earth.*) *Hint:* Sketch the situation in which there is no raised plain, and the shells land at their launch height.

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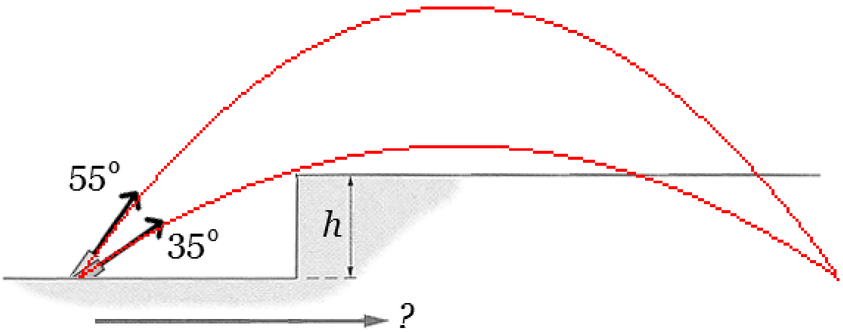
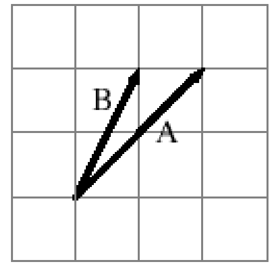
Since the angles are symmetric about 45°, the shells would have the same range if they returned to their launch height (i.e., $h = 0$). At every horizontal position on the descent, however, the shell fired at 55° will be coming down more steeply than the shell fired at 35°. To land at the same $h = 0$ position, even though it is falling more steeply, at each vertical position during the descent the shell fired at 55° must be at a greater horizontal position than the shell fired at 35°.

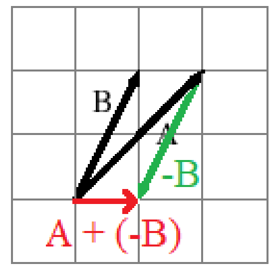
The shell fired at 55° travels a greater horizontal distance.



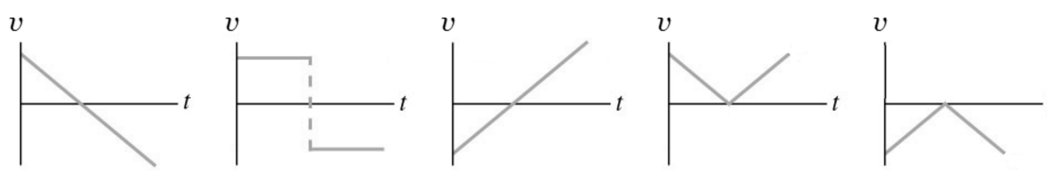
5. (8 points) Consider the two vectors $\vec{A}$ and $\vec{B}$ shown at right. Which of the following is the correct sketch for the vector difference $\vec{A}-\vec{B}$?



$\vec{A}-\vec{B}=\vec{A}+\left(-\vec{B}\right)$. Adding them graphically by placing them tip-to-tail,



6. (8 points) An object is thrown upward. It reaches a peak, and falls back downward. If upward is the positive direction, which graph shows its velocity as a function of time during this process? (*On Earth.*)



i *ii* *iii* *iv* *v*

The acceleration of an object in free fall is constant and downward. If upward is the positive direction, then the object's velocity graph must have a constant negative slope. That is graph

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7. (8 points) A car drives through a semi-circular valley at constant speed. Three motion diagrams are shown, with velocity vectors between the car's locations, and an acceleration vector at the instant the car is at the very bottom of the valley. Which diagram or diagrams, if any, shows a correct acceleration vector? (*On Earth.*)

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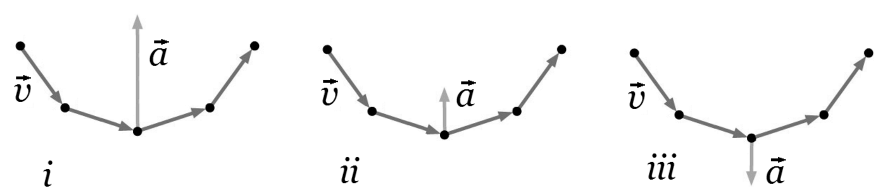
If one were to construct a Δv vector from the velocity vectors on either side of the bottom of the valley in diagram ii, one would obtain a Δv the same length and direction as the acceleration vector shown in that diagram. The acceleration vector must be in that same direction, up the page. However, velocity and acceleration are two different things, so there's no necessity that the vectors representing them have any particular length relationship.



Either diagrams i or ii could be correct.