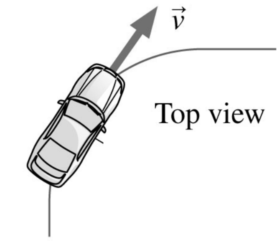
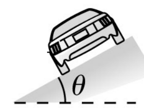

I. (16 points) Consider a road whose curve has a "reverse bank", that is, the outside of the curve is lower than the inside, as shown. If the bank angle is θ from the horizontal, and the coefficient of static friction between tires and the road is μ_s, with what maximum speed can a car travel around a curve of radius R? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants. (*On Earth.*)





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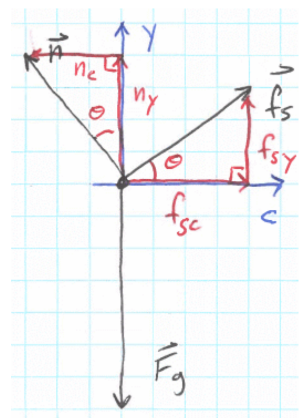
Use Newton's Second Law. Sketch a Free Body Diagram. There is a gravitational force $\vec{F}_g$ downward, a normal force $\vec{n}$ up and to the left. Since the car will slide down and to the left if it goes too fast, the static friction force $\vec{f}_s$ is up along the bank (up and to the right). Choose a coordinate system. I'll choose the c axis in the direction of the known acceleration, which is directly right toward the center of the curve. I'll choose the y axis upward. Resolve the forces into components, and write Newton's Second Law for each axis. I'll show signs explicitly, so symbols represent magnitudes. Although the mass of the car isn't a parameter defined in the problem statement, for now I'll represent it as m.



$$\sum F_y = n_y + f_{sy} - mg = ma_y = 0 \qquad \Rightarrow \qquad n\cos\theta + \mu_s n \sin\theta = mg$$

where $f_s = \mu_s n$ is valid, as we're looking for the *maximum* speed of the car. Solving for the normal force

$$n = \frac{mg}{\cos\theta + \mu_s \sin\theta}$$

Then

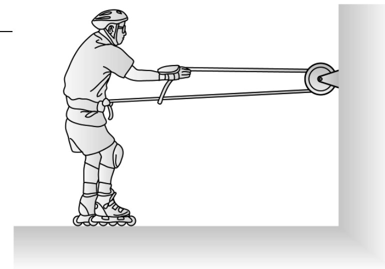
$$\sum F_c = f_{sc} - n_c = ma_c = m\frac{v^2}{R} \qquad \Rightarrow \qquad \mu_s n\cos\theta - n\sin\theta = n\left(\mu_s\cos\theta - \sin\theta\right) = m\frac{v^2}{R}$$

Substitute the expression for n found from $\sum F_y$.

$$\left(\frac{mg}{\cos\theta + \mu_s\sin\theta}\right)\left(\mu_s\cos\theta - \sin\theta\right) = mg\left(\frac{\mu_s\cos\theta - \sin\theta}{\cos\theta + \mu_s\sin\theta}\right) = m\frac{v^2}{R}$$

Solve for v. Note that it does not depend on the mass of the car.

$$v = \sqrt{gR\left(\frac{\mu_s\cos\theta - \sin\theta}{\cos\theta + \mu_s\sin\theta}\right)}$$



1. (6 points) Jorge, with mass m, wants to test his new skating helmet. He straps on his cheap skates, which have a non-negligible coefficient of rolling friction μ_r with the level floor, ties a rope to his waist, passes it around a pulley attached to the wall as shown, and pulls to accelerate from rest. Assuming the rope is horizontal except where it passes around the pulley, describe the resulting thermal energy changes. Note that "Jorge" includes his skates. (*On Earth.*)

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Mechanical energy will be transformed to thermal energy due to the dissipative force of rolling friction. Some of this thermal energy will be in Jorge (including his skates) and some will be in the floor. But we don't know how it will be divided between the two.

The floor's thermal energy and Jorge's thermal energy increase by a combined total of $\mu_r mgd$.

II. (16 points) In Jorge's excitement, he pulls harder and harder as he moves. In fact, if he starts at the origin and $+x$ is to the right, the magnitude of his force depends on position according to

$$F = F_0 \left(\frac{x}{d}\right)^2$$

where F_0 is a constant and d is his initial distance from the wall. With what speed does he strike the wall? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants.

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Use the Energy Principle, $W_{\text{ext}} = \Delta K + \Delta E_{\text{th}}$. Choose a system consisting of Jorge and the floor, as the thermal energy change in that system can be determined. The only external force doing work on this system is Tension, which is the same as the force Jorge applies (Newton's Third Law), but acts twice, once at Jorge's hands, and once at his waist.

$$\int \vec{F} \cdot d\vec{s} = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) + f_r\,\Delta s \qquad \Rightarrow \qquad \int_0^d 2F_0 \left(\frac{x}{d}\right)^2 dx = \tfrac{1}{2}mv_f^2 + \mu_r n d$$

Newton's Second Law shows us that the normal force $\vec{n}$ balances the gravitational force, so

$$\frac{2F_0}{d^2} \int_0^d x^2\,dx = \frac{2F_0}{d^2}\left[\frac{x^3}{3}\right]_0^d = \frac{2F_0}{3d^2}\left[d^3\right] = \frac{2}{3}F_0 d = \tfrac{1}{2}mv_f^2 + \mu_r mgd$$

Solve for the final speed.

$$v_f = \sqrt{\frac{2}{m}\left(\frac{2}{3}F_0 d - \mu_r mgd\right)}$$

2. (6 points) The block of mass m accelerates up a frictionless ramp. Describe the tension in the vertical portion of the rope farthest to the right, attached to the ceiling. (*On Earth.*)

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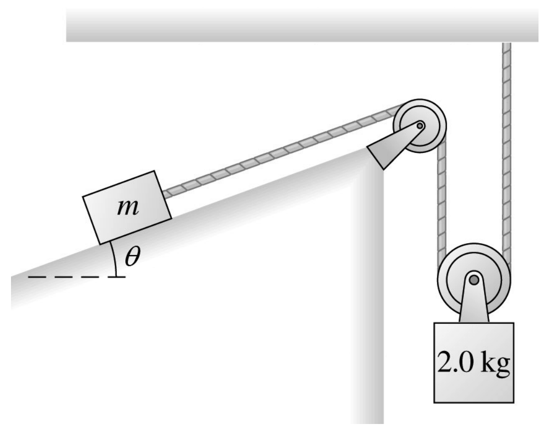

Since the block of mass m accelerates up the ramp, the 2.0 kg block accelerates downward. The net force on the block must be downward, so the upward force must be less than $mg = (2.0\,\text{kg})\left(9.8\,\text{m/s}^2\right) = 19.6\,\text{N}$. The tension, which is the same everywhere in the rope, acts twice on the 2.0 kg block, so

It is less than 9.8 N.

III. (16 points) In the problem above, the frictionless ramp makes an angle $\theta = 26°$ with the horizontal, and the block of mass m slides up it with an acceleration magnitude $1.7\,\text{m/s}^2$. What is the value of the mass m?

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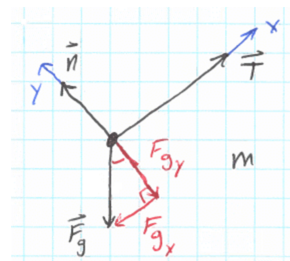
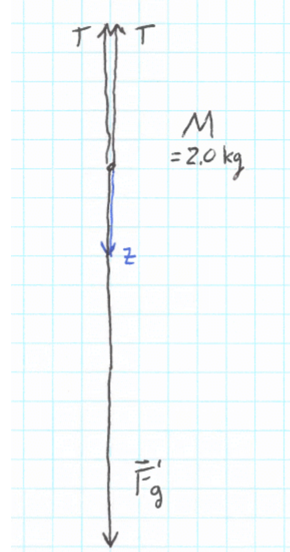


Use Newton's Second Law. Consider first the block of mass m. Sketch a Free Body Diagram. There is a tension $\vec{T}$, a normal force $\vec{n}$, and a gravitational force $\vec{F}_g$. Choose a coordinate system. I'll chose $+x$ in the known direction of the acceleration, up along the ramp, and $+y$ perpendicular out of the ramp. In this case, we need apply Newton's Second Law only to the x axis. I'll show signs explicitly, so symbols represent magnitudes.

$$\sum F_x = T - F_{gx} = ma_x \qquad \Rightarrow \qquad T = ma_x + mg\sin\theta = m\left(a_x + g\sin\theta\right)$$



Consider next the 2.0 kg block. I'll use M to represent that mass. Sketch a Free Body Diagram. There are *two* tensions $\vec{T}$, and a gravitational force $\vec{F}_g'$. Choose a coordinate system. I'll choose $+z$ downward. Apply Newton's Second Law. Again, I'll show signs explicitly, so symbols represent magnitudes. Note that with this choice of coordinate system, $a_z = +a_x/2$

$$\sum F_z = F_g' - 2T = Ma_z \qquad \Rightarrow \qquad Mg - 2T = Ma_x/2$$

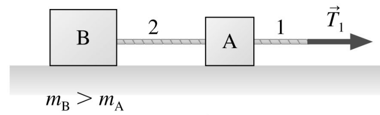
Substitute the expression for T,

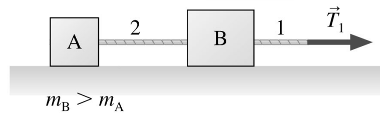
$$Mg - 2\left(ma_x + mg\sin\theta\right) = Mg - 2m\left(a_x + g\sin\theta\right) = Ma_x/2$$

and solve for m.

$$m = \frac{M\left(g - a_x/2\right)}{2\left(a_x + g\sin\theta\right)} = \frac{(2.0\,\text{kg})\left(\left(9.8\,\text{m/s}^2\right) - \left(1.7\,\text{m/s}^2\right)/2\right)}{2\left(\left(1.7\,\text{m/s}^2\right) + \left(9.8\,\text{m/s}^2\right)\sin 26°\right)} = 1.5\,\text{kg}$$

3. (8 points) Blocks A and B are connected by rope 2 and pulled along a frictionless surface by tension T_1. The mass of block B is greater than the mass of block A. The tension T_1 may be applied to block A, as in the upper illustration, or it may be applied to block B as in the lower illustration. Compare the tension magnitude in the connecting rope 2 for these two situations. (*On Earth.*)





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Since the total mass of A and B is the same in the two situations, and the net external force on the system consisting of A and B is the same in the two situations (it's tension T_1), the acceleration must be the same in the two situations.

When T_1 is applied to block A, tension T_2 must cause the greater mass of B to achieve that acceleration. Therefore,

The tension in rope 2 is greater when T_1 is applied to block A.

4. (8 points) Sugi pushes on the crate which, due to static friction, remains at rest on level ground. Which of these is a "force pair", in the sense of Newton's Third Law, for this situation? (*On Earth.*)

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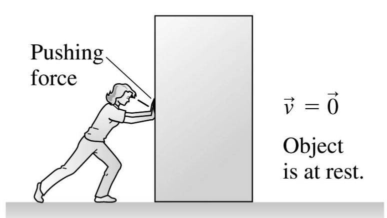
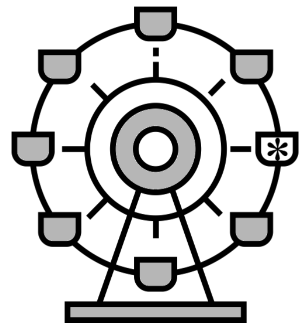
The forces that constitute a pair must be the same kind of force, and between the same two objects.

Sugi's push force on the crate, and the crate's push force on Sugi.



5. (8 points) The Ferris Wheel rotates counter-clockwise with constant angular speed. At a particular instant, there is a car directly to the right of the hub, marked with an asterisk. What is the direction of the force exerted by the seat on a person riding in that car at that instant? (*On Earth.*)



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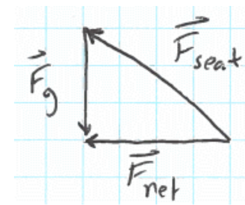
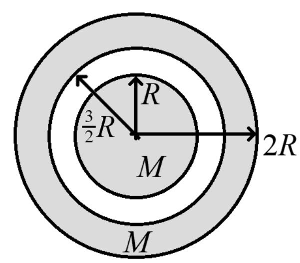
The person is moving in a circle at constant speed. The net force on them must be toward the center of the circle, directly to their left.

That net force is the (vector) sum of two forces—the force of gravity, and the force from the seat. A sketch of the vector sum $\vec{F}_{\text{net}} = \vec{F}_g + \vec{F}_{\text{seat}}$ shows that the seat force must be



Upward and to the left.

6. (8 points) Imagine a very strange planet, made of two concentric pieces. The inner piece is solid, with mass M and radius R. The outer piece is hollow, with mass M, inner radius $3R/2$, and outer radius $2R$. What is the acceleration due to gravity at the inner surface of the outer piece (that is, at radius $3R/2$)?



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The magnitude of the force of gravity is

$$F_g = Gm_1m_2/r^2 = m_2g \qquad \Rightarrow \qquad g = Gm_1/r^2$$

In this case, the point in question is inside the outer hollow piece, so that piece has no net effect. The inner solid piece acts like a point in its own center.

$$g = \frac{GM}{(3R/2)^2} = \frac{4}{9}\frac{GM}{R^2}$$

7. (8 points) Two spheres, A and B, are made of the same material, have the same radius, and have the same drag coefficient. Sphere B is hollow, though, so it has half the mass of sphere A (that is, $M_B = M_A/2$). Each is falling through the air at its own terminal speed. Compare the rate of thermal energy increase of sphere A and the air, to that of sphere B and the air. (*On Earth, do NOT neglect drag!*)

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At terminal speed, the drag force D balances the gravitational force mg.

$$A: \quad \tfrac{1}{2}C\rho A v_A^2 = M_A g \qquad B: \quad \tfrac{1}{2}C\rho A v_B^2 = M_B g = M_A g/2 \qquad \text{So} \quad v_A = v_B\sqrt{2}$$

The rate of energy transformation is power.

$$P = \frac{dE}{dt} = \frac{d}{dt}\left[\vec{F}\cdot\vec{s}\right] = \vec{F}\cdot\left[\frac{d\vec{s}}{dt}\right] = \vec{F}\cdot\vec{v}$$

In this case the next external force on a sphere-air system is the force of gravity. So $F_A = 2F_B$. With $v_A = v_B\sqrt{2}$, we find

$$P_A = 2\sqrt{2}P_B$$