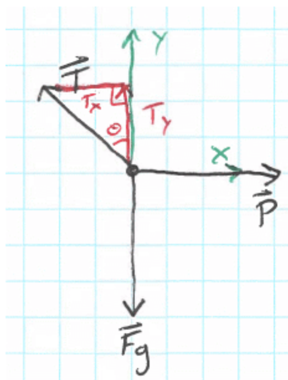
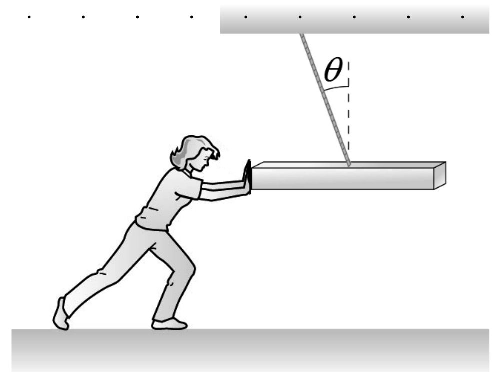
I. (16 points) Angelina pushes horizontally on a steel rail of mass m hanging by a rope from the ceiling. With what force magnitude must she push to hold the rail in place with its rope an angle θ from the vertical? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants. (*On Earth.*)





Use Newton's Second Law. Sketch a Free Body Diagram. There is a gravitational force $\vec{F}_g$, a push force $\vec{P}$, and a tension force $\vec{T}$. Choose a coordinate system. I've chosen $+x$ to the right, and $+y$ upward. With that choice, the tension force must be resolved into components.

Write Newton's Second Law for each axis. I'll show signs explicitly, so symbols represent magnitudes. The rail is in equilibrium ("held in place") so its acceleration is zero.

$$\sum F_x = P - T_x = ma_x = 0 \qquad \Rightarrow \qquad P = T\sin\theta \qquad \Rightarrow \qquad T = P/\sin\theta$$

$$\sum F_y = T_y - F_g = ma_y = 0 \qquad \Rightarrow \qquad T\cos\theta = mg$$

So

$$\left[\frac{P}{\sin\theta}\right]\cos\theta = mg \qquad \Rightarrow \qquad P = mg\tan\theta$$

II. (16 points) A car on a circular track of radius R accelerates from rest at time $t = 0$ with constant tangential acceleration magnitude $a_{\rm T}$. At what time is the magnitude of its acceleration $3a_{\rm T}/2$? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants. (*On Earth.*)

. . . .

The acceleration has two components. The tangential component is given to be $a_{\rm T}$. The centripetal component depends on the tangential speed, according to $a_c = v_{\rm T}^2/R$.

Since the tangential acceleration is constant, the tangential speed can be found using constant-acceleration kinematics, in the "tangential direction". Note that the car starts from rest.

$$v_{\rm T} = v_{\rm T0} + a_{\rm T}\,\Delta t = a_{\rm T}\,\Delta t$$

Since the tangential and centripetal components of the acceleration must be perpendicular, the magnitude of the acceleration is related to them by the Pythagorean Theorem.

$$a_{\rm total}^2 = a_{\rm T}^2 + a_c^2 \qquad \Rightarrow \qquad \left(\frac{3a_{\rm T}}{2}\right)^2 = \left(\frac{v_{\rm T}^2}{R}\right)^2 + a_{\rm T}^2 = \left(\frac{(a_{\rm T}\,\Delta t)^2}{R}\right)^2 + a_{\rm T}^2$$

Solve for Δt.

$$\frac{9a_{\rm T}^2}{4} = \frac{a_{\rm T}^4\,\Delta t^4}{R^2} + a_{\rm T}^2 \qquad \Rightarrow \qquad \frac{5a_{\rm T}^2}{4} = \frac{a_{\rm T}^4\,\Delta t^4}{R^2} \qquad \Rightarrow \qquad \Delta t^4 = \frac{5R^2}{4a_{\rm T}^2}$$

So

$$\Delta t^2 = \sqrt{\frac{5R^2}{4a_{\rm T}^2}} = \frac{R\sqrt{5}}{2a_{\rm T}} \qquad \Rightarrow \qquad \Delta t = \sqrt{\frac{R\sqrt{5}}{2a_{\rm T}}}$$

1. (6 points) At the time you determined in the problem above, what is the direction of the car's acceleration?

. . . .

The car's acceleration has a tangential component forward, and a centripetal component toward the center. So the acceleration is

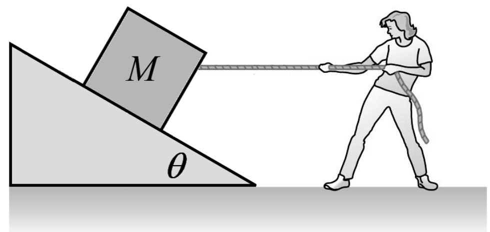
Somewhere between directly forward and toward the center of the track.

2. (6 points) Angelina pulls horizontally with force magnitude P on a box of mass M as it moves down a fixed ramp that makes an angle θ with the horizontal, as shown. Describe the normal force magnitude n exerted on the box by the ramp. (*On Earth.*)

. . . .

If Angelina weren't pulling on the box, the normal force would have magnitude $mg\cos\theta$, balancing the perpendicular component of the gravitational force. Since her pull is away from the surface, she is reducing the normal force.

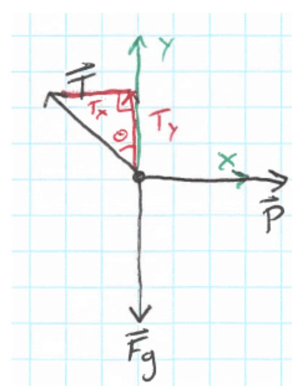
$$n < mg\cos\theta$$



III. (16 points) If the coefficient of kinetic friction between the box and the ramp in the problem above is μ_k, what is the magnitude of the acceleration of the box?

. . . .

Use Newton's Second Law. Sketch a Free Body Diagram. There is a gravitational force $\vec{F}_g$, a pull force $\vec{P}$, a normal force $\vec{n}$, and a kinetic friction force f_k. Choose a coordinate system. I've chosen $+x$ down along the ramp because that's the direction of the acceleration, and $+y$ up away from the plane. With that choice, the pull force and gravitational forces must be resolved into components.



Write Newton's Second Law for each axis. I'll show signs explicitly, so symbols represent magnitudes. The box does not accelerate perpendicular to the ramp.

$$\sum F_x = P_x + F_{gx} - f_k = ma_x \qquad \Rightarrow \qquad P\cos\theta + mg\sin\theta - \mu_k n = ma_x$$

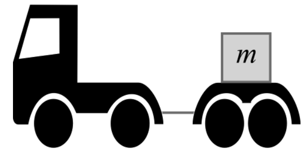
$$\sum F_y = P_y - F_{gy} + n = ma_y = 0 \qquad \Rightarrow \qquad n = mg\cos\theta - P\sin\theta$$

Substitute the expression for n, found from the y equation, into the x equation, and solve for the acceleration.

$$P\cos\theta + mg\sin\theta - \mu_k\left(mg\cos\theta - P\sin\theta\right) = ma_x$$

$$a_x = \frac{P\left(\cos\theta + \mu_k\sin\theta\right) + mg\left(sin\theta - \mu_k\cos\theta\right)}{m}$$

3. (8 points) A truck towing a small trailer is stopped at a traffic light. A box of mass m is on the trailer, as shown. When the light turns green, the truck, trailer, and box accelerate leftward together. Even though the box is not attached to the trailer, it does not slide. What force is responsible for the acceleration of the box? (*On Earth.*)



. . . .

Since the box accelerates to the left, the net force must be to the left. The tension in the hitch between the truck and the trailer does not act on the box. Since the box does not slide on the trailer, the friction force between them is static. So, the box accelerates because of

Static friction to the left.

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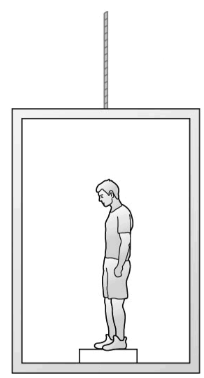
4. (8 points) Zach is descending in his elevator at increasing speed. Using the gravitational definition, his weight is w_{grav}. Using the operational definition, his weight is w_{op}. If possible, compare the magnitudes of w_{grav} and w_{op}. (*On Earth.*)

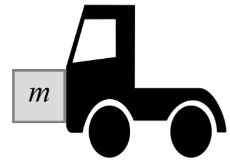
. . . .



Using the gravitational definition, Zach's weight is mg. If Zach is accelerating downward, then the net force on him must be downward. The magnitude of the normal force holding him up must be less than the magnitude of the downward gravitational force. The magnitude of the normal force is what we'd measure if we weighed him (the operational definition of weight).

w_{grav} is greater than w_{op}

5. (8 points) The truck accelerates to the left, exerting a normal force on the box of mass m, as shown. There is a minimum acceleration magnitude $a_{\min}$ for which the friction force $f_{\min}$ on the box prevents it from sliding down the front of the truck. If the truck doubles its acceleration to $a_2 = 2a_{\min}$, what is the friction force f_2 on the box? (*On Earth.*)



. . . .

Since the box does not accelerate vertically, there must be no net vertical force on it. The static friction force must balance the same gravitational force in each case.

$$f_2 = f_{\min}$$

6. (8 points) You're the passenger in an old car with slick vinyl seats and no seat belts. The driver makes a sharp left turn, and you slide rightward until you hit the door. What force is responsible for your acceleration to the right?

. . . .

No force.

Since you slide, there cannot be a static friction force. There may be a small kinetic friction force, but it would be leftward. Neither $m\vec{a}$ nor the centripetal force are a force. With (nearly) no net force on you, you go (nearly) straight while the car turns left beneath you.

7. (8 points) Two solid spheres, A and B, are made of the same material and have the same drag coefficient. Sphere B has twice the radius of sphere A (that is, $R_B = 2R_A$), so sphere B has eight times the mass of sphere A (that is, $M_B = 8M_A$). Compare the magnitudes of their accelerations at an instant when they are falling through the air with the same speed. (*On Earth, do NOT neglect drag!*)

. . . .

The gravitational force on sphere A is down, and the drag force is up. Letting down be the $+y$ direction,

$$\sum F_y = mg - D = ma_y \qquad \Rightarrow \qquad a_y = g - \frac{C\rho A v^2}{2m}$$

If the cross-sectional area A increases by a factor of 4 (because the radius doubles) and the mass increases by a factor of 8, then the term subtracted from g is halved, increasing the acceleration. But how much it increases would depend on the values of C, ρ, and v.

Acceleration of sphere B is greater than that of sphere A, but
how much greater cannot be determined with the information provided.