
I. (16 points) While driving through the mountains, you stop to see the view from a scenic overlook. Parking your car, you check your map and start walking toward the overlook, a distance $d = 880\,\text{m}$ away, in the direction $\theta = 17°$ east of north. Halfway there (that is, after walking 440 m), you realized you'd misread your map! The overlook was actually a distance $d = 880\,\text{m}$ from the parking lot, in the direction $\theta = 17°$ **north of east!** How far must you walk to get to the overlook from where you are now, and in what direction? (*On Earth.*)

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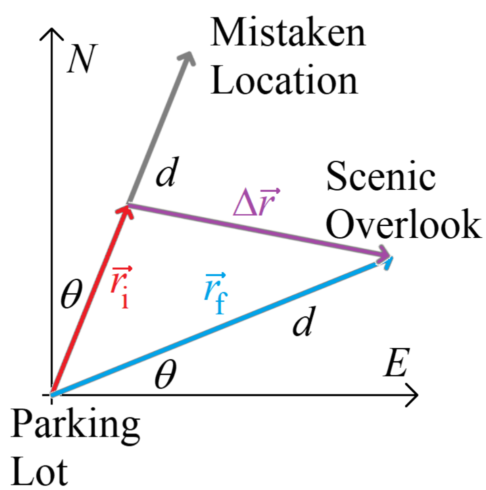

Letting $\vec{r}_\text{i}$ be your displacement while walking in the wrong direction, and $\vec{r}_\text{f}$ be the displacement to the scenic overlook, the sketch shows

$$\vec{r}_\text{f} = \vec{r}_\text{i} + \Delta\vec{r} \qquad \Rightarrow \qquad \Delta\vec{r} = \vec{r}_\text{f} - \vec{r}_\text{i}$$

where $\Delta\vec{r}$ is the answer to the question. In terms of components

$$\begin{aligned}\Delta r_\text{x} &= r_\text{fx} - r_\text{ix} = r_\text{f}\cos\theta - r_\text{i}\sin\theta \\ &= (880\,\text{m})\cos 17° - \left(\frac{880\,\text{m}}{2}\right)\sin 17° = 842\,\text{m} \\ \Delta r_\text{y} &= r_\text{fy} - r_\text{iy} = r_\text{f}\sin\theta - r_\text{i}\cos\theta \\ &= (880\,\text{m})\sin 17° - \left(\frac{880\,\text{m}}{2}\right)\cos 17° = -163\,\text{m}\end{aligned}$$

You must walk a distance

$$\Delta r = \sqrt{(\Delta r_\text{x})^2 + (\Delta r_\text{y})^2} = \sqrt{(842\,\text{m})^2 + (-163\,\text{m})^2} = 731\,\text{m}$$

in the direction

$$\phi = \tan^{-1}\left(\frac{\Delta r_\text{y}}{\Delta r_\text{x}}\right) = \tan^{-1}\left(\frac{-163\,\text{m}}{842\,\text{m}}\right) = -12.9°$$

which is

$$730\,\text{m} \quad \text{at} \quad 13° \quad \text{south of east}$$

II. (16 points) A texting driver looks up and sees a stalled truck stopped in the road straight ahead. They apply their brakes 35 m from the truck, but they hit the truck 3.8 s later, traveling at 7.5 m/s. What is the magnitude of their acceleration while braking? (*On Earth.*)

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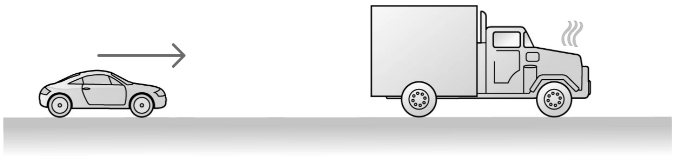
This problem involves constant-acceleration kinematics. It is one-dimensional, so signs will indicate direction, and vector notation is unnecessary.



$$x_f = x_i + v_i\,\Delta t + \tfrac{1}{2}a\,(\Delta t)^2$$

and

$$v_f = v_i + a\,\Delta t$$

Since the initial velocity is neither known nor asked, eliminate it.

$$v_i = v_f - a\,\Delta t \qquad \Rightarrow \qquad x_f = x_i + (v_i = v_f - a\,\Delta t)\,\Delta t + \tfrac{1}{2}a\,(\Delta t)^2 = x_i + v_f\,\Delta t - \tfrac{1}{2}a\,(\Delta t)^2$$

Solve for the acceleration.

$$a = \frac{x_f - x_i - v_f\,\Delta t}{-\frac{1}{2}\,(\Delta t)^2}$$

Let the car begin braking at the origin, with positive x to the right. Then

$$a = \frac{(+35\,\mathrm{m}) - (0\,\mathrm{m}) - (+7.5\,\mathrm{m/s})\,(3.8\,\mathrm{s})}{-\frac{1}{2}\,(3.8\,\mathrm{s})^2} = -0.90\,\mathrm{m/s^2}$$

The sign makes sense, as positive was defined to the right, and velocity becomes less and less rightward. But that's a consequence of the coordinate system that was chosen. The magnitude is independent of coordinate system.

$0.90\,\mathrm{m/s^2}$

1. (6 points) In the problem above, describe the angle between the car's velocity and acceleration while braking.

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Since the car is slowing, but not changing direction, the velocity and acceleration must be in opposite directions.

The angle between the velocity and acceleration is 180°.

III. (16 points) An object moves in one dimension with a position x that depends on time t according to

$$x = At^4 + Bt^2 + C$$

where $A = 0.20\,\mathrm{m/s^4}$, $B = -0.40\,\mathrm{m/s^2}$, and $C = 0.80\,\mathrm{m}$. What is the object's average acceleration between times $t = -1.5\,\mathrm{s}$ and $t = 2.0\,\mathrm{s}$?

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Since the object is moving in one dimension, signs indicate direction, and vector notation is unnecessary.

The average acceleration is the difference in the instantaneous velocities, divided by the time interval,

$$a_{\mathrm{avg}} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

The instantaneous velocity is related to the position,

$$v = \frac{dx}{dt} = \frac{d}{dt}\left[At^4 + Bt^2 + C\right] = 4At^3 + 2Bt$$

So

$$v_f = 4At_f^3 + 2Bt_f = 4\left(0.20\,\mathrm{m/s^4}\right)(2.0\,\mathrm{s})^3 + 2\left(-0.40\,\mathrm{m/s^2}\right)(2.0\,\mathrm{s}) = 4.8\,\mathrm{m/s}$$

and

$$v_i = 4At_i^3 + 2Bt_i = 4\left(0.20\,\mathrm{m/s^4}\right)(-1.5\,\mathrm{s})^3 + 2\left(-0.40\,\mathrm{m/s^2}\right)(-1.5\,\mathrm{s}) = -1.5\,\mathrm{m/s}$$

so

$$a_{\mathrm{avg}} = \frac{v_f - v_i}{t_f - t_i} = \frac{(4.8\,\mathrm{m/s}) - (-1.5\,\mathrm{m/s})}{(2.0\,\mathrm{s}) - (-1.5\,\mathrm{s})} = 1.8\,\mathrm{m/s^2}$$

2. (6 points) In the problem above, is the object at the origin at time $t = 0$? Is at rest at time $t = 0$?

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When $t = 0$, $x = C \neq 0$, so the object is not at the origin. But since the expression for position has no t^1 term, there will be no constant term in the expression for v. When $t = 0$, $v = 0$.

At time $t = 0$, the object is not at the origin, but it is at rest.

3. (8 points) A cannon is a distance d from a cliff with height h. It is aimed 30° above the horizontal, and fires a shell that lands on the plain below, as shown. If the canon is aimed 60° above the horizontal and fires a shell with the same launch speed, how should the canon be positioned so this shell lands in the same position as the one fired at 30°? (*On Earth.*)

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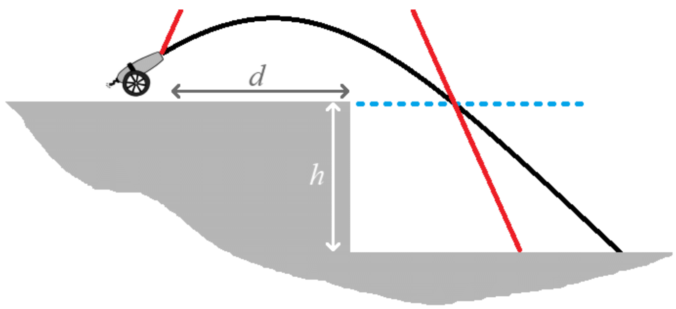
If the ground were level, and the cannon kept in the same position, the shell fired at 60° would land in the same place as the shell fired at 30°. But firing on to the plain below would result in the 60° shell falling short. For it to land in the same position as the shell fired at 30°,

The cannon should be moved closer to the edge of the cliff (that is, to the right).



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4. (8 points) An airline pilot wants to fly directly north from San Francisco, California, to Eugene, Oregon. There is a steady wind from the northwest, so they point their 'plane at an angle θ west of north. On the southward return flight, the same steady wind blows from the northwest, so they point their 'plane at an angle ϕ west of south. If it can be determined, how are θ and ϕ related? (*On Earth.*)

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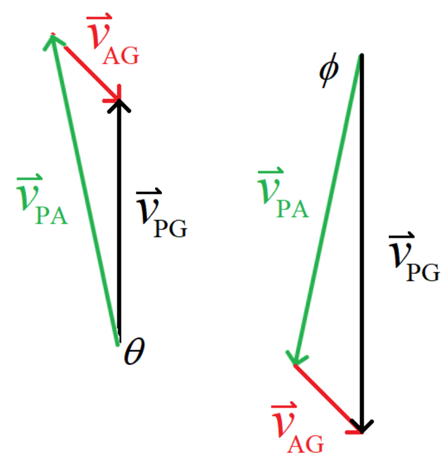
In both cases, the E-W component of the 'plane's velocity through the air must be equal in magnitude but opposite in direction to the E-W component of the wind's velocity over the ground. This makes the E-W component of the 'plane's velocity over the ground equal to zero (that is, it flies directly north or south).

Since the magnitude of the velocity of the 'plane through the air is the same in each case, and the E-W component is the same in each case, then the angle between the velocity of the 'plane through the air and its N-S component must be the same in each case.

$$\phi = \theta$$



5. (8 points) Consider an object moving with the motion diagram shown. What is the direction of the object's average acceleration around point 3?

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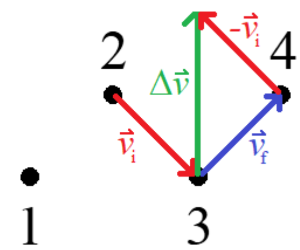

The average acceleration is the difference in the instantaneous velocities, divided by the time interval,

$$\vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{\vec{v}_f + (-\vec{v}_i)}{\Delta t}$$

As Δt is positive, $\vec{a}_{\text{avg}}$ is in the same direction as $\Delta\vec{v}$, which is constructed graphically in the figure. The direction of the average acceleration around point 3 is

Up the page.

6. (8 points) When ball 1 is released from the top of the ramp, it has speed v when it reaches the bottom. When ball 2 is launched up the ramp with speed v, it stops when it reaches the top. Consider the situation in which ball 1 is released from the top, and ball 2 is launched with speed v from the bottom at the same time. If it can be determined, where is their meeting point in relation to the midpoint of the ramp? (*On Earth.*)

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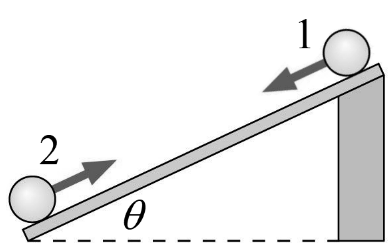
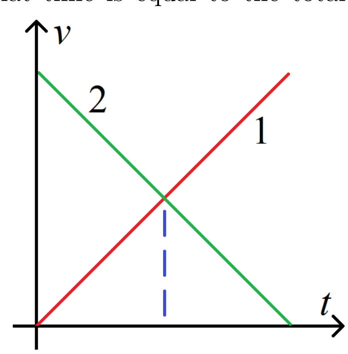
Let positive be down the ramp for ball 1, and up the ramp for ball 2. Consider the graph of their velocities as a function of time. If it takes each ball a time t to make its trip up or down the ramp, the two balls have the same speed at time $t/2$, as shown. The area under each curve to that time $t/2$ represents the distance each ball has traveled to that time. Notice that the sum of the areas at that time is equal to the total area under one curve. Since the two balls, together, have traveled a distance equal to the total distance, the balls must meet at time $t/2$. At that time, the area under the curve for ball 2 is greater than the area under the curve for ball 1. Ball 2, which was launched up the ramp, has traveled a greater distance than ball 1 by the time they meet.





They meet above the midpoint of the ramp.

7. (8 points) An object moving in one dimension has an acceleration that depends on time, as shown. If the object is at rest when time $t = 0$, at what point does the object have its maximum speed?

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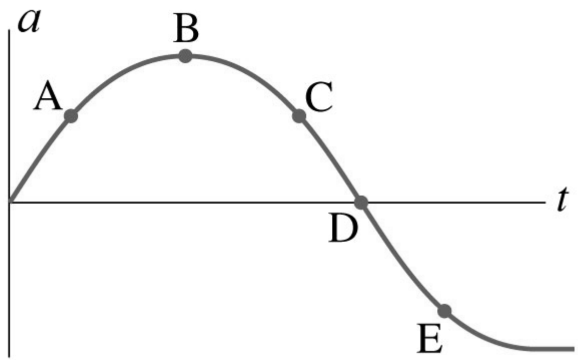

As the object is moving in one dimension, the sign of direction and velocity indicates their direction. Acceleration changes the velocity of an object according to

$$\Delta v = \int a\, dt$$

that is, when acceleration is graphed as a function of time, the change in velocity is the area under the curve.

In this case, the object starts at rest. As time increases, the area under the curve increases up to point D, so the velocity (and speed) increases. Beyond point D, the area under the curve is negative, so the velocity (and the speed) begins to decrease. Maximum speed, then, occurs at point

D