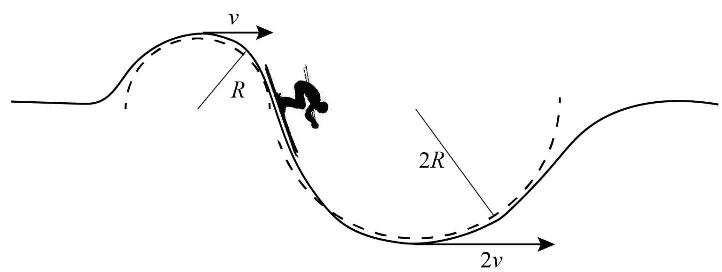
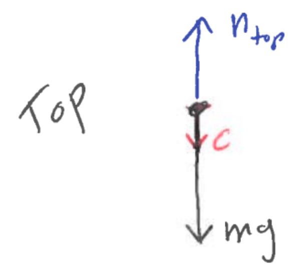

I. (16 points) A skier goes over a small hill (of radius R) and through a larger trough (of radius $2R$) on the other side. At the top of the hill their speed is v; at the bottom of the trough they're moving faster, with speed $2v$). Their apparent weight at the top of the hill is $W_{\text{top}} = 883\,\text{N}$; at the bottom of the trough it's $W_{\text{bot}} = 1180\,\text{N}$. Find the mass of the skier. (*On Earth.)*

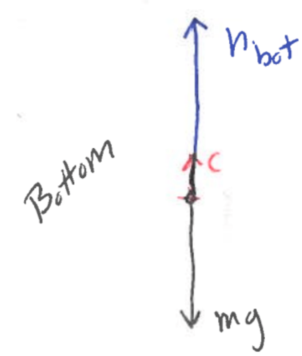


Use Newton's Second Law. Sketch a Free Body Diagram of the skier at the top of the hill with radius R. There is a gravitational force mg downward and a normal force n_{top} upward. Note that this normal force is the skier's apparent weight. Choose a coordinate system. I'll choose one with positive in the direction of the known acceleration, toward the center of the skier's circular path, or downward. Write Newton's Second Law for that axis (the sum of the forces is ...). I'll show signs explicitly, so symbols represent magnitudes.



$$\sum F_c = mg - n_{\text{top}} = ma_c = \frac{mv_{\text{top}}^2}{r_{\text{top}}} = \frac{mv^2}{R}$$

Now for the trough. Again, there is a gravitational force mg downward and a normal force n_{bot} upward. I'll choose an axis with positive in the direction of the known acceleration, toward the center of the skier's circular path, or upward.



$$\sum F_c = n_{\text{bot}} - mg = ma_c = \frac{mv_{\text{bot}}^2}{r_{\text{bot}}} = \frac{m\,(2v)^2}{2R} = 2\frac{mv^2}{R}$$

So

$$\frac{mv^2}{R} = mg - n_{\text{top}} = \frac{n_{\text{bot}} - mg}{2}$$

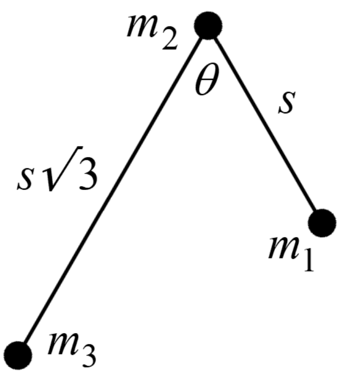
Solve for m.

$$2mg - 2n_{\text{top}} = n_{\text{bot}} - mg \qquad \Rightarrow \qquad 3mg = n_{\text{bot}} + 2n_{\text{top}}$$

So

$$m = \frac{n_{\text{bot}} + 2n_{\text{top}}}{3g} = \frac{1180\,\text{N} + 2\,(883\,\text{N})}{3\,(9.81\,\text{m/s}^2)} = 1.00 \times 10^2\,\text{kg}$$

1. (6 points) Three point masses, $m_1 = 1.0\,\text{kg}$, $m_2 = 2.0\,\text{kg}$, and $m_3 = 3.0\,\text{kg}$ are positioned as shown, with the distance $s = 11\,\text{cm}$. Compare the magnitude of the gravitational force $F_{1\text{on}2}$ of mass 1 on mass 2 with the magnitude of the gravitational force $F_{3\text{on}2}$ of mass 3 on mass 2.



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m_3 has three times the mass of m_1, but it's $\sqrt{3}$ times farther from m_2. Since the gravitational force follows an inverse-square law, those effects exactly cancel.

$$F_{1\text{on}2} = F_{3\text{on}2}$$

II. (16 points) In the problem above, the angle $\theta = 60.0°$. What is the magnitude of the gravitational force on mass m_2? *NOT on Earth!*

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Since $F_{1\text{on}2} = F_{3\text{on}2}$ the forces are symmetric about an axis up and down the page (let's call it y). The components of the gravitational force left and right on the page cancel. The net gravitational force is twice the y component of the force from one of m_1 or m_3. Let's find the force from m_1.

$$F_{1\text{on}2} = G\frac{m_1 m_2}{r^2_{1\text{to}2}} = G\frac{m_1 m_2}{s^2}$$

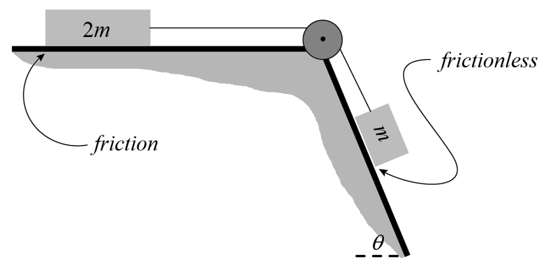
The y component of this force is

$$F_{1\text{on}2y} = G\frac{m_1 m_2}{s^2}\cos(\theta/2)$$

and the net force is

$$\begin{aligned} F_{\text{net}} &= 2F_{1\text{on}2y} = 2G\frac{m_1 m_2}{s^2}\cos(\theta/2) \\ &= 2\left(6.673\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{(1.0\,\text{kg})\,(2.0\,\text{kg})}{(0.11\,\text{m})^2}\cos\left(\frac{60.0°}{2}\right) = 1.9\times10^{-8}\,\text{N} = 19\,\text{nN} \end{aligned}$$

III. A block of mass $2m$, on a horizontal surface with coefficients of kinetic and static friction μ_k and μ_s, respectively, is pulled by a block of mass m that slides down a frictionless slope that makes an angle θ with the horizontal. Find the magnitude of the tension in the rope in terms of parameters defined in the problem, and physical or mathematical constants. You may assume that the slope is steep enough that the blocks do slide. (*On Earth.*)



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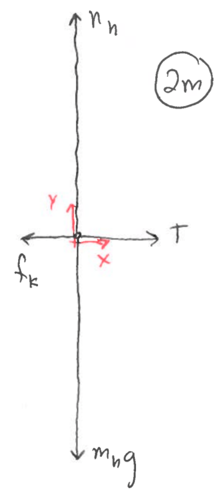
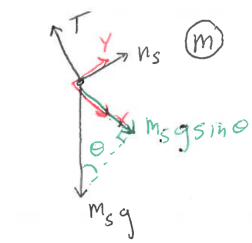
Use Newton's Second Law. Sketch a Free Body Diagram of the block on the horizontal surface. There is a gravitational force $m_h g$ downward and a normal force n_h upward. There's a leftward kinetic friction force f_k and a rightward tension T. Choose a coordinate system. I'll choose y vertically upward, and x positive in the direction of the known acceleration, toward the right. Write Newton's Second Law for each axis (the sum of the forces is ...). I'll show signs explicitly, so symbols represent magnitudes.



$$\sum F_y = n_h - m_h g = m_h a_y = 0 \quad \Rightarrow \quad n_h = m_h g = 2mg$$

$$\sum F_x = T - f_k = m_h a_x \quad \Rightarrow \quad T - \mu_k n_h = 2ma_x \quad \Rightarrow \quad T - \mu_k (2mg) = 2ma_x$$

Now for the block on the slope. There is a gravitational force $m_s g$ downward and a normal force n_s perpendicular to the slope (up and to the right). There's a tension T parallel to the slope (up and to the left). Choose a coordinate system. I'll choose y perpendicular to the slope, and x positive in the direction of the known acceleration, parallel to the slope (down and to the right). With this choice, the gravitational force must be resolved into components. Note that this choice of axes makes a_x for the block on the slope the same as that for the block on the horizontal surface.



$$\sum F_x = m_s g_x - T = m_s a_x \quad \Rightarrow \quad mg\sin\theta - T = ma_x$$

The two x equations have two unknowns, T and a_x. Eliminate a_x.

$$a_x = \frac{mg\sin\theta - T}{m} = \frac{T - 2\mu_k mg}{2m} \quad \Rightarrow \quad 2mg\sin\theta - 2T = T - 2\mu_k mg \quad \Rightarrow \quad 2mg\sin\theta + 2\mu_k mg = 3T$$

So

$$T = \frac{2mg}{3}\left(sin\theta + \mu_k\right)$$

2. (6 points) Let the coefficient of static friction on the horizontal surface in the problem above be 1/4. If the slope is insufficiently steep, the blocks will not slide at all. At what angle θ above the horizontal does this happen?

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See Free Body Diagrams above. When the blocks don't slide, their accelerations are zero.

$$2m: \quad \sum F_x = T - f_k = m_h a_x = 0 \quad \Rightarrow \quad T = \mu_k n_h = \mu_k 2mg$$

$$m: \quad \sum F_x = m_s g_x - T = m_s a_x = 0 \quad \Rightarrow \quad mg\sin\theta' = T = \mu_k 2mg$$

So

$$\sin\theta' = 2\mu_k = 2\,(1/4) = 1/2 \quad \Rightarrow \quad \theta' = \sin^{-1}(1/2) = 30°$$

3. (8 points) A car is driving around a banked curve as shown. The car's trajectory lies in a horizontal plane, with the center of the curve to the left. What are the directions of the acceleration of the car, and the frictional force on the car?

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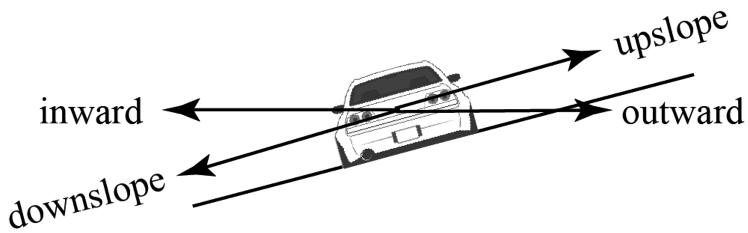
The acceleration must be toward the center of the car's circular path, which is inward. But we don't know whether friction is preventing a slow car from sliding down into the center of the curve, or preventing a fast car from sliding up out of the curve.



Acceleration is inward; friction could be upslope or downslope.

4. (8 points) The newly discovered planet X is composed of material that is half as dense as the Earth, while its diameter is three times the Earth's. How does the acceleration due to gravity at the surface of planet X (g_X) compare to the acceleration due to gravity on Earth (g_E)?

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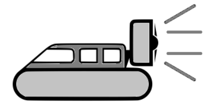
The force of gravity on an object at a planet's surface is

$$F = G\frac{Mm}{r^2} = mg \qquad \text{so} \qquad g = G\frac{M}{r^2} = G\frac{\rho V}{r^2} = G\frac{\rho \frac{4}{3}\pi r^3}{r^2} = \frac{4}{3}G\rho\pi r$$

The gravitational acceleration, then, is proportional to both the density ρ and the radius r. Since planet X has half the density and three times the diameter (and therefore radius) of Earth,

$$g_X = 3g_E/2$$

5. (8 points) A hovercraft, powered by a fan at its stern, is heading south. It turns 90° to the east, slowing as it does so, as shown in the top views below. When it is halfway around its turn, how should it be oriented?



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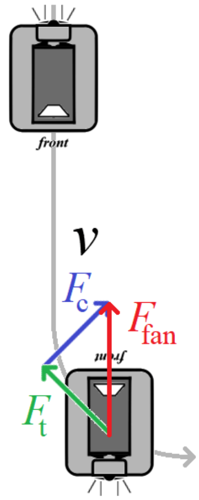
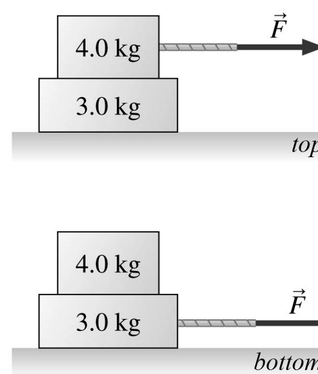
If the hovercraft is to both turn and slow, the fan force F_{fan} must have both a component F_{c} toward the center of the turn, and a component F_{t} tangential opposite the velocity. The only choice satisfying those requirements is



Orientation *v*.

6. (8 points) A 4.0 kg block and a 3.0 kg block are stacked on a level frictionless surface, as shown. A horizontal force $\vec{F}$ may be applied to the top block, or that same horizontal force $\vec{F}$ may be applied to the bottom block. In either case, friction between the blocks keeps them moving together. (*On Earth.*)



When the force $\vec{F}$ is applied to the bottom block, what is the direction of the friction force on the bottom block? How does the magnitude of the friction force on the bottom block compare to the magnitude on it when the force $\vec{F}$ is applied to the top block?

When the force $\vec{F}$ is applied to the bottom block, the friction force on the bottom block . . .

. . . .

When the force $\vec{F}$ is applied to the bottom block, static friction from the bottom block on the top block is the force accelerating the top block to the right. Newton's Third Law tells us that the friction force from the top block on the bottom block must be to the left.

The two-block combination has the same acceleration in both situations (same total mass, same net external force). Each block, individually, must have that same acceleration. Since the top block has a greater mass than the bottom block, the friction force to provide it with that acceleration must be greater than the friction force that accelerates the lower-mass bottom block when $\vec{F}$ is applied to the top block. So

When the force $\vec{F}$ is applied to the bottom block, the friction force on the bottom block . . .

is **to the left**, and its magnitude is **greater than** when $\vec{F}$ is applied to the top block.

7. (8 points) Four blocks, with masses 1.0, 2.0, 3.0, and 4.0 kg, are tied together with ideal cords as shown, and pulled to the right on a frictionless level surface by horizontal pull force $\vec{P}$. Compare the tension magnitude T in the cord connecting the 3.0 and 4.0 kg blocks, with the magnitude of the pull force. (*On Earth.*)

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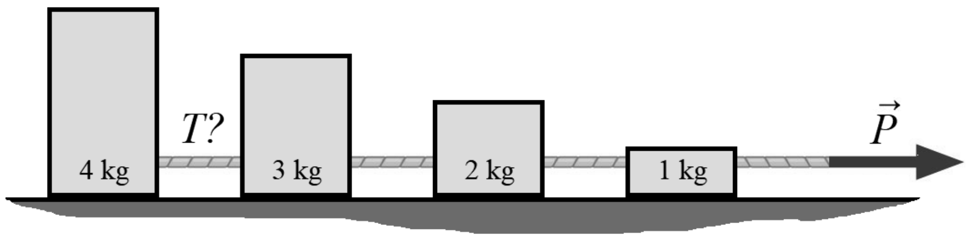

Use Newton's Second Law. The mass of the four blocks together is 10 kg, so the pull force causes an acceleration of

$$a = \frac{F}{m} = \frac{P}{10\,\mathrm{kg}}$$

which must also be the acceleration of each block individually. The tension T provides that acceleration to the 4 kg block.

$$a = \frac{F}{m} = \frac{P}{10\,\mathrm{kg}} = \frac{T}{4\,\mathrm{kg}} \qquad \Rightarrow \qquad T = \frac{4\,\mathrm{kg}}{10\,\mathrm{kg}} P = 0.40P$$