
I. (16 points) A block of mass $m = 6.0\,\mathrm{kg}$ slides upward along a ceiling under the influence of a vertical push force $\vec{P}$ (magnitude $75\,\mathrm{N}$). The coefficient of kinetic friction between the block and the ceiling is $\mu_{\mathrm{k}} = 0.35$. If the ceiling makes an angle $\theta = 25°$ with the horizontal, what is the acceleration (magnitude and direction) of the block? (*On Earth.*)

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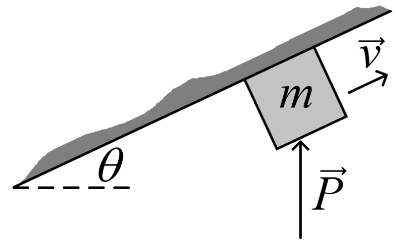
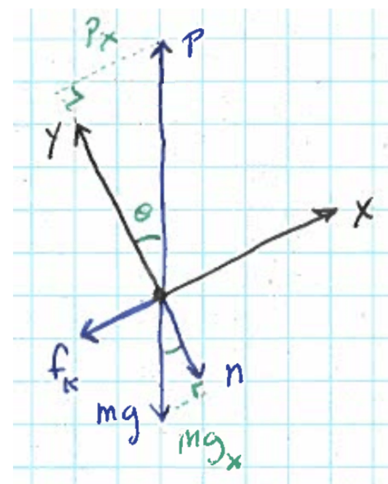


Use Newton's Second Law. Sketch a Free Body Diagram. There is a normal force $\vec{n}$, the upward push force $\vec{P}$, a kinetic friction force $\vec{f}_k$ opposite the velocity, and a gravitational force $m\vec{g}$ downward. Choose a coordinate system. I'll choose one that has the x axis parallel up along the ceiling (a possible direction of the acceleration) and the y axis up perpendicular to the ceiling (a direction in which the acceleration must be zero). Note which angles in the FBD are θ.

Write Newton's Second Law (the sum of the forces is ...) for each axis. I'll show signs explicitly, so symbols represent magnitudes.



$$\sum F_x = P_x - mg_x - f_k = ma_x \quad \Rightarrow \quad P\sin\theta - mg\sin\theta - \mu_k n = ma_x$$

$$\sum F_y = P_y - mg_y - n = ma_y = 0 \quad \Rightarrow \quad n = P\cos\theta - mg\cos\theta$$

Substitute the expression for n found from the y components of the forces, into the expression for the x components of the forces.

$$P\sin\theta - mg\sin\theta - \mu_k\left(P\cos\theta - mg\cos\theta\right) = ma_x$$

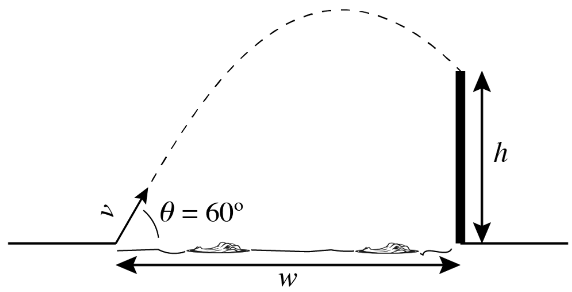
Solve for the acceleration.

$$\begin{aligned} ma_x &= P\sin\theta - mg\sin\theta - \mu_k P\cos\theta + \mu_k mg\cos\theta \\ &= P\left(\sin\theta - \mu_k\cos\theta\right) + mg\left(\mu_k\cos\theta - \sin\theta\right) \\ a_x &= \frac{P}{m}\left(\sin\theta - \mu_k\cos\theta\right) + g\left(\mu_k\cos\theta - \sin\theta\right) \\ &= \frac{75\,\mathrm{N}}{6.0\,\mathrm{kg}}\left(\sin 25° - 0.35\cos 25°\right) + \left(9.81\,\mathrm{m/s^2}\right)\left(0.35\cos 25° - \sin 25°\right) \\ &= 0.28\,\mathrm{m/s^2} \end{aligned}$$

Since the positive x axis was chosen up along the ceiling, this represents an acceleration of

$0.28\,\mathrm{m/s^2}$ up along the ceiling

II. (16 points) You need to jump over a wall (of height h) that stands behind a moat (of width w). If you launch yourself at a 60° angle above the horizontal, what is the minimum jumping speed that will allow you to clear the wall? Simplify your answer using the fact that $\sin 60° = \sqrt{3)}/2$ and $\cos 60° = 1/2$. (*On Earth.*)



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This is a projectile motion problem, which is a special case of constant-acceleration kinematics, with the acceleration on one axis being zero, and the acceleration on the other being due to gravity. I'll chose the x axis to be horizontal with positive to the right, and the y axis vertical with positive upward. Let the origin be at the launch point.

$$x = x_0 + v_{0x}\,\Delta t + \tfrac{1}{2}a_x\,(\Delta t)^2 \quad \Rightarrow \quad w = 0 + v_0\cos\theta\,\Delta t + \tfrac{1}{2}\,(0)\,(\Delta t)^2 \quad \Rightarrow \quad \Delta t = \frac{w}{v_0\cos\theta}$$

and

$$y = y_0 + v_{0y}\,\Delta t + \tfrac{1}{2}a_y\,(\Delta t)^2 \quad \Rightarrow \quad h = 0 + v_0\sin\theta\,\Delta t - \tfrac{1}{2}g\,(\Delta t)^2$$

Substitute the expression found for Δt,

$$h = v_0\sin\theta\left(\frac{w}{v_0\cos\theta}\right) - \tfrac{1}{2}g\left(\frac{w}{v_0\cos\theta}\right)^2$$

and solve for v_0.

$$h\cos^2\theta = w\sin\theta\cos\theta - \tfrac{1}{2}g\left(\frac{w}{v_0}\right)^2$$

$$\frac{gw^2}{v_0^2} = 2\left(w\sin\theta\cos\theta - h\cos^2\theta\right)$$

$$v_0^2 = \frac{gw^2}{2\left(w\sin\theta\cos\theta - h\cos^2\theta\right)} = \frac{gw^2}{2\left(w\left(\sqrt{3}/2\right)(1/2) - h\,(1/2)^2\right)} = \frac{gw^2}{2\left(w\sqrt{3}/4 - h/4\right)} = \frac{2gw^2}{w\sqrt{3} - h}$$

So

$$v_0 = \sqrt{\frac{2gw^2}{w\sqrt{3} - h}}$$

1. (6 points) If the wall is too high, you will not be able to clear it (no matter how fast you jump). As a function of your jump angle θ above the horizontal, what is the highest wall H you can possibly clear? *Hint:* you can re-solve the previous problem again for a general launch angle θ, but it is also possible to figure this out by simple geometric reasoning.

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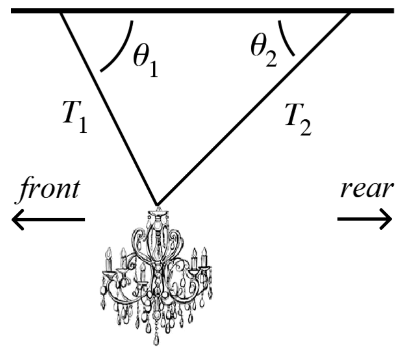
Imagine jumping *infinitely* fast. You'd reach the wall in zero time, before your trajectory had any time to fall below a straight line. After traveling horizontally a distance w, you'd be a height H above the ground.

$$\tan\theta = H/w \qquad \text{so} \qquad H = w\tan\theta$$

III. A 53 kg chandelier in a luxury train is suspended by two wires, which make angles $\theta_1 = 60.0°$ and $\theta_2 = 45.0°$ with respect to the ceiling, as shown. Find the tension magnitudes T_1 and T_2 while the train is stationary at a station. (*On Earth.*)



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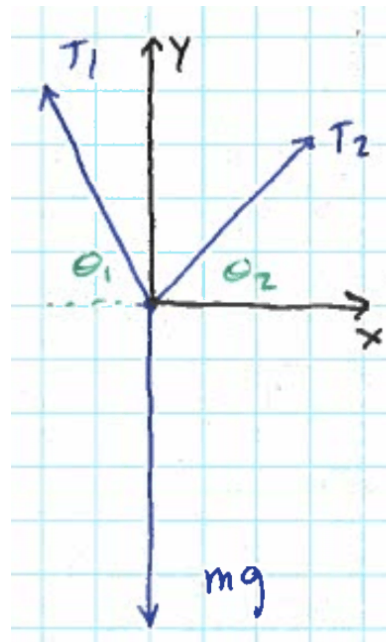
Use Newton's Second Law. Sketch a Free Body Diagram. There two tension forces, $\vec{T_1}$ and T_2 at angles θ_1 and θ_2 below the horizontal, and a gravitational force $m\vec{g}$ downward. Choose a coordinate system. I'll choose one that has the x axis horizontal to the right, and the y axis vertical upward.

Write Newton's Second Law (the sum of the forces is ...) for each axis. I'll show signs explicitly, so symbols represent magnitudes.



$$\sum F_x = T_{2x} - T_{1x} = ma_x = 0 \quad \Rightarrow \quad T_2 \cos\theta_2 - T_1 \cos\theta_1 = 0$$

$$\sum F_y = T_{2y} + T_{1y} - mg = ma_y = 0 \quad \Rightarrow \quad T_2 \sin\theta_2 + T_1 \sin\theta_1 = mg$$

That's two equations with two unknowns (T_1 and T_2). I'll eliminate T_2 solve the x equation for it and substituting into the y equation.

$$T_2 \cos\theta_2 = T_1 \cos\theta_1 \quad \Rightarrow \quad T_2 = T_1 \frac{\cos\theta_1}{\cos\theta_2}$$

Then

$$\left(T_1 \frac{\cos\theta_1}{\cos\theta_2}\right) \sin\theta_2 + T_1 \sin\theta_1 = mg$$

$$T_1 \left(\frac{\cos\theta_1}{\cos\theta_2} \sin\theta_2 + \sin\theta_1\right) = mg$$

$$T_1 (\cos\theta_1 \sin\theta_2 + \sin\theta_1 \cos\theta_2) = mg \cos\theta_2$$

So

$$T_1 = \frac{mg\cos\theta_2}{\cos\theta_1 \sin\theta_2 + \sin\theta_1 \cos\theta_2} = \frac{(53\,\mathrm{kg})\left(9.81\,\mathrm{m/s^2}\right)\cos 45.0°}{\cos 60.0° \sin 45.0° + \sin 60.0° \cos 45.0°} = 380.6\,\mathrm{N}$$

Substitute this value of T_1 into the expression previously found for T_2.

$$T_2 = T_1 \frac{\cos\theta_1}{\cos\theta_2} = (380.6\,\mathrm{N}) \frac{\cos 60.0°}{\cos 45.0°} = 269.1\,\mathrm{N}$$

To two significant figures,

$$T_1 = 380\,\mathrm{N} \quad \text{and} \quad T_2 = 270\,\mathrm{N}$$

2. (6 points) The train leaves the station, accelerating to the left. What happens to the tensions in the wires?

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If the chandelier accelerates to the left, the net horizontal force must be to the left. This could be accomplished by T_1 increasing or T_2 decreasing. However, the net vertical force cannot change, so if one tension increases, the other must decrease.

T_1 increases. T_2 decreases.

3. (8 points) You push a book horizontally against the wall with just enough force force to prevent it from sliding down. The coefficients of static and kinetic friction between the book and the wall are the same as between the book and your hand: 0.8 and 0.6, respectively. (*On Earth.*)

If you push on the book twice as hard, what happens to amount of frictional force on the book?

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The book does not accelerate vertically, so the net vertical force is zero. The upward static friction force and the downward gravitational force must have the same magnitude. Pushing harder on the book doesn't change this. The friction force,

It stays the same.

4. (8 points) When a sphere of radius R and mass m is dropped through the atmosphere, it reaches a terminal speed v_{T}. When it is towed at constant speed behind a horizontally-flying airplane, the tow-rope is at an angle of 45° below the horizontal. What is the speed of the airplane? (*On Earth, do NOT neglect drag.*)

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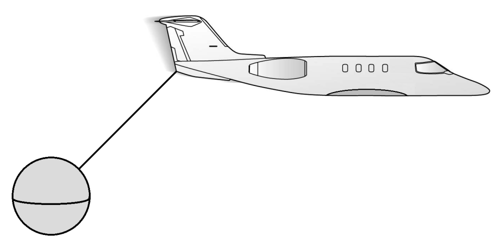
If the tow-rope is 45° below the horizontal, then the horizontal and vertical components of the tension must have the same magnitude. The horizontal component balances the drag force, and the vertical component balances the gravitational force. Therefore, the drag force and gravitational force must have the same magnitude. This is the condition for traveling at the terminal speed.



v_{T}

5. (8 points) Zouhair's mass is 75 kg. When he stands on a bathroom scale in an elevator, the reading is 700 N. Describe the motion of the elevator. (*On Earth.*)

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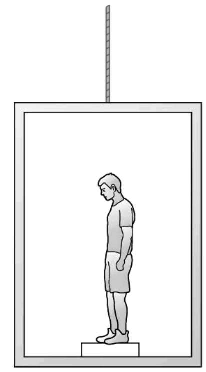

The gravitational force on Zouhair is $mg = (75\,\text{kg})\left(9.8\,\text{m/s}^2\right) = 735\,\text{N}$ downward. The 700 scale reading is the normal force upward on him. The net force on Zouhair, then, is 35 N downward. His acceleration must be downward. The elevator can't remain stationary with a non-zero acceleration, but it could either be moving up and slowing, or moving down with increasing speed. The elevator,

It could be moving up or down, but can't remain stationary.

6. (8 points) An airplane pilot wishes to fly directly north. Because of a wind from the west, however, if he points the nose of his aircraft directly north, he finds himself flying over the ground at an angle θ east of north (*Fig. 1*). To compensate, the pilot points the nose of his aircraft at an angle ϕ west of north, so he flies over the ground directly to the north (*Fig. 2*). How does the angle ϕ compare to the angle θ, and how does the time required to reach his destination compare to the time required in still air?

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Note that the missing side of the velocity triangle is the velocity of the air over the ground. The velocity of the plane through the air is a leg of the triangle in Fig. 1, while it is the hypotenuse in Fig. 2. Therefore, the velocity of the plane over the ground in Fig. 2 must be less than the velocity of the plane through the air in Fig. 2 *and* in Fig. 1. The angle ϕ, then, must be greater than the angle θ. Since the velocity of the plane over the ground in Fig. 2 is only a component of the velocity of the plane though the air, then the plane travels more slowly toward its destination then it would in still air.

$\phi > \theta$ and the time is **greater than** that required in still air.

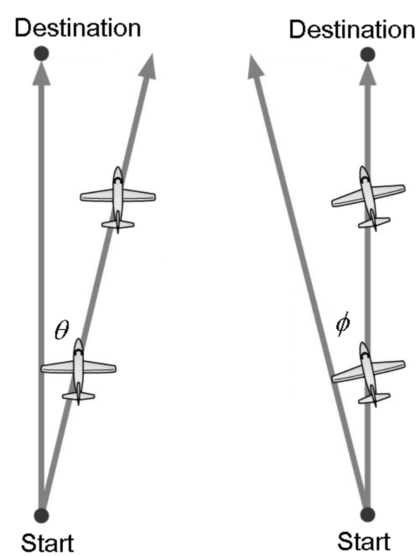

Fig. 1 *Fig. 2*

7. (8 points) A pendulum swings to the left. It stops momentarily at position 1 on the extreme left, then swings back to the right through positions 2 through 5. At the instant it is passing rightward through position 4, what is the direction of the bob's acceleration? (*On Earth.*)

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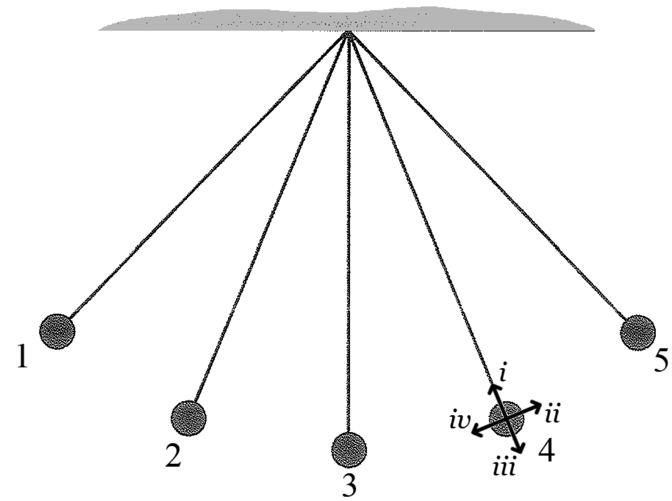

As the bob swings rightward through position 4, it is slowing. The tangential component of its acceleration, then, must be opposite its velocity, or in direction *iv*. But since the bob is moving in a circular arc, its acceleration also has a centripetal component, v^2/r, toward the center of the arc in direction 1. With these as its components, the acceleration of the bob must be directed

Between directions *i* and *iv*.