
I. (16 points) It's a hot day at the park, and you'd like a drink of water. You know there's a water fountain (let's call it "A") 160 m north and 55 m east of your current location. But there's a closer one ("B"), 116 m away, 21° north of East. After walking to this closer one, though, you find it is out of order. How far, and in what direction, must you walk directly from "B" to "A"? *Be sure to specify, clearly, the reference for your direction!*

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You've walked to foundtain "B" a displacement $\vec{B}$. You must now walk a displacment $\vec{C}$ to get to fountain "A" at $\vec{A}$. Symbolically,

$$\vec{B} + \vec{C} = \vec{A} \qquad \text{so} \qquad \vec{C} = \vec{A} - \vec{B}$$

In the north direction,

$$C_n = A_n - B_n = A_n - B\sin\theta = (160\,\mathrm{m}) - (116\,\mathrm{m})\sin 21° = 118.4\,\mathrm{m}$$

In the east direction,

$$C_e = A_e - B_e = A_e - B\sin\theta = (55\,\mathrm{m}) - (116\,\mathrm{m})\cos 21° = -53.3\,\mathrm{m}$$

Use the Pythagorean Theorem to find the magnitude of the distance you must walk.

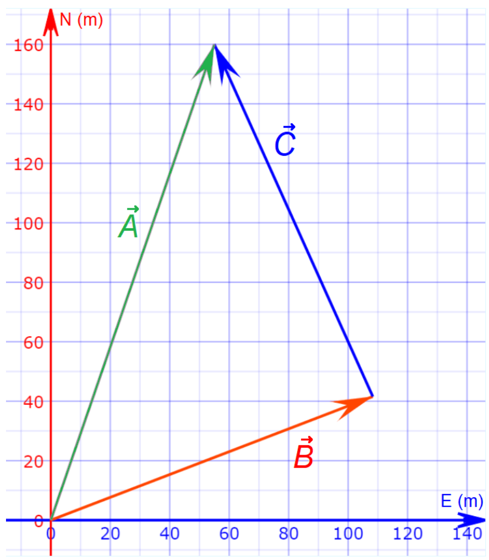
$$\left|\vec{C}\right| = \sqrt{C_e^2 + C_n^2} = \sqrt{(-53.3\,\mathrm{m})^2 + (118.4\,\mathrm{m})} = 129.9\,\mathrm{m}$$

Find the direction using $\tan\phi = C_n C_e$, so

$$\phi = \tan^{-1}\left(\frac{C_n}{C_e}\right) = \tan^{-1}\left(\frac{118.4\,\mathrm{m}}{-53.3\,\mathrm{m}}\right) = -65.6°$$

But (as can be seen in the diagram) the inverse tangent yields the wrong quadrant when C_e is negtive. Adding 180° to the direction, you must walk

130 m at 114° N of E



1. (6 points) A toy car is rolling along a horizontal workbench, when at time $t = 0$ it encounters a large patch of epoxy that is in the process of drying. The epoxy slows down the car, and as it dries this effect becomes larger and larger: quantitatively, the epoxy produces an acceleration $a(t) = kt$, directed opposite to the car's motion, with $k = 0.10\,\text{m/s}^3$.

Which equation, if any, shows the relationship between the car's position, velocity, and acceleration as it moves through the epoxy?

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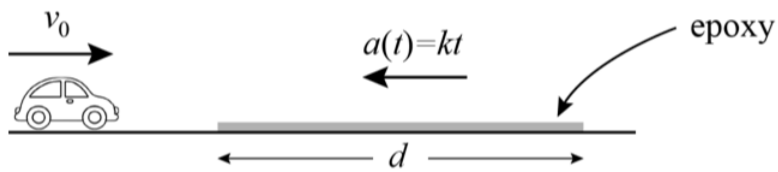

The acceleration of the car depends on time, so it is not constant. No constant-accelation kinematics equation can describe the car's motion.

None of the others is correct.

II. (16 points) In the problem above, if you launch the car with a speed of $0.20\,\text{m/s}$, at what distance into the epoxy does the car come to a halt?

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In one dimension, signs represent direction, so vector notation isn't necessary. If we let the initial velocity of the car be positive, its acceleration is negative, $-kt$. As $a = dv/dt$, an expression for the velocity can be found.

$$v = \int dv = \int a\,dt = \int -kt\,dt = -k\frac{t^2}{2} + v_0$$

The time at which the car stops ($v = 0$) can be found:

$$0 = -k\frac{t^2}{2} + v_0 \qquad \Rightarrow \qquad k\frac{t^2}{2} = v_0 \qquad \Rightarrow \qquad t = \sqrt{\frac{2v_0}{k}}$$

Then, as $v = dx/dt$, an expression for position can be found.

$$x = \int dx = \int v\,dt = \int \left(-k\frac{t^2}{2} + v_0\right) dt = -k\frac{t^3}{3 \cdot 2} + v_0 t + x_0$$

Substuting the expression for time, the displacement into the epoxy is

$$\Delta x = x - x_0 = -k\frac{t^3}{6} + v_0 t = -k\frac{(2v_0/k)^{3/2}}{6} + v_0\sqrt{\frac{2v_0}{k}}$$

$$= -\frac{0.10\,\text{m/s}^3}{6}\left(\frac{2\,(0.20\,\text{m/s})}{0.10\,\text{m/s}^3}\right)^{3/2} + (0.20\,\text{m/s})\sqrt{\frac{2\,(0.20\,\text{m/s})}{0.10\,\text{m/s}^3}} = 0.27\,\text{m}$$

III. A steel ball (mass m) is dropped from rest from a height h above the ground. At the same instant, a brass ball (mass $2m$), is launched upward with a velocity v_0, from ground level. If the steel ball is dropped from too large a height, the balls won't collide before the brass ball returns to the ground. Find the maximum drop height h for which the balls collide in the air. Express your answer in terms of parameters defined in the problem, and physical or mathematical constants. (*On Earth.*)

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When the steel ball is dropped from the maximum height, it spends the maximum time in the air. Therefore, it must hit the brass ball instantaneously before the brass ball lands on the ground.

Use constant-acceleration kinematics. Choosing positive upward, we can find the time the brass ball is in the air:

$$v = v_0 + a\,\Delta t \qquad \text{where} \qquad a = -g \quad \text{and} \quad v = -v_0 \qquad \text{so} \qquad -v_0 = v_0 - g\,\Delta t \qquad \Rightarrow \qquad \Delta t = \frac{2v_0}{g}$$

Remember that the brass ball's motion is symmetric about it's highest point, so it lands with the same speed it had at launch ($v = -v_0$).

The steel ball must be in the air for this same time. Still with positive upward,

$$x = x_0 + v_0\,\Delta t + \tfrac{1}{2}a\,(\Delta t)^2 \qquad \text{where} \qquad a = -g \quad \text{and} \quad v_0 = 0 \qquad \text{so} \qquad x = x_0 - \tfrac{1}{2}g\,(\Delta t)^2$$

If we choose the origin at ground level, then $x = 0$ and $x_0 = h$. Substitute the time found above.

$$0 = h - \tfrac{1}{2}g\,(\Delta t)^2 \qquad \Rightarrow \qquad h = \tfrac{1}{2}g\,(\Delta t)^2 = \tfrac{1}{2}g\left(\frac{2v_0}{g}\right)^2 = \frac{2v_0^2}{g}$$

2. (6 points) How would your answer for the maximum drop height h change if the mass of the brass ball is increased from $2m$ to $4m$?

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Acceleration is the same for all objects in free-fall.

h is unchanged.

3. (8 points) The position x of an object moving in one dimension is shown as a function of time t. At about what time, if any, is the instantaneous velocity of the object equal to its average velocity over the time interval 0 to 4.5 s?

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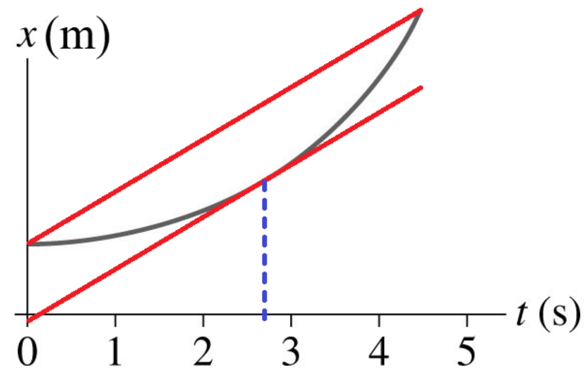
The average velocity is the displacement over the time, represented by the upper red line. If the instantaneous velocity, a tangent to the position curve, is to have the same value, it must have the same slope. This is represented by the lower red line, which is parallel (same slope) to the upper one. This line is tangent to the position curve

At about $t = 3\,\mathrm{s}$.



4. (8 points) The velocity v of an object moving in one dimension is shown as a function of time t. At what times in the range from zero to C is the object's speed increasing?

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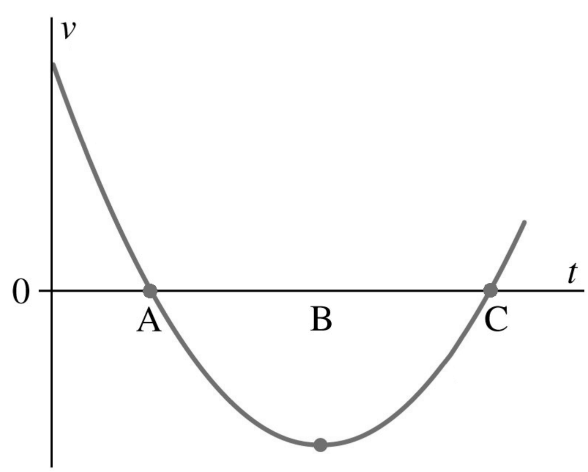
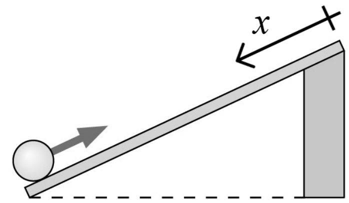
For an object's speed to be increasing, its velocity and acceleration must be in the same direction. In one dimension, this means they must have the same sign.

Between times zero and A, the velocity is positive, but the acceleration (slope of the velocity-tiime graph) is negative. Between times B and C the velocity is negative, but the acceleration is positive. The velocity and acceleration have the same sign (it happens to be negative)

Only between times A and B.



5. (8 points) An object is given an initial velocity up an inclined plane. It rolls halfway up the plane before it stops and rolls back down. If the origin is at the top of the plane with the positive direction down the plane, what is the graph of the object's velocity as a function of time? (*On Earth.*)



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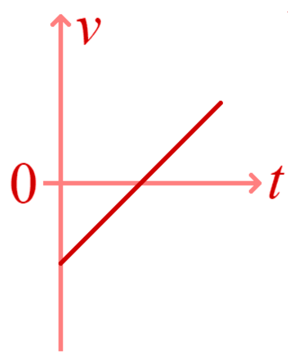
The object's initial velocity is negative (it's going up the plane, while positive is defined downward). It has constant acceleration, so the slope of the velocity-time graph must be constant (a straight line).



6. (8 points) Car A is traveling at constant velocity. Car B is at rest at the origin, and begins to travel with constant acceleration when car A passes at time $t = 0$. At what time, if at all, does car B catch up to car A?

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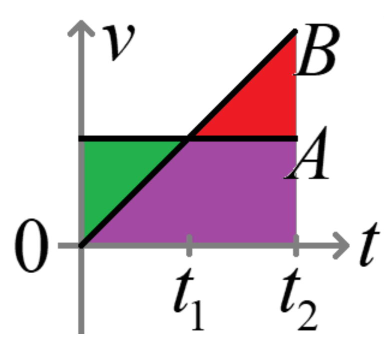
As $\Delta x = \int v\, dt$, displacement is the area under the curve on a velocity-time graph. Car B catches car A when their displacements are the same. The displacement of car A is represented by the rectangular area (green and purple). The displacement of car B is represented by the triangular area (green and red). The purple area is common to both cars, so car B's red area must equal car A's green area. This is achieved

At time t_2.



7. (8 points) Vector $\vec{A}$ has magnitude 1.0 m and vector $\vec{B}$ has magnitude 4.0 m, as shown. What is the magnitude of $3\vec{A} + \vec{B}$?

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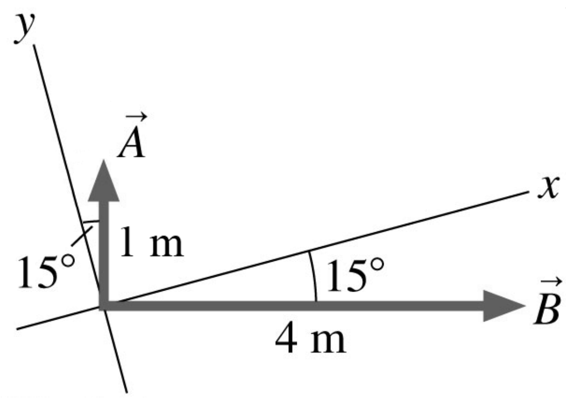

The magnitude of vectors is independant of coordinate system. $3\vec{A}$ and $\vec{B}$ are two legs of a 3-4-5 right triangle. The sum is the hypotenuse,

5.0 m