1. (16 points) An object moves in one dimension with a position that varies with time according to

$$x = At^4 + Bt^2 + C$$

where $A = 2\,\mathrm{m/s^4}$, $B = 3\,\mathrm{m/s^2}$, and $C = 4\,\mathrm{m}$. What is the average acceleration of the object from $t = -1\,\mathrm{s}$ to $t = +2\,\mathrm{s}$?

. . . .

The average acceleration is defined as

$$\vec{a}_{\mathrm{avg}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

The velocity can be found from the position by

$$\vec{v} = \frac{d\vec{x}}{dt} = \frac{d}{dt}\left(At^4 + Bt^2 + C\right) = 4At^3 + 2Bt$$

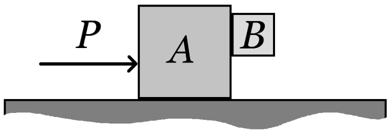
So

$$a_{\mathrm{avg}} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\left(4\left(2\,\mathrm{m/s^4}\right)\left(+2\,\mathrm{s}\right)^3 + 2\left(3\,\mathrm{m/s^2}\right)\left(+2\,\mathrm{s}\right)\right) - \left(4\left(2\,\mathrm{m/s^4}\right)\left(-1\,\mathrm{s}\right)^3 + 2\left(3\,\mathrm{m/s^2}\right)\left(-1\,\mathrm{s}\right)\right)}{\left(+2\,\mathrm{s}\right) - \left(-1\,\mathrm{s}\right)}$$

$$= \frac{\left(\left(64\,\mathrm{m/s}\right) + \left(12\,\mathrm{m/s}\right)\right) - \left(\left(-8\,\mathrm{m/s}\right) + \left(-6\,\mathrm{m/s}\right)\right)}{3\,\mathrm{s}} = \frac{\left(76\,\mathrm{m/s}\right) - \left(-14\,\mathrm{m/s}\right)}{3\,\mathrm{s}} = \frac{90\,\mathrm{m/s}}{3\,\mathrm{s}}$$

$$= 30\,\mathrm{m/s^2}$$

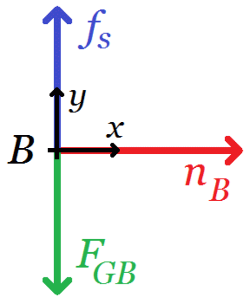
2. (16 points) Block A in the illustration, with mass $m_A = 5.6\,\text{kg}$, slides along a horizontal frictionless plane. Block B, with mass $m_B = 1.2\,\text{kg}$ has a coefficient of static friction $\mu_s = 0.75$ and a coefficient of kinetic friction $\mu_k = 0.45$ with the front face of block A. What is the minimum magnitude of a horizontal push force, P, on block A so that block B does not slide downward? (*On Earth.*)



. . . .

Use Newton's Second Law. One possible approach is to analyze block B by itself, then to analyze the AB combination.

Turning first to block B, draw a Free-Body Diagram, showing the normal force n_B from A, the gravitational force F_{GB}, and the force of static friction f_s. Choose a coordinate system. In the horizontal direction:



$$\sum F_x = n_B = m_B a_x$$

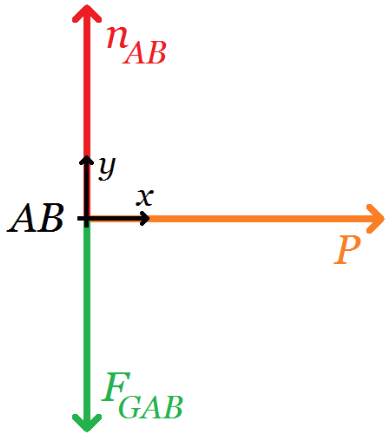
In the vertical direction:

$$\sum F_y = f_s - F_{GB} = m_B a_y = 0$$

For the minimum push force P, the force of static friction will be at its maximum value, so $f_s = \mu_s n_B$. Substitute the expression for n_B found from the horizontal equation.

$$\mu_s n_B - m_B g = 0 \qquad \Rightarrow \qquad \mu_s m_B a_x = m_B g$$

Look at the AB combination to find an expression for the acceleration. The Free-Body Diagram shows the normal force n_{AB} from the Earth's surface, the gravitational force F_{GAB}, and the push P. In the horizontal direction:



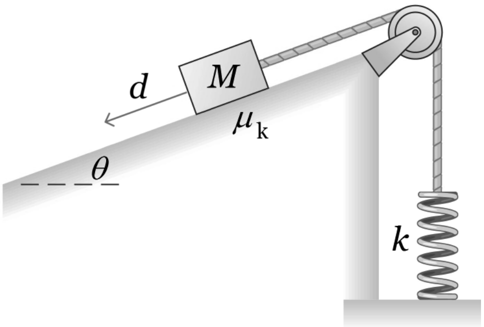
$$\sum F_x = P = m_{AB} a_x \qquad \Rightarrow \qquad a_x = \frac{P}{m_A + m_B}$$

Substitute this expression for a_x and solve for P.

$$\mu_s m_B \left(\frac{P}{m_A + m_B} \right) = m_B g \qquad \Rightarrow \qquad \frac{P}{m_A + m_B} = \frac{g}{\mu_s}$$

$$P = \frac{(m_A + m_B)\, g}{\mu_s} = \frac{(5.6\,\text{kg} + 1.2\,\text{kg})\,(9.81\,\text{m/s}^2)}{0.75} = 89\,\text{N}$$

3. (16 points) A block of mass $M = 15\,\text{kg}$ is held at rest on a frictionless slope that makes an angle $\theta = 12°$ with the horizontal. The block is attached to a vertical spring by an ideal cord that passes over an ideal pulley on a frictionless axle. The spring has Hooke's Law (spring) constant $k = 28\,\text{N/m}$ and is at its natural length (neither stretched nor compressed). Find the speed of the block when it has travelled a distance $d = 72\,\text{cm}$ down along the slope after release. (*On Earth.*)



Use the Energy Principle:

$$W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{th}}$$

Define the system to be the block, spring, Earth, and slope surface. Then there will be no work done by external forces. No internal forces change the thermal energy within the system. Gravitational and elastic (spring) potential energies must be considered.

$$0 = \left(\tfrac{1}{2}Mv_f^2 - \tfrac{1}{2}Mv_i^2\right) + (Mgy_f - Mgy_i) + \left(\tfrac{1}{2}k\,(\Delta s_f)^2 - \tfrac{1}{2}k\,(\Delta s_i)^2\right) + 0$$

The block starts at rest, so $v_i = 0$. Let $y_f = 0$, which makes $y_i = d\sin\theta$. The spring is initially at its natural length, so $\Delta s_i = 0$. When the block has moved a distance d, the spring will be stretched a distance d, so $\Delta s_f = d$.

$$0 = \left(\tfrac{1}{2}Mv_f^2 - 0\right) + (0 - Mgd\sin\theta) + \left(\tfrac{1}{2}kd^2 - 0\right) = \tfrac{1}{2}Mv_f^2 - Mgd\sin\theta + \tfrac{1}{2}kd^2$$

Solve for v_f:

$$\begin{aligned} v_f &= \sqrt{2gd\sin\theta - (kd^2/M)} \\ &= \sqrt{2\,(9.81\,\text{m/s}^2)\,(0.72\,\text{m})\sin 12° - \left((28\,\text{N/m})\,(0.72\,\text{m})^2/15\,\text{kg}\right)} \\ &= 1.4\,\text{m/s} \end{aligned}$$

4. ($4\frac{1}{3}$ points) If

$$\vec{A} = (6\hat{\imath} - 1\hat{\jmath})\,\text{m/s} \qquad \text{and} \qquad \vec{B} = (5\hat{\imath} - 2\hat{\jmath})\,\text{m/s}$$

what is the magnitude of $\vec{A} - 2\vec{B}$?

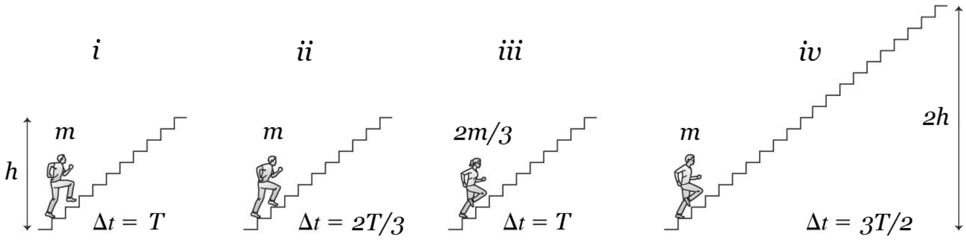
. . . .

$$\vec{A} - 2\vec{B} = (6\hat{\imath} - 1\hat{\jmath})\,\text{m/s} - 2\left((5\hat{\imath} - 2\hat{\jmath})\,\text{m/s}\right) = (6\hat{\imath} - 1\hat{\jmath})\,\text{m/s} - \left((10\hat{\imath} - 4\hat{\jmath})\,\text{m/s}\right) = (-4\hat{\imath} + 3\hat{\jmath})\,\text{m/s}$$

Use the Pythagorean Theorem to find the magnitude

$$\left|\vec{A} - 2\vec{B}\right| = \sqrt{(-4\,\text{m/s})^2 + (+3\,\text{m/s})^2} = \sqrt{16\,\text{m}^2/\text{s}^2 + 9\,\text{m}^2/\text{s}^2} = \sqrt{25}\,\text{m/s} = 5\,\text{m/s}$$

5. ($4\frac{1}{3}$ points) Four students with the indicated masses run up their staircases in the indicated times. Rank, from greatest to least, their power outputs.



. . . .

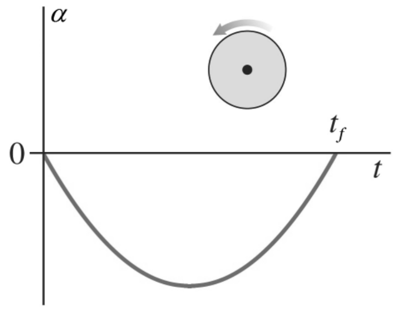
Power is the time rate of energy transformation, $P = dE/dt$ or $\Delta E/\Delta t$. For each of the students, the energy transformation is an increase in the gravitational potential energy of the student-Earth system.

$$P_i = \frac{mgh}{T} \qquad P_{ii} = \frac{mgh}{2T/3} = \frac{3}{2}\left(\frac{mgh}{T}\right) \qquad P_{ii} = \frac{(2m/3)\,mgh}{T} = \frac{2}{3}\left(\frac{mgh}{T}\right) \qquad P_{iv} = \frac{mg\,(2h)}{3T/2} = \frac{4}{3}\left(\frac{mgh}{T}\right)$$

so

$$ii > iv > i > iii$$

6. ($4\frac{1}{3}$ points) A wheel is rotating counterclockwise with initial angular velocity ω_0. Let this direction be positive. It is then given a non-uniform angular acceleration, α, shown in the graph, from time $t = 0$ to time $t = t_f$. How does the *magnitude* of the angular velocity, ω_f, at time t_f, compare to the *magnitude* of ω_0?



. . . .

At time $t = 0$ the angular velocity of the wheel is positive. The graph shows that the wheel has negative angular acceleration from from time $t = 0$ to time $t = t_f$ (that is the only relevant information on the graph). Therefore, the wheel's angular velocity will be less positive at time $t = t_f$ than it was at time $t = 0$. There are many ways that condition can be satisfied, including having a lower positive angular velocity, an angular velocity of zero, or any negative angular velocity.

This cannot be determined from the information provided.

7. ($4\frac{1}{3}$ points) Two blocks, one with mass m and one with mass $2m$, are traveling along level frictionless tracks with the same momenta. Identical applied forces $\vec{F}_A$ will be used to bring each block to a stop. Compare the *times* required to stop the blocks.

. . . .

The blocks have the same initial momentum, and have the same final momentum (zero). Therefore, they have the same momentum change, $\Delta\vec{p}$. As the impulse on the blocks, $\vec{J}$, is also the momentum change, the same impulse must be applied to the blocks. Knowing

$$\vec{J} = \Delta\vec{p} = \vec{F}\,\Delta t$$

The time to stop the block with mass $2m$ is **the same as** that to stop the block with mass m.

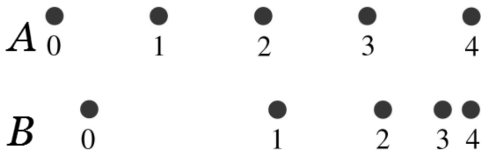
8. ($4\frac{1}{3}$ points) The motion diagrams for two objects, A and B are illustrated. Describe the acceleration of each object.

. . . .

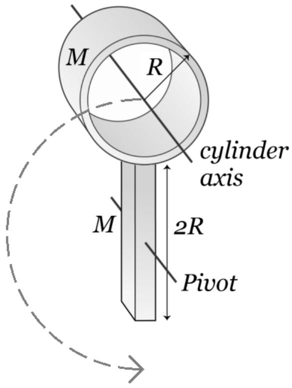
Object A moves the same distance along a straight line in each time interval. Its velocity is constant.

Object B moves decreasing distances to the right along its straight line in each time interval. It is slowing down.

Object A has an acceleration of zero, while object B has an acceleration to the left.



9. ($4\frac{1}{3}$ points) A cylindrical hoop of mass M and radius R is attached to the end of a thin rod. The rod also has mass M, and has length $2R$. A frictionless horizontal axle passes through the center of the rod, and the resulting object is positioned vertically in unstable equilibrium, as illustrated. A gentle nudge causes the object to begin rotating. How does the net torque magnitude on the object about the pivot change as it rotates from the illustrated position of unstable equilibrium to the position of stable equilibrium? (*On Earth.*)



. . . .

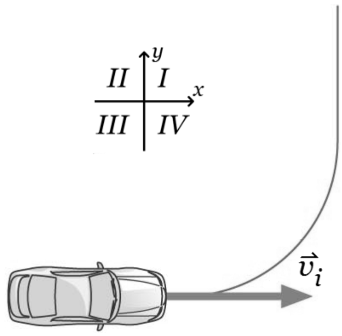
Remember

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad \Rightarrow \qquad \tau = rF \sin\theta$$

The magnitudes of both $\vec{r}$ and $\vec{F}$ are constant. $\vec{r}$ points from the pivot point to the place the force is applied (*i.e.*, the center of mass). Therefore, θ is initially $180°$ and decreases to $0°$. In the process, $\sin\theta$ increases from 0 to 1, and then decreases to 0 again, so

The torque magnitude first increases, then decreases.

10. ($4\frac{1}{3}$ points) The automobile in the top-down illustration is traveling in the $+x$ direction with velocity $\vec{v}_i$. It then turns to travel in the $+y$ direction *at constant speed*. What direction, if any, is the impulse on the car for this process?



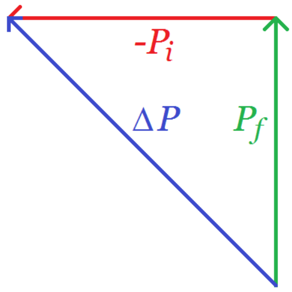
. . . .

Remember that the impulse on an object is the change in momentum of the object

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

This relationship is illustrated to the right. Note that Δp is directed

Somewhere in quadrant II.



11. ($4\frac{1}{3}$ points) A satellite with mass m travels around the Earth with angular speed ω_E in a circular orbit with radius R. A satellite with mass $10m$ travels around Mars with angular speed ω_M in a circular orbit with the same radius R. The mass of Mars is one-tenth the mass of the Earth. Compare the angular speeds of the two satellites.

. . . .

Use Newton's Second Law. The only force on the satellite as it orbits is the force of gravity from the planet. Choose a coordinate system with the r axis toward the center of the orbit. Letting M represent the mass of the planet and m represent the mass of the satellite,

$$\sum F_r = G\frac{Mm}{R^2} = ma_r = m\frac{v^2}{R} = mR\omega^2 \qquad \Rightarrow \qquad \omega = \sqrt{\frac{GM}{R^3}}$$

Note that the mass of the satellite is irrelevant. Then

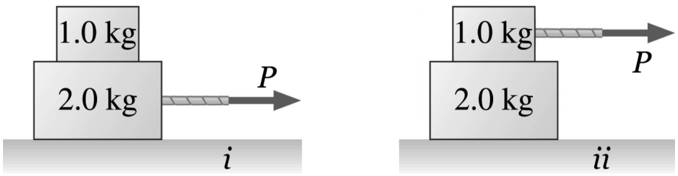
$$\omega_E = \sqrt{\frac{GM_E}{R^3}} \qquad \text{and} \qquad \omega_M = \sqrt{\frac{GM_M}{R^3}} = \sqrt{\frac{GM_E/10}{R^3}}$$

So

$$\omega_M = \omega_E/\sqrt{10}$$

12. ($4\frac{1}{3}$ points) A block with a mass of 2.0 kg lies on a frictionless horizontal surface. A block with a mass of 1.0 kg is placed on top of it. A rope will be attached to either the bottom block (situation i) or the top block (situation ii) and pulled with a horizontal force magnitude P. The coefficient of static friction between the blocks is sufficient that the blocks do not slide relative to each other in either situation. Compare the force of static friction between the blocks f_i in situation i, with the force of static friction between the blocks f_{ii} in situation ii.

. . . .



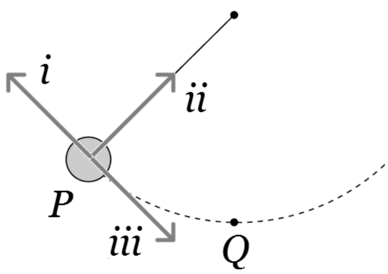
Consider the combined 3.0 kg object. The same net horizontal force P is applied in each case, so the acceleration is the same in each case.

In Case i, the friction force between the blocks gives that acceleration to the 1.0 kg object. In Case ii, the friction force between the blocks gives that acceleration to the 2.0 kg object. Since twice the mass is being given that acceleration in Case ii, the friction force must be twice as much.

$$f_{ii} = 2f_i$$

13. ($4\frac{1}{3}$ points) The pendulum in the illustration is swinging back and forth. What is the direction of its acceleration at the highest point of its swing (position P)? (*On Earth.*)

. . . .

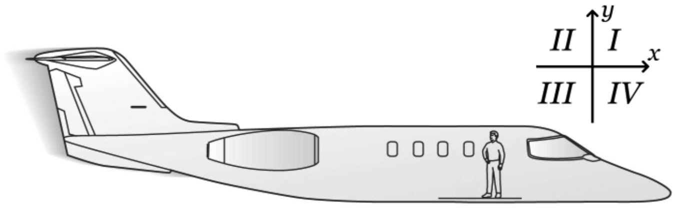


At the end of the swing (position P), the pendulum is momentarily stopped. Its centripetal acceleration, v_t^2/R, is zero, so the total acceleration has no component in direction ii. The pendulum, however, does not remain stopped. Its tangential velocity begins to increase in the iii direction, so the acceleration at this instant is in . . .

Direction iii.

14. ($4\frac{1}{3}$ points) Carlos is a skydiver. He jumps out of the illustrated airplane, which is traveling horizontally at a speed greater than Carlos' terminal speed. Immediately after jumping out of the airplane, and before reaching his terminal speed, what is the direction of Carlos' acceleration? (*On Earth.* Do NOT neglect drag!)

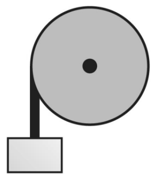
. . . .



Immediately after jumping out of the airplane, there will be a gravitational force on Carlos in the $-y$ direction. His velocity will be in the $+x$ direction, so there is a drag force in the $-x$ direction. As he slows horizontally and speeds up vertically, Carlos' velocity vector shifts from the $+x$ direction to quadrant IV, so the drag force shifts into quadrant II. However, the vertical component of the drag force will not balance the gravitational force until he reaches (vertical) terminal speed. The net force on Carlos, then, must be in quadrant III, so his acceleration must be directed

Somewhere in quadrant III.

15. ($4\frac{1}{3}$ points) An object with circular cross-section has mass M and radius R. An ideal cord is wrapped around it and tied to a block of mass m, as illustrated. If it matters, which of these four uniform objects with circular cross-section from the Table of Moments of Inertia should be selected to *maximize* the acceleration magnitude of the block upon release?



. . . .

Consider the rotational version of Newton's Second Law, $\vec{\tau} = I\vec{\alpha}$. To maximize the acceleration of the block, the angular acceleration of the circular object must be maximized by minimizing the rotational inertia. Of the objects with circular cross-section in the Table, then,

The object should be a solid sphere.