
I. (16 points) Asteroids have a density of $2500\,\mathrm{kg/m^3}$ (this is also the typical density of rocks on Earth). An Olympian can run at $10.0\,\mathrm{m/s}$. What's the maximum radius of a spherical asteroid, so that the Olympian can get into orbit just by running?

. . . .

Use Newton's Second Law. In orbit, the only force on the Olympian is that of gravity, and the acceleration of the Olympian is centripetal. Letting the mass of the asteroid be M and the mass of the Olympian be m,

$$\sum F_c = F_G = ma_c \qquad \Rightarrow \qquad G\frac{Mm}{r^2} = m\frac{v^2}{r} \qquad \Rightarrow \qquad \frac{GM}{r} = v^2$$

where the orbit is just above the surface with radius r.

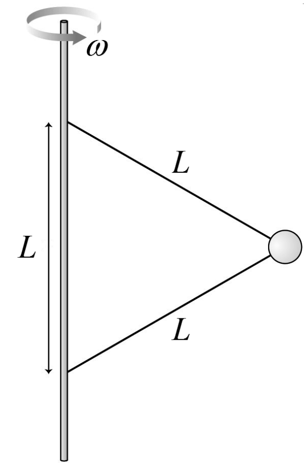
The mass of the asteroid is related to its density ρ and volume $V = \frac{4}{3}\pi r^3$, so

$$\frac{G\rho 4\pi r^3}{3r} = v^2 \qquad \Rightarrow \qquad G\rho 4\pi r^2 = 3v^2$$

Solve for r.

$$r = \sqrt{\frac{3v^2}{4\pi\rho G}} = \sqrt{\frac{3\,(10.0\,\mathrm{m/s})^2}{4\pi\,(2500\,\mathrm{kg/m^3})\,(6.673\times10^{-11}\,\mathrm{N{\cdot}m^2/kg^2})}} = 1.2\times10^4\,\mathrm{m} = 12\,\mathrm{km}$$

II. (16 points) Two strings, each of length $L = 1.5\,\text{m}$ tie a sphere to a rotating shaft, as shown, so the sphere revolves in a horizontal circle. If the tension in the upper string is *twice* the tension in the lower string, with what constant angular speed ω is the shaft rotating? (*On Earth.*)



. . . .

Use Newton's Second Law. Sketch a Free Body Diagram for the sphere and choose a coordinate system. I'll choose $+y$ upward, and $+c$ toward the center to the left. The sphere has a gravitational force mg downward, a tension force T up and to the left, and a tension force $T/2$ down and to the left. Write Newton's Second Law for each axis. I'll show signs explicitly, so symbols represent magnitudes. Representing the mass of the sphere by m, and letting $\theta = 30°$,

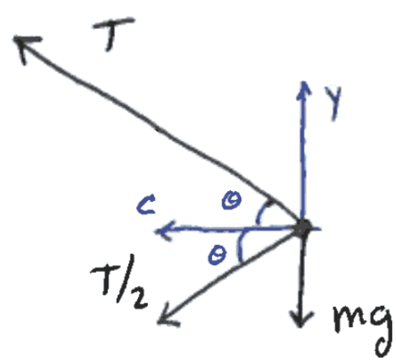
$$\sum F_y = T_y - T_y/2 - mg = ma_y = 0 \qquad \Rightarrow \qquad T\sin\theta - (T/2)\sin\theta = mg$$

$$\text{so} \qquad (T/2)\sin\theta = mg \qquad \Rightarrow \qquad T = 2mg/\sin\theta = 4mg$$

$$\sum F_c = T\cos\theta + (T/2)\cos\theta = ma_c = mr\omega^2 = mL\cos\theta\,\omega^2 \qquad \Rightarrow \qquad 3T/2 = mL\omega^2$$

Substitute the expression for T found from the y axis calculation, into the expression found from the c axis calculation, and solve for ω.

$$3\,(4mg)\,/2 = mL\omega^2 \qquad \Rightarrow \qquad 6g = L\omega^2 \qquad \Rightarrow \qquad \omega = \sqrt{6g/L} = \sqrt{6\,(9.81\,\text{m/s}^2)\,/1.5\,\text{m}} = 6.3\,\text{rad/s}$$



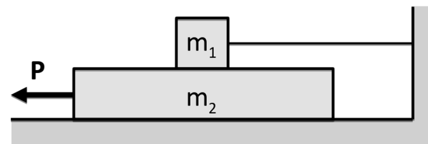
1. (6 points) If the sphere in the problem above has mass m and the tensions in the upper and lower strings are T and $T/2$, respectively, what is the apparent weight of the sphere as it revolves?

. . . .

The apparent weight is the magnitude of the supporting force. Often, the supporting force is a normal force, but in this case the supporting force is the vector sum of the two tensions. Only one of the offered choices is the magnitude of a vector sum.

$$\sqrt{[(3T/2)\cos 30°]^2 + [(T/2)\sin 30°]^2}$$

III. (16 points) A block of mass m_1 rests on top of a block of mass m_2. There is friction on all surfaces with coefficient of static friction μ_s and coefficient of kinetic friction μ_k. A force $\vec{P}$ pulls on the lower box as shown. Calculate the magnitude of the force P such that the box of mass m_2 moves with constant speed in terms of parameters defined in the problem, and physical or mathematical constants. (*On Earth.*)



. . . .

Use Newton's Second Law. For each block, sketch a Free Body Diagram and choose a coordinate system. I'll choose $+y$ upward, and $+x$ to the left.

The top block has a gravitational force $m_1 g$ downward, a normal force n_1 upward from the bottom block, a tension force T to the right, and a kinetic friction force f_1 to the left from the bottom block. Write Newton's Second Law for each axis. I'll show signs explicitly, so symbols represent magnitudes.

$$\sum F_y = n_1 - m_1 g = m_1 a_y = 0 \qquad \Rightarrow \qquad n_1 = m_1 g$$

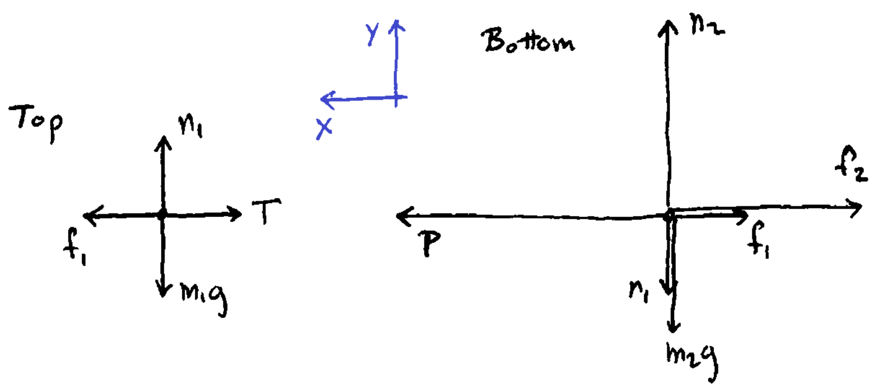
$$\sum F_x = f_1 - T = m_1 a_x = 0 \qquad \Rightarrow \qquad T = f_1 = \mu_k n_1 = \mu_k m_1 g$$

The bottom block has a gravitational force $m_2 g$ downward, a normal force n_1 downward from the top block, a normal force n_2 upward from the "floor", a pull force P to the left, a kinetic friction force f_1 to the right from the bottom block, and a kinetic friction force f_2 to the right from the "floor". Again, write Newton's Second Law for each axis.

$$\sum F_y = n_2 - n_1 - m_2 g = m_2 a_y = 0 \qquad \Rightarrow \qquad n_2 = n_1 + m_2 g = m_1 g + m_2 g = (m_1 + m_2)\, g$$

$$\sum F_x = P - f_1 - f_2 = m_2 a_x = 0 \qquad \Rightarrow \qquad P = f_1 + f_2 = \mu_k n_1 + \mu_k n_2 = \mu_k m_1 g + \mu_k (m_1 + m_2)\, g$$

$$= \mu_k (2m_1 + m_2)\, g$$



2. (6 points) If the magnitude of force $\vec{P}$ were small enough in the above problem, neither the upper nor lower block would move. In that situation, what is the magnitude of the static friction force acting on the *upper* block?

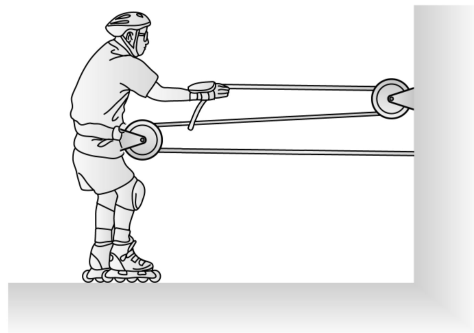
. . . .

The net force on the top block is always zero. When the bottom block moves, the force of kinetic friction is balanced by the tension in the string. When the bottom block is stationary, the force of static friction is balanced by the tension in the string.

It has the same magnitude as the tension in the string.

3. (8 points) Jorge is wearing his frictionless roller skates, and pulling on a rope. He has passed this rope around a pulley attached to the wall, then around another pulley tied to his waist, and then fastened it to the wall, as shown. If Jorge has mass m, and pulls with force magnitude F, what is the magnitude of his acceleration a across the level floor? You may assume that the rope is horizontal where it is not passing around a pulley. *(On Earth.)*

. . . .



Since Jorge's hands pull on the rope with force magnitude F, the rope pulls on his hands with force magnitude F (Newton's Third Law). Therefore, the tension in the rope has magnitude F, which acts on Jorge in three places: at his hands, and above and below the pulley on his waist. Applying Newton's Second Law in the horizontal direction,

$$\sum F_{\text{horiz}} = 3F = ma \qquad \Rightarrow \qquad a = 3F/m$$

---

4. (8 points) Europa is one of Jupiter's moons. It has 1/124 the mass of the Earth and its radius is 1/4 that of the Earth. With this information, and remembering that the free fall acceleration at the Earth's surface is $9.8\,\text{m/s}^2$, calculate the free fall acceleration on the surface of Europa.

. . . .

The weight of an object is the gravitational force on it.

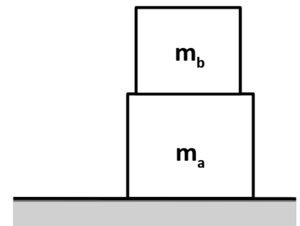
$$mg = G\frac{Mm}{r^2} \qquad \Rightarrow \qquad g = \frac{GM}{r^2}$$

If Europa as 1/124 the mass of the Earth and 1/4 the radius of the Earth, the free fall acceleration at its surface must be $\frac{1/124}{(1/4)^2}$ the free fall acceleration at the Earth's surface.

$$\frac{1/124}{(1/4)^2}\left(9.8\,\text{m/s}^2\right) = 1.3\,\text{m/s}^2$$

5. (8 points) There is a normal force, pointing up, that the Earth exerts on bottom block "a". What is the other force in the action-reaction pair with this force? *(On Earth.)*

. . . .

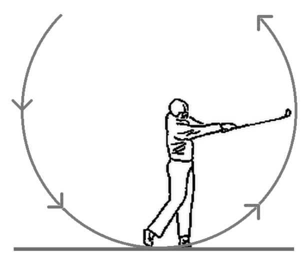


The two forces in an action-reaction pair must be the same kind of force, act between the same two objects, and be in opposite directions. So, a normal force the Earth exerts on block "a", pointing up, must be

A normal force that block "a" exerts on the Earth, pointing down

6. (8 points) A golfer swings his club in a vertical circle, hitting the ball to the right at the bottom of the swing. As he follows through, the club rises and slows. In what direction is the acceleration of the club's head at the moment shown? (*On Earth.*)

. . . .



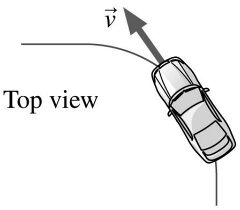
The club is moving in a circle, so its acceleration has a radial component toward the center (leftward). It is also slowing, so its acceleration has a tangential component opposite its upward velocity (so, downward). The vector sum of these components is

Approximately toward the golfer's feet.

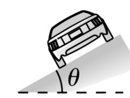
7. (8 points) The car rounds the banked curve at the maximum speed it can do so without sliding. Given that there is a frictional force between the tires and the road surface, which of these are forces on the car with a non-zero component in the direction of its acceleration? (*On Earth.*)

   i. The centripetal force
   ii. The gravitational force
   iii. The frictional force
   iv. The normal force

Top view



Rear view



. . . .

The direction of the car's acceleration is toward the center of its circular path, which is horizontal. The "centripetal force" isn't a force, and the gravitational force is vertical. However, the frictional force is down the bank and the normal force is perpendicular to the bank, giving both of those forces horizontal components.

Just *iii* and *iv*.