Solutions

Printed Name

Nine-digit GT ID

signature

Spring 2019 **PHYS 2211 ABC** **Test 04**

Test Form: **4A**

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on yourtest, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- The standard formula sheet is on the back of this page, which may be removed from the quiz form if you wish, but it must be submitted.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Fill in bubbles for your Multiple Choice answers darkly and neatly.

1 ⓐ ⓑ ⓒ ⓓ ⓔ

2 ⓐ ⓑ ⓒ ⓓ ⓔ

3 ⓐ ⓑ ⓒ ⓓ ⓔ

4 ⓐ ⓑ ⓒ ⓓ ⓔ

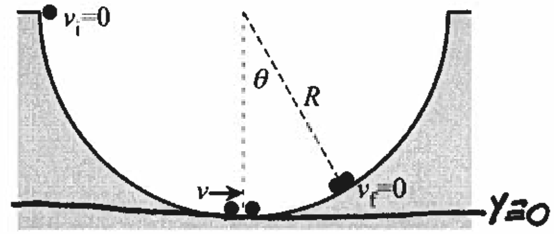
5 ⓐ ⓑ ⓒ ⓓ ⓔ

6 ⓐ ⓑ ⓒ ⓓ ⓔ

The following problem will be hand-graded. Show all supporting work for this problem.

[1] *(20 points)* **Tonk Hawk has lost his touch.** Professional skateboarder Tony Hawk is attempting a stunt in a half-pipe. Starting from rest at the top of the pipe, he rolls (without friction) to the bottom of the pipe, where he accidentally collides with a video cameraman…who ends up after the collision on the skateboard with Tony, holding on for dear life.

Assuming the cameraman has the same mass as Tony, how far up the other side of the pipe will they roll before coming to a stop? Express your answer as a numerical angle θ measured from the vertical (see figure). Record your answer to three-digit precision.



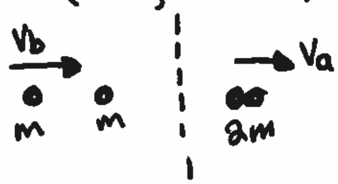
① Negligible rolling friction: mechanical energy conserved on the way down

• choose $y = 0$ at bottom of pipe: $Y_i = +R$ and $Y_f = 0$

$K_i + U_i = K_f + U_f \longrightarrow 0 + mgR = \frac{1}{2}mv_f^2 + 0 \qquad \boxed{v_f^2 = 2gR}$

② Collision with cameraman: perfectly inelastic same V_{after}

(where, for Tony, $V_{before} = \sqrt{2gR}$



$\vec{P}_i = \vec{P}_f \longrightarrow \langle +mV_b \rangle = \langle +2mV_a \rangle$

$V_a = V_b/2 = \sqrt{2gR}/2$

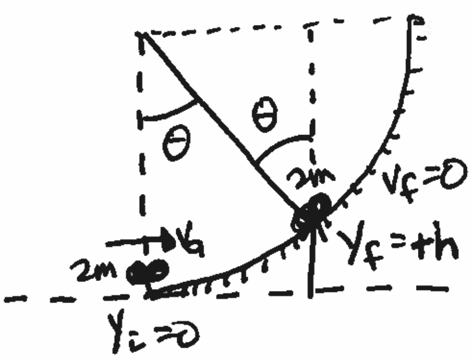
$\boxed{V_a = \sqrt{\frac{gR}{2}}}$

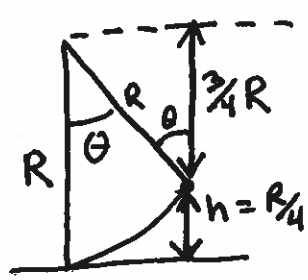
③ Rising back up the pipe

→ again, energy is conserved

$K_i + U_i = K_f + U_f \rightarrow \frac{1}{2}(2m)V_a^2 + 0 = 0 + (2m)gh$

$\rightarrow h = \frac{V_a^2}{2g} = \frac{1}{2g}\left(\frac{gR}{2}\right) = \boxed{\frac{R}{4}}$ max height h





④ Angle θ can be found from:

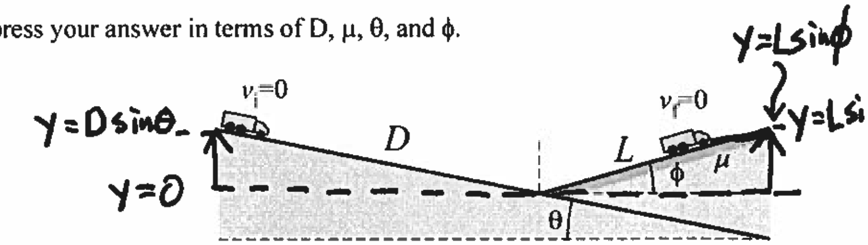
$\cos\theta = \frac{adj}{hyp} = \frac{\frac{3}{4}R}{R} = \frac{3}{4}$

$\boxed{\theta = \cos^{-1}\left(\frac{3}{4}\right) = 41.4^\circ}$

The following problem will be hand-graded. *Show all supporting work for this problem.*

[II] *(20 points)* A truck is initially at rest on a steep mountain road inclined at angle θ when its brakes fail. It begins rolling (with negligible friction) down the incline, picking up speed. After rolling a distance *D*, the driver encounters a runaway truck ramp that is inclined *upward* at an angle ϕ and has a combination of sand and gravel that provide an effective friction coefficient μ. The driver swerves onto the ramp, and travels a total length *L* along the ramp in coming to a stop.

Determine the stopping length *L* for the truck. Express your answer in terms of D, μ, θ, and ϕ.



start/End at rest:

$K_i = K_f = 0$

or $\Delta K = 0$

→ Apply energy principle $\Delta E_{sys} = \cancel{\text{Work}} \to 0$

$$\underbrace{\cancel{\Delta K}}_{0} + \Delta U_g + \Delta E_{Th} = 0$$

but $U_{gi} = mgy_i = \boxed{+mgD\sin\theta}$

$U_{gf} = mgy_f = \boxed{+mgL\sin\phi}$

$\Delta U_g = U_{gf} - U_{gi} = mgL\sin\phi - mgD\sin\theta$

To find ΔE_{Th}, we need $\Delta E_{Th} = +f_k L$ → what is f_k on ramp?

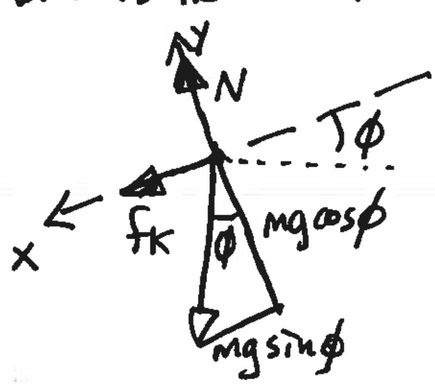
→ apply 2nd law on ramp:

$\sum \vec{F}_y = m\vec{a}_y = 0$

$\langle +N \rangle + \langle -mg\cos\phi \rangle = 0$

$\to N = mg\cos\phi$

so $\boxed{f_k = \mu mg\cos\phi}$ then $\Delta E_{Th} = \mu mgL\cos\phi$



Hence, Energy Principle says:

$$(+mgL\sin\phi - mgD\sin\theta) + (+\mu mgL\cos\phi) = 0$$

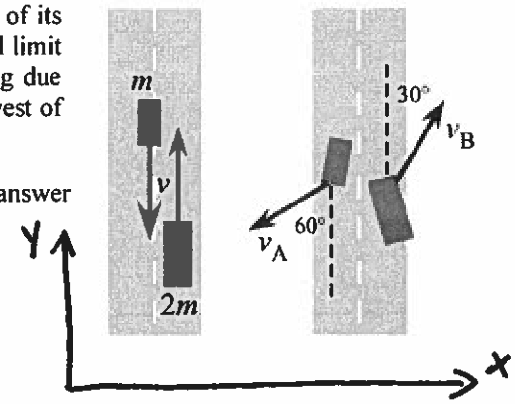
$$\cancel{mg}L(\sin\phi + \mu\cos\phi) = \cancel{mg}D\sin\theta$$

$$\boxed{L = \frac{D\sin\theta}{\sin\phi + \mu\cos\phi}}$$

The following problem will be hand-graded. *Show all supporting work for this problem.*

[III] *(20 points)* A near-head-on collision occurs when one car drifts slightly out of its lane and "clips" an oncoming van. Both vehicles were travelling at the speed limit v, with car A (mass m) travelling due south, and van B (mass $2m$) travelling due north. As a result of the impact, car A veers off the road in a direction 60° west of south while van B veers off the road in a direction 30° east of north.

Find the speeds of each vehicle immediately after the collision. Express each answer as a numerical multiple of the speed limit, v.



Collision: momentum $\vec{P}$ is conserved

$\Rightarrow \vec{P}_{ix} = \vec{P}_{fx}$ and $\vec{P}_{iy} = \vec{P}_{fy}$

① Initial momentum is easy:

$$\boxed{\vec{P}_{ix} = 0} \quad \Sigma\vec{P}_{iy} = \langle -mv\rangle_A + \langle +2mv\rangle_B \rightarrow \boxed{\Sigma\vec{P}_{iy} = \langle +mv\rangle}$$

② Final momentum requires vector decomposition

$$\vec{V}_A = \langle -V_A \sin 60°\rangle\hat{\imath} + \langle -V_A \cos 60°\rangle\hat{\jmath} \quad \boxed{\vec{V}_A = -\tfrac{\sqrt{3}}{2}V_A\hat{\imath} - \tfrac{1}{2}V_A\hat{\jmath}}$$

$$\vec{V}_B = \langle +V_B \sin 30°\rangle\hat{\imath} + \langle +V_B \cos 30°\rangle\hat{\jmath} \quad \boxed{\vec{V}_B = +\tfrac{1}{2}V_B\hat{\imath} + \tfrac{\sqrt{3}}{2}V_B\hat{\jmath}}$$

③ $\vec{P}_{fx} = \vec{P}_{ix} \longrightarrow \langle -\tfrac{\sqrt{3}}{2}mV_A\rangle + \langle +\tfrac{1}{2}(2m)V_B\rangle = 0 \rightarrow \boxed{V_B = \tfrac{\sqrt{3}}{2}V_A}$ (*)

④ $\vec{P}_{fy} = \vec{P}_{iy} \longrightarrow \langle -\tfrac{1}{2}mV_A\rangle + \langle +\tfrac{\sqrt{3}}{2}(2m)V_B\rangle = \langle +mV\rangle$

$$-\tfrac{1}{2}\cancel{m}V_A + \tfrac{\sqrt{3}}{2}(2\cancel{m})\left(\tfrac{\sqrt{3}}{2}V_A\right) = \cancel{m}V$$

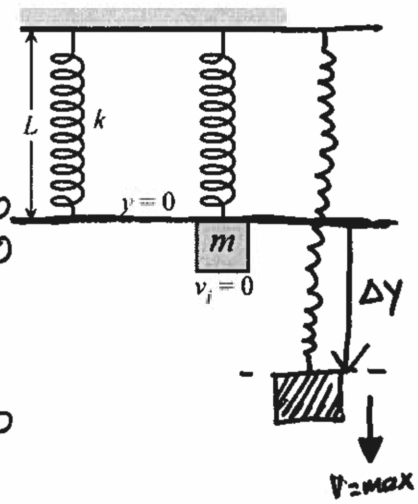
$$-\tfrac{1}{2}V_A + \tfrac{3}{2}V_A = V \longrightarrow \boxed{V_A = V}$$

⑤ Plug this result into (*):

$$V_B = \tfrac{\sqrt{3}}{2}V_A = \tfrac{\sqrt{3}}{2}(V) \longrightarrow \boxed{V_B = \tfrac{\sqrt{3}}{2}V}$$

Question value 8 points

(01) A spring having natural length L and elastic constant k is suspended from the ceiling. A mass m is then attached to the end of the spring, and the system is released from rest. What is the kinetic energy of the block as it passes through its equilibrium position?



(a) $\boxed{K = \frac{(mg)^2}{2k}.}$

(b) $K = \frac{1}{2}kL^2.$

(c) $K = mgL$

(d) $K = \frac{(mg)^2}{k}.$

(e) $K = -mgL$

① Block passing through equilibrium:

$\Sigma \vec{F} = 0$

$\langle +F_s \rangle + \langle -F_g \rangle = 0$

$-k\Delta y - mg = 0$

$\boxed{\Delta y = -\frac{mg}{k}}$

F_s, F_g $(a=0)$

$U_g = 0$
$U_s = 0$

Now, solve energy problem: $K_f + U_{gf} + U_{sf} = \cancel{K_i}^{0} + \cancel{U_{gi}}^{0} + \cancel{U_{si}}^{0}$

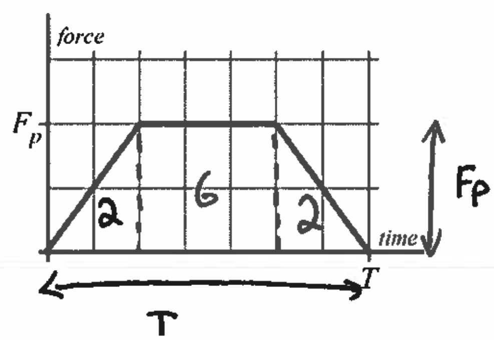
$K + mgy_f + \frac{1}{2}k(\Delta y_f)^2 = 0$

$K + mg\left(-\frac{mg}{k}\right) + \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = 0$

$K = \frac{m^2g^2}{k} - \frac{1}{2}\frac{m^2g^2}{k} \rightarrow \boxed{K = \frac{(mg)^2}{2k}}$

Question value 8 points

(02) A pitcher throws a fastball toward home plate, and the batter swings and hits a line-drive straight back at the pitcher. A sensor array in the ball measures the force-versus-time curve shown at right, where F_P is the peak force delivered by the bat, and T is the total duration of the contact. What was the magnitude of the *time-averaged* force exerted by the bat on the ball?



(a) $F_{avg} = \frac{5}{8}F_P$

(b) $F_{avg} = \frac{6}{7}F_P$

(c) $F_{avg} = \frac{8}{7}F_P$

(d) $F_{avg} = \frac{7}{10}F_P$

(e) $\boxed{F_{avg} = \frac{5}{7}F_P}$

Time averaged Force
→ same impulse as real force

$J = F_{av}\Delta t = \int F\Delta t$

$F_{av}\Delta t = $ (area under force curve)

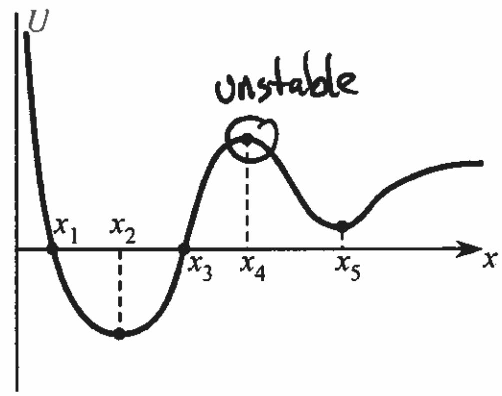
Note: one box has:
- height $F/2$
- width $T/7$
- area $FT/14$

so $F_{av} = \frac{1}{\Delta t}$ (area under force curve)

$= \frac{1}{T}$ (10 boxes, from graph)

$F_{av} = \frac{1}{T} \cdot 10\left[\frac{FT}{14}\right] = \frac{10}{14}F = \boxed{\frac{5}{7}F}$

The next two questions involve the following situation:
A particle moves along the x-axis, subject to a single conservative force. The potential energy curve for that force is shown at right.



Question value 4 points

(03) At which of the indicated postitions will the particle be in an *unstable* equilibrium?

(a) At x_2 and x_5.

(b) At non of the indicated positions.

(c) At x_1 and x_3.

$\boxed{\text{(d) At } x_4 \text{ only.}}$

(e) At x_2, x_4 , and x_5.

Equilibrium: $\vec{F}_{net} = 0$

but for conservative forces,

$\vec{F} = -\frac{dU}{dx}$ → so: equilibrium where

$\frac{dU}{dx} = 0 = \boxed{\text{max or min of PE function}}$

• minima = stable equilibria

$\boxed{\text{• maxima = unstable equilibria}}$

(small energy nudge causes object to leave)

Question value 4 points

(04) Suppose that in the vicinity of position x_2 , the potential energy curve can be modelled by the functional expression:

$$U(x) \approx -A + B(x - x_2)^2$$

What will be the force experienced by the particle when it is at position $x = x_2 / 2$?

(a) $\vec{F} = \langle +B\,(x_2)^2/4 \rangle$

(b) $\vec{F} = \langle +A - B\,(x_2)^2/4 \rangle$

$\boxed{\text{(c) } \vec{F} = \langle +B\,x_2 \rangle}$

(d) $\vec{F} = \langle -A + B\,(x_2)^2/4 \rangle$

(e) $\vec{F} = \langle -B\,x_2 \rangle$

$\vec{F} = \langle -\frac{dU}{dx} \rangle_{at\ x = \frac{x_2}{2}}$] Force as "gradient" of PE function

so $\vec{F} = \langle -\frac{d}{dx}(A + B(x-x_2)^2) \rangle_{x = x_2/2}$

$= \langle -(0 + 2B(x - x_2)) \rangle_{x = x_2/2}$

$= \langle -(0 + 2B(-x_2/2)) \rangle$

$= \langle -(-Bx_2) \rangle$

$$\boxed{\vec{F} = \langle +Bx_2 \rangle}$$

Question value 8 points

(05) A golfer on a Par 3 hole scores a hole-in-one. From the tee she hits the ball eastward and overshoots the cup, striking at A in the figure. However there is enough backspin on the ball for it to bounce *backward*, and then roll into the cup. What is the direction of the impulse delivered to the ball at the initial impact point (position A)?

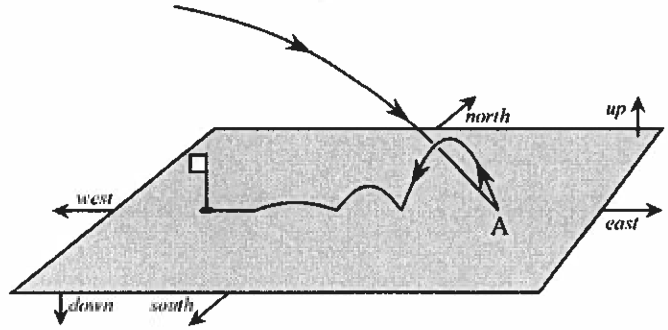
(a) The impulse is directed eastward and up.

(b) The impulse is directed westward and up.

(c) We can't determine impulse, because no information about speed was given.

(d) The impulse is directed due westward.

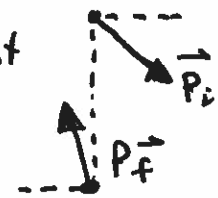
(e) The impulse is directed straight upward.



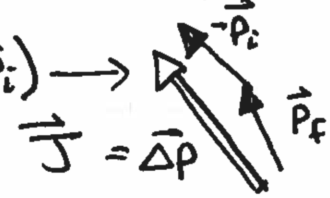
$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$

→ $\vec{p}_i$ = down and east

→ $\vec{p}_f$ = up and west



$\vec{p}_f - \vec{p}_i = \vec{p}_f + (-\vec{p}_i) \longrightarrow$



$\boxed{\vec{J} = \text{up and west}}$

Question value 8 points

(06) In a perfectly elastic collision between two objects…

(a) kinetic energy is conserved but momentum is lost.

(b) momentum is conserved but kinetic energy is *gained*

(c) both momentum and kinetic energy are conserved.

(d) momentum is conserved but kinetic energy is lost.

(e) both momentum and kinetic energy are lost.

① collision: momentum is conserved
(true for all collisions where external forces can be neglected)

② perfectly elastic: a special case for some collisions
→ Kinetic energy is conserved, too.