Solutions

Printed Name

Nine-digit GT ID

signature

Spring 2019 **PHYS 2211 ABC** **Test 03**

Test Form:

3A

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on yourtest, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- The standard formula sheet is on the back of this page, which may be removed from the quiz form if you wish, but it must be submitted.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Fill in bubbles for your Multiple Choice answers darkly and neatly.

1 ⓐ ⓑ ⓒ ⓓ ⓔ

2 ⓐ ⓑ ⓒ ⓓ ⓔ

3 ⓐ ⓑ ⓒ ⓓ ⓔ

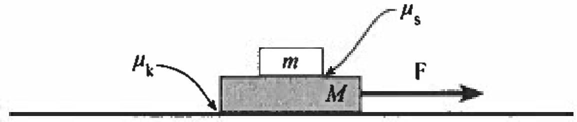
4 ⓐ ⓑ ⓒ ⓓ ⓔ

5 ⓐ ⓑ ⓒ ⓓ ⓔ

6 ⓐ ⓑ ⓒ ⓓ ⓔ

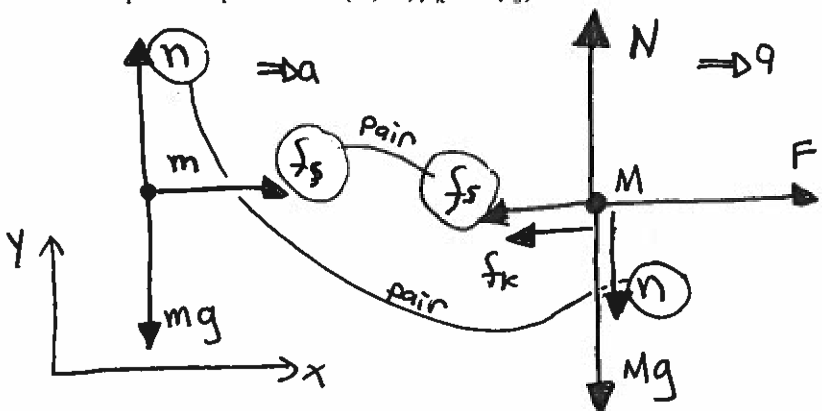
The following problem will be hand-graded. Show all supporting work for this problem.

[1] *(20 points)* A block of mass M is being pulled along a rough floor (kinetic coefficient of friction μ_k) with a horizontal force of magnitude F. A second block of mass m is perched on top of the first, with static coefficient of friction μ_s between the blocks. Assume the top block does not slip along the bottom block.



1. Draw separate free body diagrams for each block. Label all forces clearly and uniquely. Indicate on the diagrams any forces that comprise the two halves of a Newton's third law pair.

2. Determine the maximum force F that you can apply without causing block m to slip. Express your answer in terms of problem parameters (m, M, μ_k and μ_s) and fundamental constants.



f_s = static friction between blocks
n = normal force between blocks (bottom of m / top of M)

These are both third law pairs

Other forces:
grav forces Mg, mg: by Earth
Kinetic friction f_k: floor on M
normal force N: floor on M
pulling force F: on M
[Paired with forces on Earth/floor]

block m

- $\sum F_y = 0 = \langle +n \rangle + \langle -mg \rangle$ so $\boxed{n = mg}$ ①
- $\sum \vec{F}_x = m\vec{a}_x \rightarrow \langle +f_s \rangle = m\langle +a \rangle$ $\boxed{f_s = ma}$ ②

block M

- $\sum \vec{F}_y = 0 = \langle +N \rangle + \langle -Mg \rangle + \langle -n \rangle \longrightarrow \boxed{N = Mg + n = Mg + mg}$ ③ (using ①)
- $\sum \vec{F}_x = M\vec{a}_x \longrightarrow \boxed{\langle +F \rangle + \langle -f_k \rangle + \langle -f_s \rangle = M\langle +a \rangle}$ ④

when pulled with max force: $F \rightarrow F_{max}$, $a \rightarrow a_{max}$ and $f_s \rightarrow f_{s,max} = \mu_s n$

Then we have $f_s = ma \rightarrow \mu_s n = m a_{max} \rightarrow \mu_s mg = m a_{max} \longrightarrow \boxed{a_{max} = \mu_s g}$ ⑤

and $f_k = \mu_k N = \mu_k [Mg + mg] = \mu_k [M + m] g$

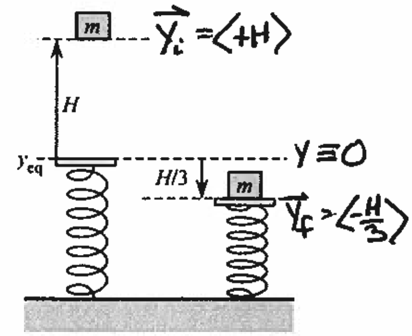
Sub what we knew into ④:

$$F_{max} = f_k + f_s + Ma_{max} = \mu_k (M+m) g + \mu_s mg + M(\mu_s g)$$

$$\boxed{F_{max} = (\mu_k + \mu_s)(M+m) g}$$

The following problem will be hand-graded. *Show all supporting work for this problem.*

[II] *(20 points)* A crate of mass m is held at a height H above the equilibrium position of a rigid vertical spring having elastic constant k. The crate is released from rest and falls onto the spring, compressing it by a distance $H/2$ before momentarily coming to rest. Assume that drag is negligible as the crate falls.



1. Identify all forces that act on the crate after it is released, and write out expressions the work done by each of those forces as the crate falls from its highest to lowest point.

2. Use the Energy Principle to find an expression for the elastic constant k for the spring. Express your answer in terms of m, H, and g.

As problem states — we ignore drag. So forces acting are

① Grav force acts down throughout full displacement

– choosing $\vec{y} \equiv 0$ at spring's equilibrium: $\Delta\vec{y} = \vec{Y}_f - \vec{Y}_i = \langle -\frac{H}{3} \rangle - \langle +H \rangle$

so $\Delta\vec{y} = \langle -\frac{4}{3}H \rangle$

→ grav force $= \langle -mg \rangle =$ constant, so $W_g = \vec{F}_g \cdot \Delta\vec{y}$

$= \langle -mg \rangle \cdot \langle -\frac{4}{3}H \rangle$

$\boxed{W_g = +\frac{4}{3}mgH}$

② Spring force acts, but only from the moment crate makes contact, at $\vec{y} = \vec{y}_{eq} = 0$

so $W_s = \int_0^{y_f} F_s(y)\,dy$ [work by a variable force]

relative to $y_{eq} = 0$, spring force is $F_s(y) = -ky$

$$W_s = \int_0^{-H/3} (-ky)\,dy = \left[-k\frac{y^2}{2}\right]_{y=0}^{y=-H/3}$$

$$\boxed{W_s = -\frac{1}{2}k\left(\frac{H}{3}\right)^2}$$

Now, use Energy Principle $\Delta E_{sys} = W_{ext}$

with system = "crate only", $E_{sys} \to K$ and $W_{ext} \to W_s + W_g$

[earth and spring are "external" to system]

$$\Delta K = W_s + W_g$$

$\Delta K = 0$: crate starts at rest, ends at rest, $K_i = 0 = K_f$, $\Delta K \equiv 0$

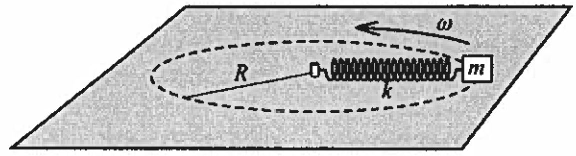
$$0 = -\frac{1}{2}k\frac{H^2}{9} + \frac{4}{3}mgH$$

$$\frac{1}{2}kH^{\cancel{2}} = 12mg\cancel{H}$$

$$\boxed{k = \frac{24mg}{H}}$$

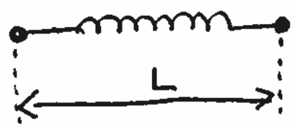
The following problem will be hand-graded. *Show all supporting work for this problem.*

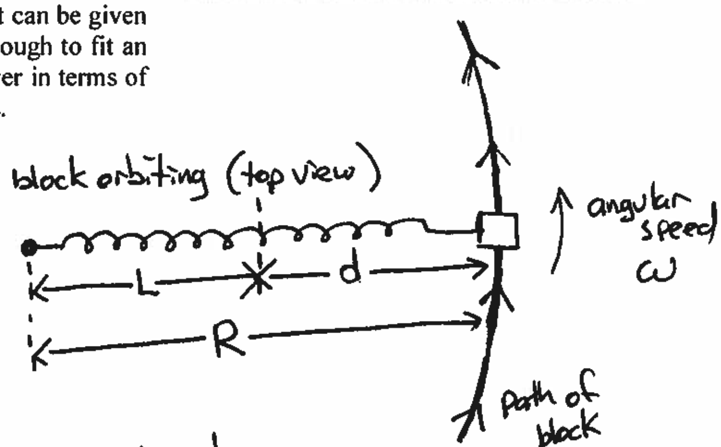
[III] *(20 points)* A block of mass m is placed on a frictionless horizontal table. The block is attached to a central pivot point by a spring having elastic constant k and natural (i.e. unstretched) length L. The block is spun in a circular path about the pivot point with some angular speed ω. As a result of this motion, the spring is observed to stretch; the higher the angular speed, the greater the radius of the orbit.



1. Find an expression for the radius R of the block's orbit, as a function of its angular speed: $R(\omega)$. Express your answer in terms of the parameters provided above (m, L, k, and ω).

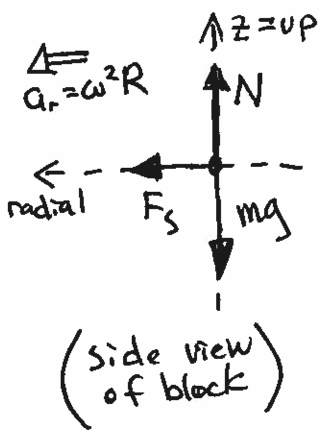
2. What is the maximum possible angular speed that can be given to the block? Assume that your table is large enough to fit an orbit of arbitrarily large size. Express your answer in terms of the parameters m, L, k, and fundamental constants.





let d = amount of stretch

→ Radius of block's circular path is $R = L + d$



So: Circular motion — spring provides radial force

$\sum \vec{F}_r = m\vec{a}_r$ where spring force is $\vec{F}_s = -k\vec{\Delta s} = \langle +kd \rangle$

so $\langle +kd \rangle = m\omega^2 R$

but if $R = L + d$, then $\boxed{d = R - L}$

$k(R-L) = m\omega^2 R$

$(k - m\omega^2)R = kL \qquad R = \dfrac{kL}{k - m\omega^2}$

or

$$\boxed{R(\omega) = \frac{k}{k - m\omega^2} L}$$

Finally: R cannot be negative (and should not be allowed to $\rightarrow \infty$)

We thus restrict the denominator in expression above to be positive

$k - m\omega^2 > 0 \qquad k > m\omega^2 \qquad \sqrt{\frac{k}{m}} > \omega$ or $\boxed{\omega \leq \sqrt{\frac{k}{m}}}$

i.e. $\omega_{max} = \sqrt{\frac{k}{m}}$

Question value 8 points

(01) You tie a lead shot of mass $m = 1.0$ kg to a string having a *yield strength* (i.e. maximum sustainable tension) $T_{max} = 30$ N. You whirl the shot in a vertical circle of radius R = 0.10 m. What is the maximum tangential speed the shot can have at the top of the circle? Use $g = 10$ m/s^2.

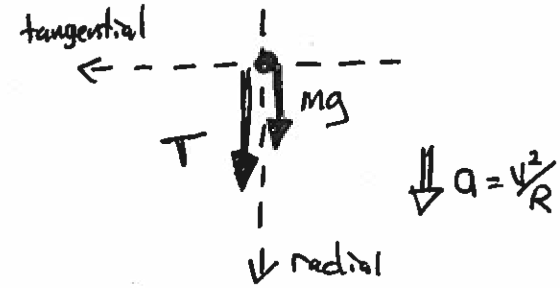
(a) $v_{max} = 1.0$ m/s

(b) $v_{max} = 1.4$ m/s

(c) $v_{max} = 1.7$ m/s

(d) $v_{max} = 0.87$ m/s

(e) $\boxed{v_{max} = 2.0 \text{ m/s}}$



$$\Sigma \vec{F}_r = m\vec{a}_r$$

$$\langle +T_{max}\rangle + \langle +mg\rangle = m\langle +v^2/R\rangle$$

$$\rightarrow v^2/R = (g + T_{max}/m)$$

$$\boxed{v = \sqrt{R(g + T_{max}/m)} = 2.0 \text{ m/s}}$$

Question value 8 points

(02) A box of mass m is sliding at constant speed v along a frictionless floor, until (at $x = 0$) it encounters an area with linearly increasing frictional force: $F_k = F_0\, x/L$. How far does it slide on the floor? That is, at what value of x does it stop?

(a) $x = \dfrac{mv^2}{F_0}$

(b) $x = \dfrac{2mv^2}{F_0}$

(c) $\boxed{x = \sqrt{\dfrac{mv^2 L}{F_0}}}$

(d) $\sqrt{\dfrac{mv^2 L}{2F_0}}$

(e) $x = \dfrac{mv^2}{2F_0}$

Work by a variable force

$$W = \int_i^f \vec{F}\cdot d\vec{s}$$

Here, we displace in positive x-direction, while friction force is in negative x-direction:

$$W_F = -\int_0^x F_k(x')\,dx' \quad \text{(using dummy variable)}$$

$$= -\frac{F_0}{L}\int_0^x x'\,dx' = -\frac{F_0}{L}\left[\frac{x'^2}{2}\right]_0^x = -\frac{1}{2}\frac{F_0}{L}x^2$$

Now apply work-KE theorem $\Delta K = W_{TOT}$ (work by all forces)

$$\cancel{K_f}^{\,0} - K_i = W_F \quad \text{(only friction does work)}$$

$$-\frac{1}{2}mv^2 = -\frac{1}{2}\frac{F_0}{L}x^2$$

$$x^2 = \frac{mv^2 L}{F_0}$$

$$\boxed{x = \sqrt{\frac{mv^2 L}{F_0}}}$$

Question value 8 points

(03) The force-vs-speed curve for a contracting frog muscle can be parameterized by the equation $F(v) = F_{max}\,(1 - (v/v_0)^2)$, where $0 < v < 2v_0$. At what speed does this muscle produce maximum power output?

(a) $v = 0$

(b) $v = v_0$

(c) $v = 2v_0$

$\boxed{\text{(d)}\ v = \frac{1}{\sqrt{3}} v_0}$

(e) $v = \frac{1}{2} v_0$

① instantaneous power delivery by a force:

$$P = \vec{F} \cdot \vec{V}$$

$$P(v) = F_{max}\left(1 - \frac{V^2}{V_0^2}\right) \cdot V = F_{max}\left(V - \frac{V^3}{V_0^2}\right)$$

maximize power: set $\frac{dP}{dV} = 0$

$$\frac{dP}{dV} = F_{max}\left(1 - \frac{3V^2}{V_0^2}\right) = 0$$

$$1 - \frac{3V^2}{V_0^2} = 0$$

$$V_0^2 = 3V^2 \rightarrow \boxed{V = \frac{1}{\sqrt{3}} V_0}$$

Question value 8 points

(04) Block M rests on a horizontal frictionless surface. It is connected to hanging mass m by an ideal cord that passes over a frictionless pulley. What will be the magnitude of the acceleration of mass m as it falls? Do not assume that $M > m$.

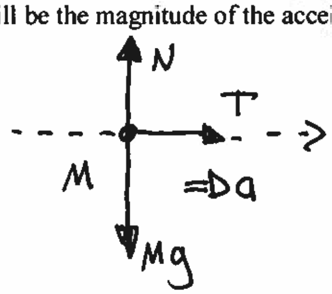
$\boxed{\text{(a)}\ a = \frac{m}{M+m} g}$

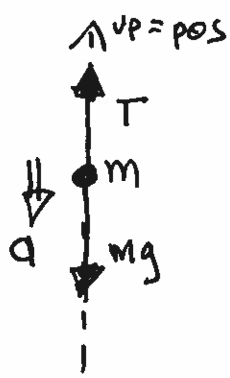
(b) $a = \frac{M}{M-m} g$

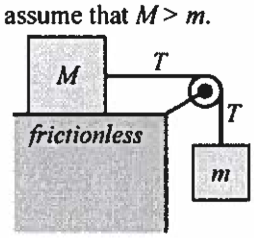
(c) $a = g$

(d) $a = \frac{m}{M-m} g$

(e) $a = \frac{M}{M+m} g$







downward acceleration

2nd Law: $\sum \vec{F}_x = M\vec{a}_x \rightarrow \boxed{\langle +T \rangle = M\langle +a \rangle}$

$\sum \vec{F}_y = m\vec{a}_y \rightarrow \boxed{\langle +T \rangle + \langle -mg \rangle = m\langle -a \rangle}$

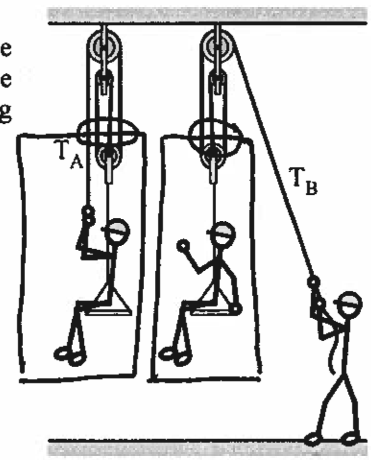
Sub 1st into 2nd, to eliminate "T"

$$Ma - mg = -ma$$

$$(M+m)a = mg \qquad \boxed{a = \frac{mg}{M+m}}$$

Question value 8 points

(05) Block and tackle. You are sitting in a bosun's chair suspended by a block and tackle (pictured), holding yourself stationary using tension T_A in the rope (left). You pass the end of the rope to your friend standing on the ground, who holds you stationary using tension T_B (right). How are T_A and T_B related?



(a) $T_B = T_A$

(b) $T_B = \frac{2}{3} T_A$

(c) $T_B = \frac{3}{4} T_A$

(d) $T_B = \frac{4}{3} T_A$

(e) $T_B = \frac{3}{2} T_A$

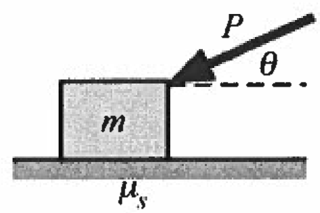
Consider lower pulley, seat, and passenger as a single system — tension in cord must support total weight W

- in case A, FOUR copies of T_A support system (three at pulley, one at hands)
- in case B, THREE copies of T_B support system (just the three at pulley)

so $4T_A = W = 3T_B \longrightarrow \boxed{T_B = \frac{4}{3} T_A}$

Question value 4 points ***EXTRA CREDIT***

(06) A block of mass m is placed on a rough horizontal surface having static friction coefficient μ_s. An external agent applies a pushing force of magnitude P, at an angle θ below the horizontal, as shown in the figure at right. The block does not move as a result of this applied force. Which of the expressions below *best* characterizes the magnitude of the static friction force acting on the block?



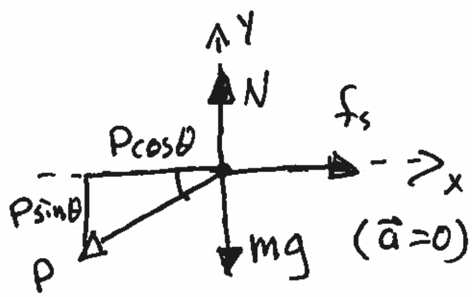
(a) $f_s = \mu_s mg$

(b) $f_s = P \cos\theta$

(c) $f_s = \mu_s [mg + P \sin\theta]$

(d) $f_s = 0$

(e) $f_s = \mu_s P \sin\theta$



$\Sigma \vec{F}_x = m\vec{a}_x = 0$ (block does not move)

$\langle -P\cos\theta \rangle + \langle +f_s \rangle = 0$

$\boxed{f_s = P\cos\theta}$