Solutions

Printed Name

Nine-digit GT ID

signature

Spring 2019 **PHYS 2211 ABC** **Test 02**

Test Form: **2A**

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on yourtest, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- The standard formula sheet is on the back of this page, which may be removed from the quiz form if you wish, but it must be submitted.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Fill in bubbles for your Multiple Choice answers darkly and neatly.

1 ⓐ ⓑ ⓒ ⓓ ⓔ

2 ⓐ ⓑ ⓒ ⓓ ⓔ

3 ⓐ ⓑ ⓒ ⓓ ⓔ

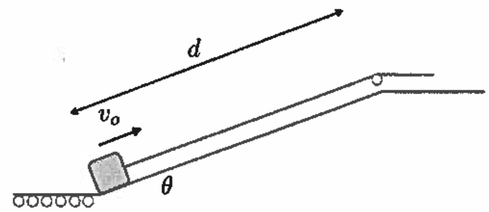
4 ⓐ ⓑ ⓒ ⓓ ⓔ

5 ⓐ ⓑ ⓒ ⓓ ⓔ

6 ⓐ ⓑ ⓒ ⓓ ⓔ

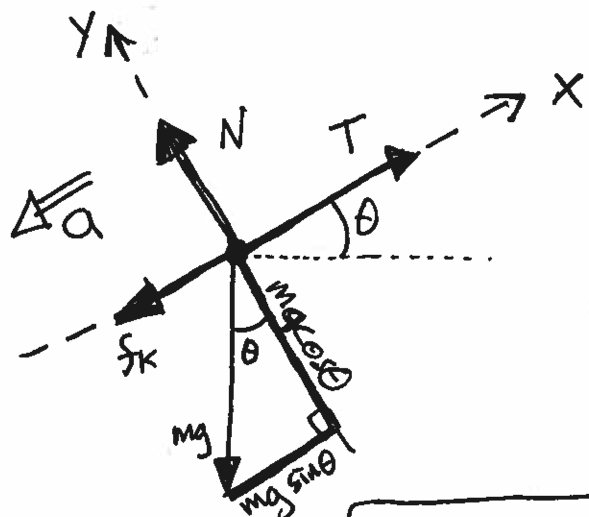
The following problem will be hand-graded. *Show all supporting work for this problem.*

[1] *(20 points)* A miner gets stones out of a quarry by pulling them up a ramp of length d inclined at an angle θ above the horizontal. At time $t = 0$, a conveyor belt delivers a stone to the bottom of the ramp with speed v_o. At that moment, the miner begins pulling with a rope so that the rock, under uniform acceleration, arrives at the top of the ramp with speed with $v = 0$. The rope that pulls the stone uphill is rated for a maximum tension T_m. That is, the rope will break if the tension is larger than this value. The static/kinetic coefficients for friction are μ_s and μ_k respectively.



1. Draw a free body diagram, select a coordinate system, write down Newton's second law as a vectorial equation, and decompose these equations along the axes of the chosen coordinate system.

2. What's the maximum rock mass that the person will be able to pull? Express your answer in terms of the quantities defined in the statement and physical or mathematical constants.



- block is in motion upslope
  ⇒ kinetic friction points downslope
- 1D kinematics on incline:
  - slowing to a stop = downslope accel
  - $V_f^2 = V_i^2 + 2\vec{a}\,\Delta\vec{x}$

  gives: $0 = V_0^2 + 2\langle -a\rangle\langle +d\rangle$

  $\boxed{a = \frac{V_0^2}{2d}}$ magnitude of acceleration

used in this vector equation (downslope)

$\Sigma\vec{F}_x = m\vec{a}_x \longrightarrow \boxed{\langle +T\rangle + \langle -f_k\rangle + \langle -mg\sin\theta\rangle = m\langle -a\rangle}$

$\Sigma\vec{F}_y = m\vec{a}_y \longrightarrow \boxed{\langle +N\rangle + \langle -mg\cos\theta\rangle = 0}$ (no $\vec{a}_y$!)

$N = mg\cos\theta$ so $f_k = \mu_k N = \mu_k mg\cos\theta$

sub this into x-equation:

$$+T - \mu_k mg\cos\theta - mg\sin\theta = -ma = -m\left(\frac{V_0^2}{2d}\right)$$

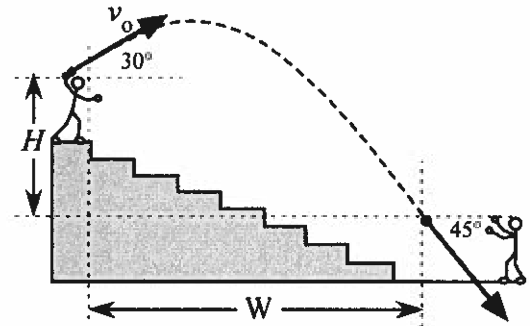
$$T = m\left[+\mu_k g\cos\theta + \sin\theta - \frac{V_0^2}{2d}\right]$$

Note that $T \to$ max value, T_m will define the maximum possible mass lifted

$$\boxed{M_{max} = \frac{T_m}{g(\mu_k\cos\theta + \sin\theta) - V_0^2/2d}}$$

The following problem will be hand-graded. *Show all supporting work for this problem.*

[II] *(20 points)* You are standing at the top of a stairwell of height H while your friend is standing at the bottom, a horizontal distance W away from you. You throw a tennis ball with speed v_o at an angle $\theta = 30°$ above the horizontal. When your friend catches the ball it is traveling in a direction $\phi = 45°$ below the horizontal.

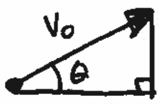


Find expressions for the height H and width W of the stairwell. Express each answer in terms of the parameters g and v_o.

Hint: start by finding an expression for the landing speed as a multiple of the launch speed. What kinematic quantity is *constant* *as the tennis ball travels?*

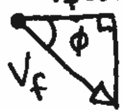
Projectile motion: $\vec{V}_x = \text{constant}$, vertical motion = free-fall

① Decompose initial / final velocity



$V_o \sin\theta = \frac{1}{2}V_o$

$V_o \cos\theta = \frac{\sqrt{3}}{2}V_o$



$V_f \cos\phi = \frac{1}{\sqrt{2}}V_f$

$V_o \sin\phi = \frac{1}{\sqrt{2}}V_f$

note $V_{yf} = V_{xf}$ (equal magnitudes)

② Invoke $\vec{V}_x = \text{constant} \rightarrow \vec{V}_{xi} = \vec{V}_{xf} \rightarrow \langle +\frac{\sqrt{3}}{2}V_o \rangle = \langle +V_{xf} \rangle$

so $\boxed{V_{xf} = \frac{\sqrt{3}}{2}V_o}$ and consequently, $\boxed{V_{yf} \text{ also } = \frac{\sqrt{3}}{2}V_o}$

③ Vertical motion: $\vec{\Delta V}_y = \langle -g \rangle \Delta t$ allows us to determine time of flight

$\vec{V}_{yf} - \vec{V}_{yi} = \langle -\frac{\sqrt{3}}{2}V_o \rangle - \langle +\frac{1}{2}V_o \rangle = \langle -g \rangle \Delta t_{TOT}$

$\rightarrow \boxed{\Delta t_{TOT} = \frac{V_o}{g}\left(\frac{\sqrt{3}+1}{2}\right)}$

④ Use time of flight to find vertical and horizontal displacements

$\vec{\Delta x} = \langle +W \rangle = \vec{V}_{ox}\Delta t_{TOT} \rightarrow W = \frac{\sqrt{3}}{2}V_o \cdot \frac{V_o}{g}\left(\frac{\sqrt{3}+1}{2}\right)$

$$\boxed{W = \frac{V_o^2}{g}\left(\frac{3+\sqrt{3}}{4}\right)}$$

$\vec{\Delta y} = \vec{V}_{oy}\Delta t + \frac{1}{2}\langle -g \rangle \Delta t^2$

<u>OR</u> (more easily):

$V_{yf}^2 = V_{yi}^2 + 2\langle -g \rangle \langle -H \rangle$

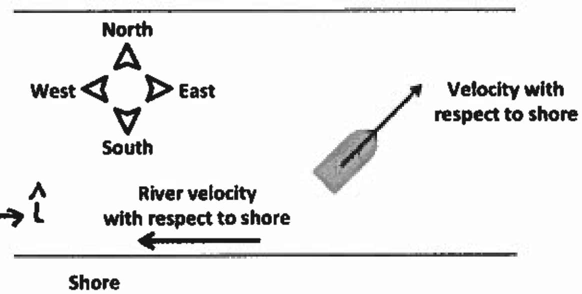
$\left(\frac{\sqrt{3}}{2}V_o\right)^2 = \left(\frac{1}{2}V_o\right)^2 + 2gH$

$\frac{3}{4}V_o^2 - \frac{1}{4}V_o^2 = 2gH$

$\frac{1}{2}V_o^2 = 2gH \longrightarrow \boxed{H = \frac{V_o^2}{4g}}$

The following problem will be hand-graded. *Show all supporting work for this problem.*

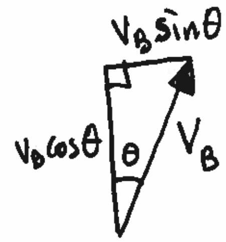
|III| *(20 points)* Relative to the shore, a canoe has a speed v_B and is moving in a direction $\theta = 30°$ east of north. The canoe is in a river that flows ~~with speed v_R~~ due west with respect to the shore. Calculate the velocity of the canoe with respect to the river, including expressions for the speed and an angle measured relative to the northward direction. Provide an answer in termes of the quantities defined in the statement.



- choose coord system with
  East $= +\hat{\imath}$
  North $= +\hat{\jmath}$
  (other choices are equally valid)

① Velocity of water relative to shore is

$$\boxed{\vec{V}_{WS} = \langle -V_R \rangle \hat{\imath} + \langle 0 \rangle \hat{\jmath}}$$



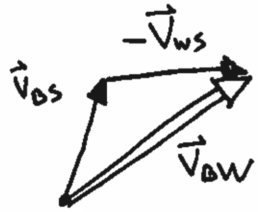
$\cos 30° = \frac{\sqrt{3}}{2}$

$\sin 30° = \frac{1}{2}$

② We are also directly given velocity of boat relative to shore

$$\vec{V}_{BS} = \langle +V_B \sin\theta \rangle \hat{\imath} + \langle +V_B \cos\theta \rangle \hat{\jmath}$$

or $$\boxed{\vec{V}_{BS} = \langle +\tfrac{1}{2} V_B \rangle \hat{\imath} + \langle +\tfrac{\sqrt{3}}{2} V_B \rangle \hat{\jmath}}$$



③ Relative Velocity Relationship says: $\vec{V}_{BW} = \vec{V}_{BS} + \vec{V}_{SW}$

we also know that $\vec{V}_{SW}$ = "shore relative to water"

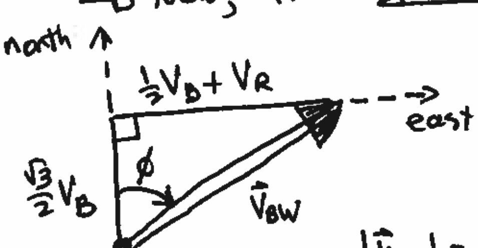
is equivalent to

$$\vec{V}_{SW} = -\vec{V}_{WS}$$

so that $\vec{V}_{BW} = \vec{V}_{BS} - \vec{V}_{WS} = \left[\langle +\tfrac{1}{2} V_B \rangle \hat{\imath} + \langle +\tfrac{\sqrt{3}}{2} V_B \rangle \hat{\jmath}\right] - \left[\langle -V_R \rangle \hat{\imath} + \langle 0 \rangle \hat{\jmath}\right]$

combining cartesian components gives $\boxed{\langle +\tfrac{1}{2} V_B + V_R \rangle \hat{\imath} + \langle \tfrac{\sqrt{3}}{2} V_B \rangle \hat{\jmath}}$

→ Now, find speed and direction



direction: $\phi = \tan^{-1}\left(\frac{opp}{adj}\right) = \tan^{-1}\left(\frac{\frac{1}{2}V_B + V_R}{\frac{\sqrt{3}}{2} V_B}\right)$

or $\boxed{\phi = \tan^{-1}\left(\frac{V_B + 2V_R}{\sqrt{3}\, V_B}\right) \text{ east of north}}$

$$|\vec{V}_{BW}| = \sqrt{\left(\tfrac{1}{2}V_B + V_R\right)^2 + \left(\tfrac{\sqrt{3}}{2} V_B\right)^2}$$

$$= \sqrt{\tfrac{1}{4} V_B^2 + V_B V_R + V_R^2 + \tfrac{3}{4} V_B^2} \Rightarrow \boxed{|\vec{V}_{BW}| = \sqrt{V_B^2 + V_B V_R + V_R^2}}$$

Question value 4 points

(01) A courier is in an express elevator, riding down from the 105th floor of a high-rise building. The elevator is descending rapidly, and must undergo a long period of deceleration as it nears the ground floor. While this is happening, how do the values for the courier's apparent mass and apparent weight compare to the normal values for mass and weight when standing on the Earth?

(a) $\cancel{m_{app} > m_{norm}}$ and $W_{app} > W_{norm}$

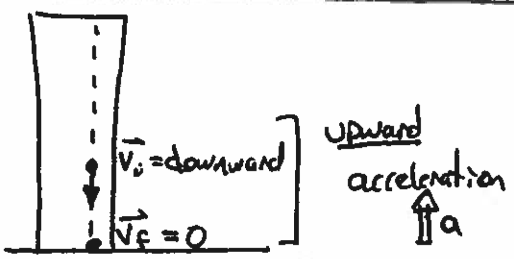
(b) $m_{app} = m_{norm}$ and $W_{app} = W_{norm}$

(c) $m_{app} = m_{norm}$ and $W_{app} < W_{norm}$

(d) $\cancel{m_{app} < m_{norm}}$ and $W_{app} = W_{norm}$

$\boxed{\text{(e) } m_{app} = m_{norm} \text{ and } W_{app} > W_{norm}}$

- Mass is an inherent inertial property of all matter, and does not change!
- Weight is a physiological perception, and can change based upon circumstances of acceleration



N = normal force by floor causes perception of "weight"

a

$\langle +N \rangle + \langle -W_{norm} \rangle = m \langle +a \rangle$

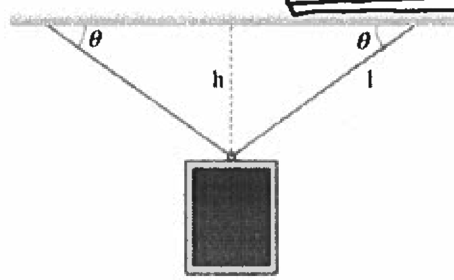
$N > W_{norm}$

W_{norm} = true grav force

$\boxed{W_{app} > W_{norm}}$

Question value 8 points

(02) A box of mass m = 10.0 kg is suspended a distance $h = 0.50$ m below the ceiling by two cables that make equal angles with the ceiling. Each cable has a length of $l = 1.0$ m. What is the tension T in each of the cables? Use $g = 10\ \text{m/s}^2$.



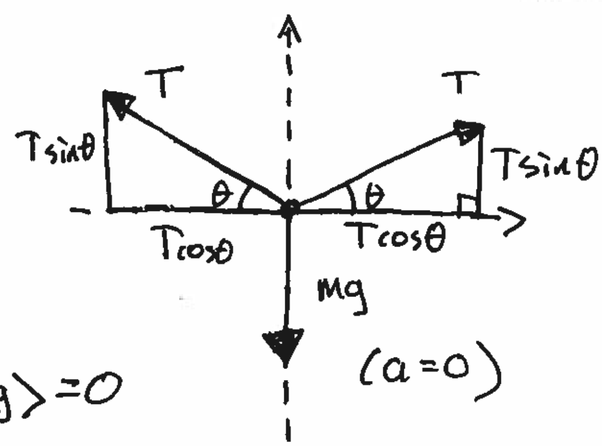
(a) 87 N

(b) 200 N

$\boxed{\text{(c) } 100\text{ N}}$

(d) 173 N

(e) 43.3 N



(a = 0)

$$\sum \vec{F}_y = 0$$

$$\langle +T\sin\theta \rangle + \langle +T\sin\theta \rangle + \langle -mg \rangle = 0$$

$$2T\sin\theta = mg$$

$$T = \frac{mg}{2\sin\theta} = \frac{mg}{2\sin 30^\circ} = \frac{mg}{2(\tfrac{1}{2})} = mg = \boxed{100\text{N}}$$

Question value 8 points

(03) A race car is on a circular track of radius 100 m. Starting from rest the car has an initial acceleration of magnitude 1.0 m/s^2. Assuming the tangential acceleration remains constant, what speed will the car have when the magnitude of the total accelearation on the car is 2.0 m/s^2?

(a) 7.60 m/s

(b) 10.0 m/s

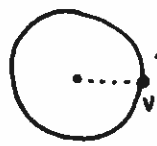
(c) 22.2 m/s

(d) 14.1 m/s

(e) 13.2 m/s

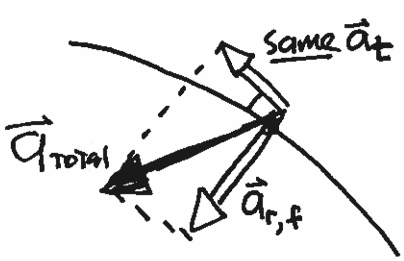
$V_i = 0$, so initial radial accel is $a_{r,i} = \frac{V_i^2}{R} = 0$

→ initially, all accel is tangential



$\vec{a}_i = \vec{a}_t$ has magnitude given as $1.0\,m/s^2$ = fixed

Later, after car builds up speed: $V_f \neq 0$ so $a_{r,f} = V_f^2/R \neq 0$



give $|\vec{a}_{TOT}| = 2.0\,m/s^2$

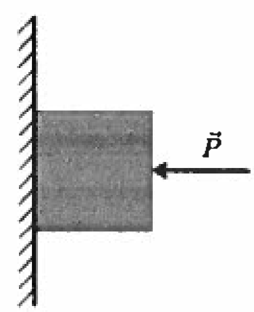
but $|\vec{a}_{TOT}| = \sqrt{a_r^2 + a_t^2}$

→ $a_r = \sqrt{a_{TOT}^2 - a_t} = 1.73\,m/s^2$

then $V_f = \sqrt{a_r \cdot R} = \boxed{13.16\,m/s}$

Question value 8 points

(04) A block of mass m has a force $\vec{P}$ applied to it, holding it stationary against a rough wall as shown at right. State the magnitude and direction of the friction force that the wall exerts on the block.



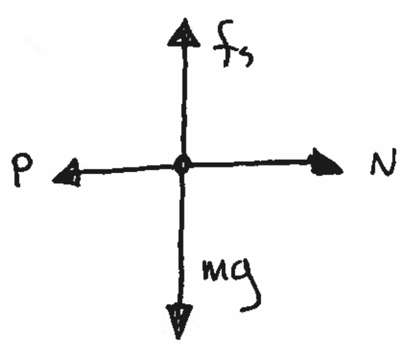
(a) $\mu_s|\vec{P}|$, upward

(b) $\mu_s mg$, upward

(c) mg, downward

(d) mg, upward

(e) $\mu_s|\vec{P}|$, downward



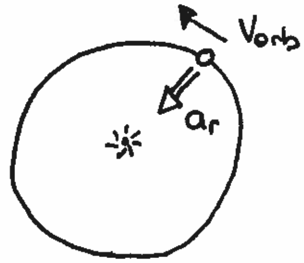
static friction holds block up, against the pull of gravity

1. f_s is an upward-directed vector
2. magnitude must balance force of gravity

$\Sigma F_y = 0 \to \langle +f_s \rangle + \langle -mg \rangle = 0 \to \boxed{f_s = mg}$

Question value 4 points

(05) The Earth orbits around the Sun in a circular orbit of $R = 1.5 \times 10^{11}$ m. What is the Earth's acceleration, as it orbits? Assume Erth's orbit is perfectly circular, and that a year lasts exactly $T = 365$ days.



(a) $\vec{a} = 1.5 \times 10^{-4}$ m/s^2 toward the Sun

(b) $\vec{a} = 6.1 \times 10^{-3}$ m/s^2 toward the Sun

(c) $\vec{a} = 1.5 \times 10^{-4}$ m/s^2 ~~away from the Sun~~

(d) $\vec{a} = 1.0 \times 10^{-2}$ m/s^2 toward the Sun

(e) $\vec{a} = 0$ because the Earth orbits at constant speed

• orbital speed $V = \frac{\text{distance}}{\text{elapsed time}} = \frac{\text{circumference}}{\text{period}} = \frac{2\pi R}{T}$

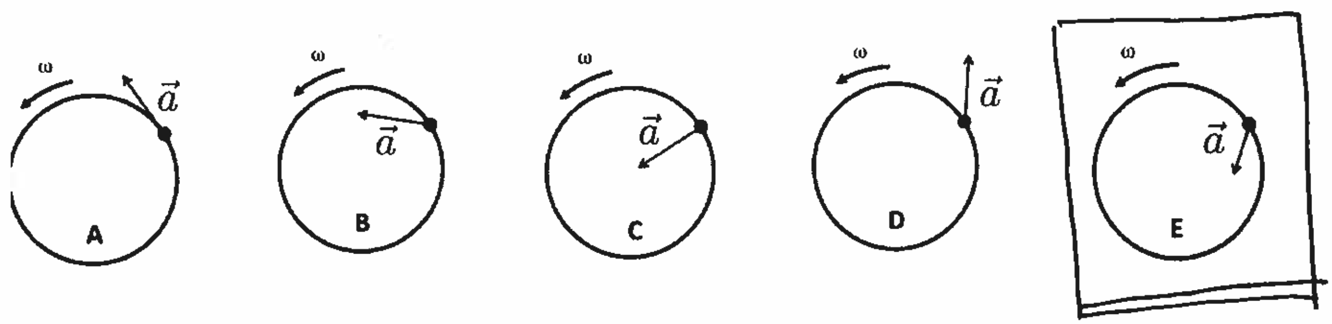
• tangential speed requires radially inward acceleration

$$a_r = V^2/R = \frac{(2\pi R/T)^2}{R} = \frac{4\pi^2}{T^2} \cdot R = \boxed{5.95 \times 10^{-3}\ m/s^2}$$

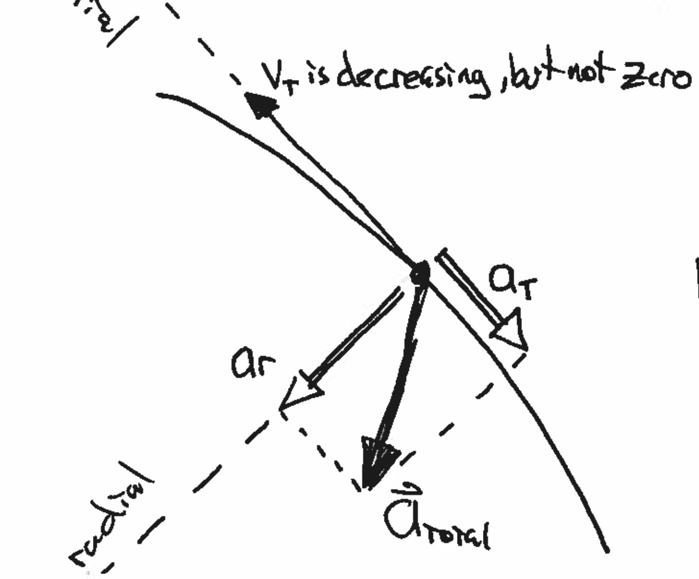
pick closest value
(rounding error)

Question value 8 points

(06) A disk is rotating counter-clockwise with some angular speed ω, when it begins to slow down. Which of the following figures *best* depicts the acceleration vector for a point of the rim of the disk, as it is slowing?



① Moving ccw and slowing down
→ tangential accel is opposite to motion



② Non-zero tangential velocity requires radially inward acceleration component

Overall acceleration vector is radially inward plus tangentially backward