Name

Physics 2211 AB

Solutions

Fall 2018

Nine-digit Tech ID

Quiz #

4D

- Put *nothing* other than your name and nine-digit Tech ID in the spaces above.
- Free-response problems are numbered I III. Show all your work clearly, including all steps and logic. Write **darkly**. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1 8. For each, select the answer most nearly correct, circle it on your quiz, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- The standard formula sheet is on the back of this page, which may be removed from the quiz form if you wish, but it must be submitted.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Your score will be posted when your quiz has been graded. Quiz grades become final when the next is given.

Fill in bubbles for your Multiple Choice answers darkly and neatly.

	a	b	c	d	e
1	ⓐ	ⓑ	ⓒ	ⓓ	ⓔ
2	ⓐ	ⓑ	ⓒ	ⓓ	ⓔ
3	ⓐ	ⓑ	ⓒ	ⓓ	ⓔ
4	ⓐ	ⓑ	ⓒ	ⓓ	ⓔ
5	ⓐ	ⓑ	ⓒ	ⓓ	ⓔ
6	ⓐ	ⓑ	ⓒ	ⓓ	ⓔ
	a	b	c	d	e

Quiz & Final Exam Formulæ & Constants

$$\vec{v} = \frac{d\vec{r}}{dt}$$
$$\vec{\omega} = \frac{d\vec{\theta}}{dt}$$
$$\vec{a} = \frac{d\vec{v}}{dt}$$
$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$
$$v_{sf} = v_{si} + a_s\,\Delta t$$
$$\omega_f = \omega_i + \alpha\,\Delta t$$
$$s_f = s_i + v_{si}\,\Delta t + \tfrac{1}{2}a_s\,(\Delta t)^2$$
$$\theta_f = \theta_i + \omega_{si}\,\Delta t + \tfrac{1}{2}\alpha\,(\Delta t)^2$$
$$s = r\theta$$
$$v = r\omega$$
$$a_t = r\alpha$$

$$\sum \vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$$
$$\sum \vec{F}_{ext} = M\vec{a}_{cm} = \frac{d\vec{P}}{dt}$$
$$\sum \vec{\tau}_{ext} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$$
$$f_{s,max} = \mu_s n$$
$$f_k = \mu_k n$$
$$a_r = \frac{v^2}{r}$$
$$\vec{w} = m\vec{g}$$
$$|\vec{F}_G| = \frac{Gm_1m_2}{|\vec{r}|^2}$$
$$D = \tfrac{1}{2}C\rho Av^2$$
$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$W = \int \vec{F} \cdot d\vec{s}$$
$$W_{ext} = \Delta K + \Delta U + \Delta E_{th}$$
$$K = \tfrac{1}{2}mv^2$$
$$K = \tfrac{1}{2}I\omega^2$$
$$U_g = mgy$$
$$U_s = \tfrac{1}{2}k\,(\Delta s)^2$$
$$U_G = -\frac{Gm_1m_2}{r}$$
$$P = \frac{dE_{sys}}{dt}$$
$$P = \vec{F} \cdot \vec{v}$$
$$\vec{J} = \int \vec{F}\,dt = \Delta\vec{p}$$
$$\vec{p} = m\vec{v}$$

$$\vec{r}_{cm} = \frac{\sum \vec{r}_i m_i}{\sum m_i}$$
$$\vec{r}_{cm} = \frac{\int \vec{r}\,dm}{\int dm}$$
$$I = \sum m_i r_i^2$$
$$I = \int r^2\,dm$$
$$I = I_{cm} + Md^2$$
$$\vec{L} = \vec{r} \times \vec{p}$$
$$\vec{L} = I\vec{\omega}$$
$$x = A\cos(\omega t + \phi_0)$$
$$\vec{a}_x = -\omega^2\vec{x}$$
$$\omega = \sqrt{k/m}$$
$$\omega = 2\pi f = \frac{2\pi}{T}$$

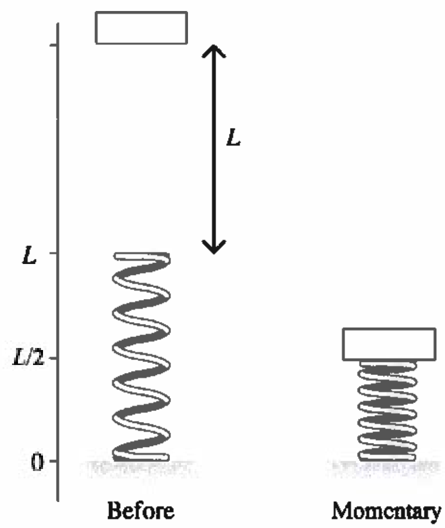
Physical Constants:

Universal Gravitation Constant $G = 6.673 \times 10^{-11}\,\mathrm{N{\cdot}m^2/kg^2}$
Gravitational Acceleration at Earth's Surface $g = 9.81\,\mathrm{m/s^2}$

Unless otherwise directed, drag is to be neglected and all problems take place on Earth, use the gravitational definition of weight, and all springs, ropes and pulleys are ideal.

You may remove this sheet from your Quiz or Exam

1. (20 points) A rigid spring having unstretched length L is oriented vertically. A block M is dropped from a height $h = L$ above the top of the spring. It is observed to compress the spring to a length $L/2$ before (momentarily) coming to a stop. If instead the block were placed gently on the spring, leaving the system in equilibrium, what would be the compressed length of the spring? Express your answer as a fraction of L.



Part A is an energy conservation problem

$$\Delta K + \Delta U = 0$$

$$K_f - K_i + U_{sf} - U_{is} + U_{Gf} - U_{Gi} = 0$$

$$0 - 0 + \frac{1}{2}k\left(\frac{L}{2}\right)^2 - 0 + Mg\frac{L}{2} - Mg\,2L = 0$$

$$k = \frac{+\frac{3}{2}MgL}{\cancel{\frac{1}{2}}\left(\frac{L}{2}\right)^2} = 12\frac{Mg}{L}$$

Part B is an application of Newton's 2nd law

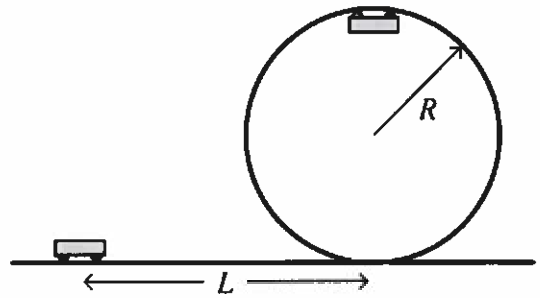
$a=0$ $\uparrow \vec{F}_s$ $\downarrow \vec{F}_g$

2nd law: $-k\Delta\vec{s} - Mg = 0$

$$\Delta s = -\frac{Mg}{k} = -\frac{L}{12}$$

The spring compresses by a distance $\frac{L}{12}$

II. (20 points) A motion cart of mass m rolls without friction on a horizontal track. A distance L ahead of the cart, there is a vertical circular loop of radius R. With what minimum speed v must the cart travel along the horizontal portion of the track, in order to complete the loop without losing contact with the track at the highest point of the loop? Express your answer symbolically in terms of the parameters m, L, R and/or g.



at top of loop

$a = a_c$ $\vec{n}$ F_g

$$F_g + n = m\frac{v_{min}^2}{R}$$

$\vec{n}$ = normal force exerted on cart by track

$$\cancel{m}\cdot g + \cancel{n} = \cancel{m}\frac{v_{min}^2}{R}$$ for minimal speed $\vec{n} = 0$

$$v_{min} = \sqrt{R\cdot g}$$

to get the speed v before cart enters the loop the loop

apply energy conservation

$$\Delta K + \Delta U = 0$$

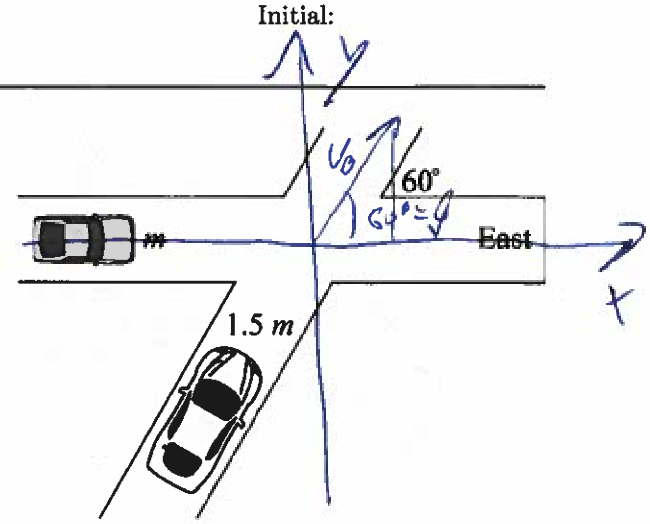
$$K_f - K_i + U_{gf} - U_{gi} = 0$$

ground is height 0

$$\frac{1}{2}\cancel{m}v_{min}^2 - \frac{1}{2}\cancel{m}v^2 + \cancel{m}\cdot g\, 2R - \cancel{m}\cdot g\cdot 0 = 0$$

$$v = \sqrt{v_{min}^2 + g\cdot 4R}$$

$$= \sqrt{R\cdot g + 4gR} = \underline{\underline{\sqrt{5R\cdot g}}}$$



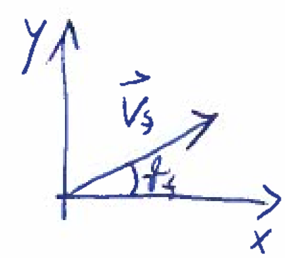
III. (20 points) A car of mass m is traveling due east on a primary road at speed v as it approaches an intersection. A second car of mass $1.5m$ is traveling with speed $2v$ along a secondary road toward the same intersection. The primary and secondary roads intersect at a 60.0° angle. The two cars collide and skid together through the intersection. What are the two cars' combined velocity immediately after they collide? Express your answer as a magnitude that is a numerical multiple of v, and a direction angle measured relative to eastward.

inelastic collision because two cars stick together

momentum conservation

$\vec{p}_A + \vec{p}_B = \vec{p}_{A+B}$ $\quad m_A = m \quad V_A = V$

$m_B = 1.5m \quad V_B = 2V$



in components

$p_x: \; m_A \cdot V_A + m_B \cdot V_B \cdot \cos\theta = (m_A + m_B)\, V_f \cdot \cos\theta_f$

$p_y: \; 0 + m_B \cdot V_B \cdot \sin\theta = (m_A + m_B)\, V_f \cdot \sin\theta_f$

$$\frac{p_y}{p_x} = \frac{m_B V_B \cdot \sin\theta}{m_A V_A + m_B V_B \cos\theta} = \tan\theta_f$$

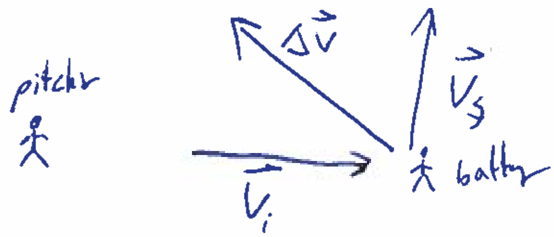
$$= \frac{1.5 \cdot 2 \cdot \sin 60°}{1 \cdot 1 + 1.5 \cdot 2 \cdot \cos 60°} = \frac{3 \cdot \sin 60°}{1 + 3\cos 60°} = \tan\theta_f \Rightarrow \underline{\underline{\theta_f = 46.1° \text{ north of east}}}$$

from p_y

$$V_f = \frac{m_B \cdot V_B \cdot \sin\theta}{(m_A + m_B) \cdot \sin\theta_f} = \frac{1.5m \cdot 2V \cdot \sin 60°}{2.5m \cdot \sin(46.1°)} = \underline{\underline{1.4\,V}}$$

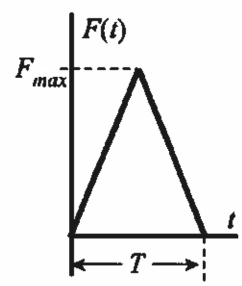
1. (8 points) A pitcher throws a baseball toward a batter at home plate. The ball arrives at the plate traveling horizontally. The batter swings at the ball, and the ball pops straight up into the air, leaving the bat with a purely vertical velocity. What is the direction of the impulse delivered to the ball by the bat, during the swing?

(a) Straight away from the pitcher.
(b) Upward and away from the pitcher.
(c) Upward and toward the pitcher.
(d) Straight upward.
(e) Straight toward the pitcher.



impulse $\vec{J} = \Delta\vec{p} \propto \Delta\vec{v}$

2. (8 points) A tennis ball of mass m is falling vertically downward. It hits the floor traveling with speed v, and rebounds straight upward with speed $\frac{1}{2}v$. It spends a total time T in contact with the floor. The graph at right displays the force exerted on the ball by the floor as a function of time, during the contact. What is the peak force (labelled as F_{max} in the diagram) during contact? (You may assume that the gravitational force mg is negligible in comparison to F_{max}.)



(a) $F_{max} = \frac{mv}{T}$
(b) $F_{max} = \frac{2mv}{3T}$
(c) $F_{max} = \frac{3mv}{T}$
(d) $F_{max} = \frac{2mv}{3T}$
(e) $F_{max} = \frac{3mv}{T}$

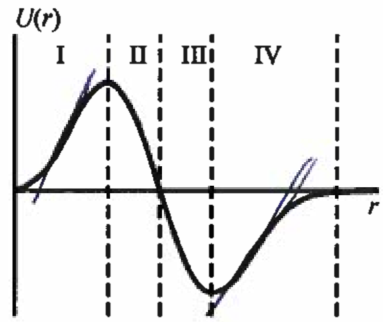
$$\Delta\vec{p} = \vec{J}$$

$$m\left(\frac{1}{2}v + v\right) = \int F \cdot dt$$

$$\frac{3}{2}mv = \frac{1}{2}T \cdot F_{max}$$

$$F_{max} = \frac{3mv}{T}$$

3. (8 points) Particle A is fixed at the origin, while particle B is free to move around at any distance r from the origin. The particles interact, subject to the potential energy curve $U(r)$ shown at right. In what regions will particle B feel a force of attraction toward particle A?



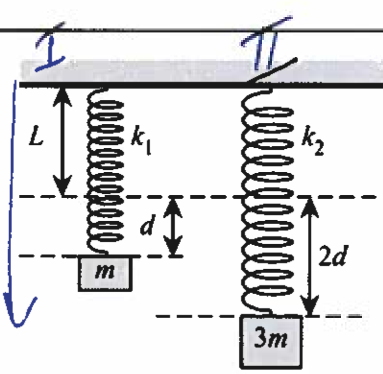
(a) In none of the regions indicated
(b) In regions I and II.
(c) In regions I and IV.
(d) In regions II and III.
(e) In regions III and IV.

$\vec{F}_r = \underbrace{-\frac{d}{dx}U(x)}_{\text{attracted to A is negative}}$

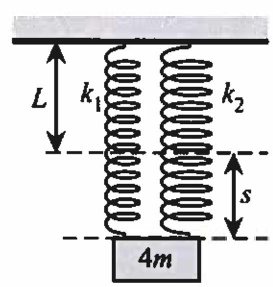
$\Rightarrow \frac{d}{dx}U(x)$ has to be positive

$\Rightarrow$ region I and IV

4. (8 points) Two springs have identical unstretched lengths L, but unequal elastic constants k_1 and k_2. The springs are suspended from the ceiling, and masses m and $3m$ are suspended from them in equilibrium, as shown in the top figure. The springs are observed to stretch by distances d and $2d$, respectively. If, instead, a mass $4m$ is suspended in equilibrium from both springs at once (bottom figure), by what distance s will the two springs stretch?



(a) $s = \frac{5}{3}d$
(b) $s = \frac{3}{2}d$
(c) $s = \frac{3}{5}d$
(d) $s = \frac{5}{4}d$
(e) $s = \frac{8}{5}d$



FBD I

$a=0$ $\uparrow F_s$ $\downarrow mg$

2nd law $mg - k_1 \Delta s = 0$

$k_1 = \frac{m \cdot g}{d}$

FBD II

$a=0$ $\uparrow F_s$ $\downarrow 3mg$

2nd law $3mg - k_2 2d = 0$

$k_2 = \frac{3}{2}\frac{mg}{d}$

FBD III

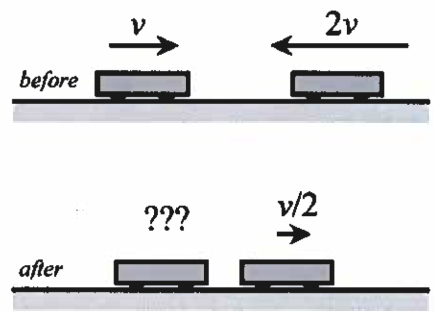
$\uparrow F_{s1} \uparrow F_{s2}$ $\downarrow 4mg$

$-k_1 \cdot s - k_2 \cdot s + 4mg = 0$

$s = \frac{4mg}{k_1 + k_2} = \frac{4mg}{\frac{mg}{d} + \frac{3}{2}\frac{mg}{d}} = \frac{8}{5}d$

5. (4 points) The figure at right displays the collision of two identical motion carts of mass m. At top is the situation immediately before contact, and at bottom is the situation immediately after contact. Motions sensors record the velocities of both carts before the collision, but one of the sensors breaks on impact and only one final velocity is recorded. What is the change in the total linear momentum of the two blocks, during the collision? You may assume that rolling friction is negligible.

(a) $\Delta\vec{P} = \langle +\frac{1}{2}mv \rangle$
(b) $\Delta\vec{P} = \langle +\frac{3}{2}mv \rangle$
(c) $\Delta\vec{P} = \langle 0 \rangle$
(d) $\Delta\vec{P} = \langle -\frac{1}{2}mv \rangle$
(e) $\Delta\vec{P} = \langle -\frac{3}{2}mv \rangle$



momentum conservation dictates

$\Delta\vec{P} = 0$

6. (4 points) In the problem above, what is the change in the total kinetic energy of the two blocks, during the collision?

(a) $\Delta K = +\frac{3}{2}mv^2$
(b) $\Delta K = +\frac{5}{4}mv^2$
(c) $\Delta K = 0$
(d) $\Delta K = -\frac{3}{2}mv^2$
(e) $\Delta K = -\frac{5}{4}mv^2$

from p-conservation

$\cancel{m}v - 2\cancel{m}v = \cancel{m}v_? + \frac{1}{2}v\cancel{m}$

$\Rightarrow v_? = -\frac{3}{2}v$

$\Delta K = K_f - K_i = \frac{1}{2}m\left(\frac{1}{2}v\right)^2 + \frac{1}{2}m\left(\frac{3}{2}v\right)^2 - \frac{1}{2}mv^2 - \frac{1}{2}m(2v)^2$

$= \frac{1}{2}mv^2\left[\frac{1}{4} + \frac{9}{4} - 1 - 4\right]$

$= -\frac{5}{4}mv^2$