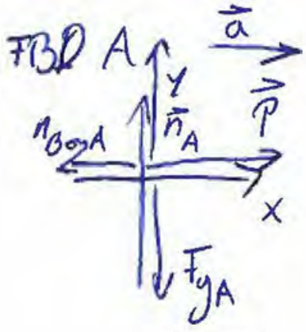
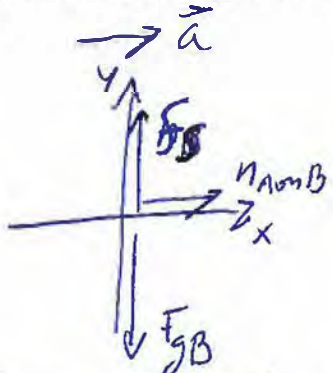
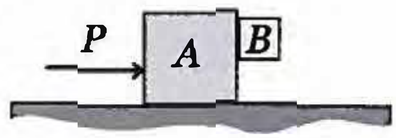
SOLUTION

1. 20 points) Block A in the illustration, with mass m_A, slides along a horizontal frictionless plane. Block B, with mass m_B, has a coefficient of static friction μ_s and a coefficient of kinetic friction μ_k with the front face of block A. What is the minimum magnitude, P, of a horizontal push force on block A so that block B does not slide downward? Express your answer in terms of parameters defined in the problem and physical or mathematical constants.







f_s is maxed out at minimal ~~spee~~ magnitude of $\vec{P}$

2nd law equations for A for B

x: $-n_{B on A} + P = m_A \cdot a$ x: $n_{A on B} = m_B \cdot a$ both objects are acceleration constrained $a_B = a_A = a$

y: $f_s - m_B \cdot g = 0$

static friction maxed out $\Rightarrow \mu_s \cdot n_{A on B} = m_B \cdot g$

I $\xRightarrow[\text{x-direction}]{\text{2nd law}}$ $\mu_s \cdot \cancel{m_B} \cdot a = \cancel{m_B} \cdot g \Rightarrow a = \dfrac{g}{\mu_s}$

with ~~and~~ 2nd law in x for A and B

$-n_{B on A} + P = m_A a$ use that $n_{B on A} = n_{A on B}$

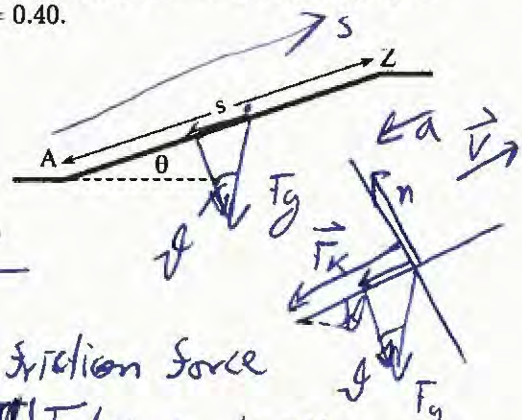
$-m_B \cdot a + P = m_A a$

II $P = a \cdot (m_B + m_A)$

I in II $\underline{\underline{P = \dfrac{g}{\mu_s}(m_B + m_A)}}$ minimal pushing force needed to keep B from falling down

II. (15 points) Abe and Zeb are carpenters constructing a ramp inclined at an angle $\theta = 18°$ above the horizontal. Abe is at the bottom of the ramp, and shoves a box of nails up toward Zeb, at the top of the ramp a distance $s = 2.5\,\text{m}$ away. Abe shoves the box with a speed v_0 that is just sufficient for it to slide all the way up to Zeb. The coefficient of friction between the box and the ramp is $\mu_k = 0.40$.

Determine the speed v_0 that will allow the box to reach the top of the ramp. **Work the problem symbolically, making numerical substitutions only at the last step.** (Credit will be deducted for numerically based solutions!)



Kinetic energy work theorem

$$\Delta K = W_f + W_{Grav}$$

$$-\frac{1}{2} m v_0^2 = -F_k \cdot s - m \cdot g \cdot \sin\theta \cdot s$$

friction force

$$|F_k| = \mu_k \cdot n = \mu_k \cdot \cos\theta \cdot m \cdot g$$

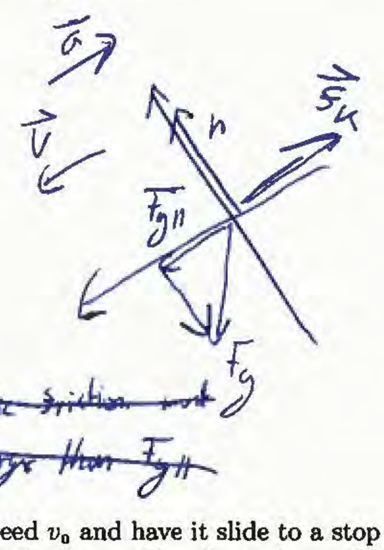
$$-\frac{1}{2} m v_0^2 = -\mu_k \cos\theta \cdot m \cdot g s - m \cdot g \cdot \sin\theta \cdot s$$

$$v_0 = \sqrt{2\mu_k g \cdot \cos\theta s + 2 g \cdot \sin\theta \cdot s}$$

$$= \sqrt{2g \cdot s\,(\mu_k \cdot \cos\theta + \sin\theta)}$$

$$= 5.8\,\frac{m}{s}$$

bottom question



~~kinetic friction must be larger than $F_{g\parallel}$~~

1. (5 points) Zeb would like to give the box of nails a shove with the same speed v_0 and have it slide to a stop just as it reaches Abe. Can Zeb do this? If so, must he remove nails from the box, add nails, or leave the number of nails the same?

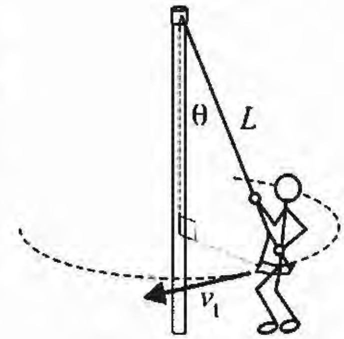
   (a) No, Zeb cannot do this, regardless of what he does with the number of nails.
   (b) Yes, Zeb can do this, but he must remove nails (and therefore mass) from the box.
   (c) Yes, Zeb can do this, but he must add nails (and therefore mass) to the box.
   (d) Yes, Zeb can do this, regardless of what he does with the number of nails.
   (e) Yes, Zeb can do this, but he must leave the number of nails unchanged.

from kinetic energy work theorem

$$s = \frac{-v_0^2}{2(-\mu_k \cos\theta g + g \sin\theta)}$$

does not depend on m!

box must slide farther than in II because friction ...

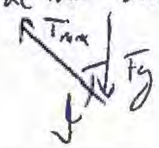


2. (5 points) A pole-swing in a childrens" playground consist of a seat attached to a central pole of height $H = 3.75\,\text{m}$ by a cord of length $L = 3.00\,\text{m}$. The swing can then be pivoted in a circle about the central pole. The cord is strong enough to sustain a maximum tension $T_{max} = 37\,\text{lb} = 1670\,\text{N}$. One night, Georgia Tech defensive tackle T.J.B̃arnes (weight $W = 333\,\text{lb} = 1480\,\text{N}$) sneaks into the playground to ride the pole-swing.

Suppose T.J. rides the swing with the maximum possible speed v_t that will not break the cord. What will be the angle θ between the cord and the pole, when T.J. rides the swing at this maximum safe speed?

(a) 37°
(b) 42°
(c) 28°
(d) 62°
(e) 48°

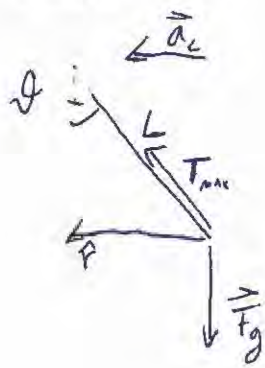
at max save speed



$$\frac{F_g}{T_{max}} = \cos\vartheta \Rightarrow \vartheta = 27.6^\circ$$

III. (15 points) In the problem above, what will be the period of T.J.'s rotation around the pole at the maximum safe speed?

FBD



2nd law

$$T_{max}\cdot\sin\vartheta = m\cdot\omega^2 R \qquad R = L\sin\vartheta$$

$$T_{max}\cdot\cancel{\sin\vartheta} = m\cdot\omega^2\cdot L\,\cancel{\sin\vartheta}$$

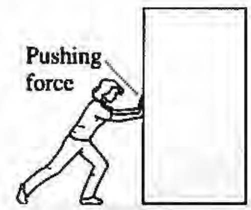
$$\omega = \sqrt{\frac{T_{max}}{m\cdot L}} \qquad m = \frac{W}{g}$$

$$= \sqrt{\frac{T_{max}\, g}{W\cdot L}}$$

from Definition of $\omega = 2\pi T$

$$\Rightarrow T = 2\pi\sqrt{\frac{W\cdot L}{T_{max}\, g}} = \underline{\underline{3.27\ s}}$$

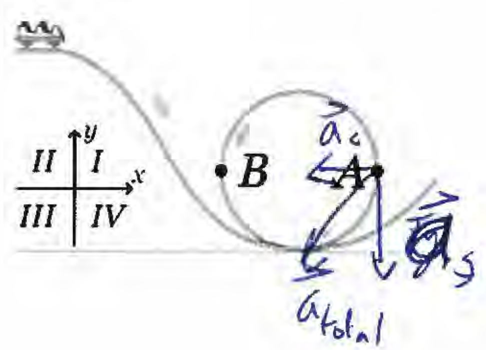
3. (6 points) A block of mass m is pushed a distance d across a horizontal floor. The pushing force $\vec{P}$ is horizontal, and the block moves at constant speed due to the coefficient of kinetic friction μ_k between the block and the floor. Describe the change in thermal energy.



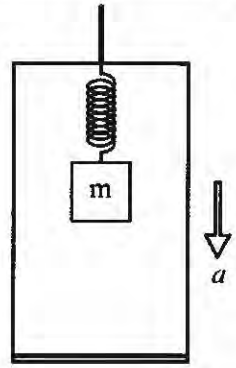
(a) The thermal energy of the block increases by Pd, while the thermal energy of the floor decreases by Pd.
(b) The thermal energy of the block increases by Pd, while the thermal energy of the floor remains unchanged as it is outside the system.
(c) The thermal energy of the floor increases by Pd, while the thermal energy of the block remains unchanged as its kinetic energy is constant.
(d) The thermal energy of the floor increases by Pd, while the thermal energy of the block decreases by Pd.
(e) The thermal energy of the block and floor together increases by a total Pd.

4. (6 points) The roller-coaster in the illustration is truly a “coaster” the car speeds up as it descends on its frictionless track, and slows down again as it ascends. Assuming it starts high enough that it completes the loop, what is the direction of the car's acceleration as it passes through point A?

(a) In the $+x$ direction.
(b) Somewhere in quadrant III.
(c) In the $-x$ direction.
(d) In the $-y$ direction.
(e) Somewhere in quadrant IV.

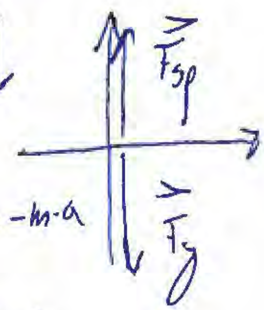


5. (10 points) A block of mass $m = 0.60\,\text{kg}$ hangs from the ceiling of an elevator by a spring of unstretched length $L_0 = 36\,\text{cm}$ having elastic constant $k = 48\,\text{N/m}$. What will be the total length of the spring when the elevator is experiencing a downward-directed acceleration of magnitude $a = 2.6\,\text{m/s}^2$?



(a) 45 cm
(b) 39 cm
(c) 48 cm
(d) 36 cm
(e) 27 cm

FBD m



$$K \cdot \Delta s - m \cdot g = -m \cdot a$$

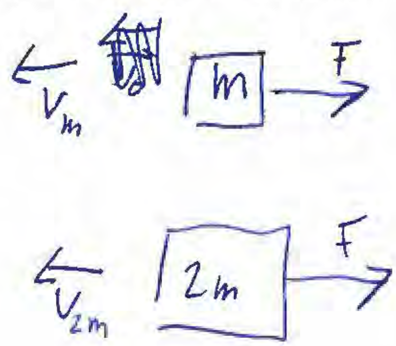
$$\Delta s = \frac{m(g-a)}{K}$$

total length $L_0 + \Delta s = 36\,cm + 9\,cm$

$= 45\,cm$

6. (6 points) Two blocks, one with mass m and one with mass $2m$, are traveling along level frictionless tracks with the same kinetic energy. Identical applied forces $\vec{F}$ will be used to bring each block to a stop. Compare the distances required to stop the blocks.

(a) The relative distances to stop the blocks **cannot be determined** from the information provided.
(b) The distance to stop the block with mass $2m$ is **the same** as that to stop the block with mass m.
(c) The distance to stop the block with mass $2m$ is **less than** that to stop the block with mass m.
(d) The distance to stop the block with mass $2m$ is **greater than** that to stop the block with mass m.



$$\Delta K = W$$

$$-K = -F \cdot S$$

because initial kinetic energy and force is the same for both objects they both stop after the same distance

7. (6 points) The inclines in the illustration are frictionless, and make angles α and β with the horizontal, where $\alpha > \beta$. The blocks have the same mass m and are initially at rest. In which direction, if any, do the two blocks move?

(a) The blocks do not move.
(b) The direction cannot be determined from the information provided.
(c) The blocks move to the left.
(d) The blocks move to the right.

The force component of gravity pointing <u>down</u> the incline is large for A than for B

$\Rightarrow$ net force to the left $\Rightarrow$ acceleration to left

8. (6 points) Two cars, A and B, on level ground have the same mass and their engines provide the same power. Each increases its speed by the same amount Δv. Car A, however, increases its speed from rest, while car B is already traveling at speed v_0 before its speed increases. Which car, if either, requires less time to complete its speed change?

(a) Which car requires less time to change its speed by Δv cannot be determined from the information provided.
(b) Car B requires less time than car A to change its speed by Δv.
(c) Car A requires less time than car B to change its speed by Δv.
(d) Both cars requires the same time to change their speeds by Δv.

$$P = \frac{dE}{dt} \Rightarrow dt = \frac{dE}{P}$$

dE for A $\quad \frac{1}{2} m \Delta v^2$

for B $\quad \frac{1}{2} m (v_0 + \Delta v)^2$

$dE_B > dE_A \Rightarrow$ B needs more time than A

or A needs less time than B