Physics 2211M
Summer 2018
Test 3

form A

First name (please write as legibly as possible within the boxes)

S	O	L	U	T	I	O	N		S	E	T					

Last name

Student nine-digit GTID

Instructions:

1) Please use **dark-colored ink** or **heavy pencil strokes**; this test will be scanned and graded electronically, and it is important for your work to be legible on the scanned page.
2) **DO NOT ERASE** any of your work and overwrite it with new work—this will interfere with the legibility of your scanned test. Please draw a line through invalid work that you wish us to disregard.
3) No not include any loose scratch work on a separate page along with your test—extra pages are not scanable. If you need additional workspace, please use the provided blank space on pages 2 or 9. Be sure to point out, on the main problem, when you have additional work on the scratch page(s).
4) For each free response question, show all work necessary to support your answer. Clearly indicate your final answer by underlining it, or boxing it in.
5) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**
6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

Please do not write above this line

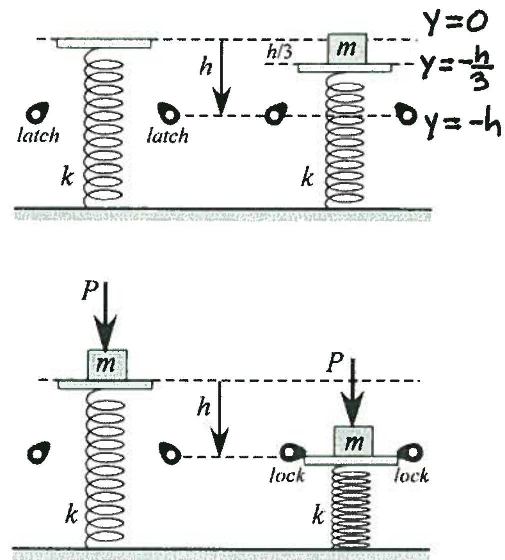
The following problem will be hand-graded. *Show all supporting work for this problem.*

[I] *(20 points)* A spring-launched cannon consists of a massless platform atop a vertical spring of elastic constant k (top left). The spring must be compressed a distance h (from its neutral position) to engage a latch mechanism that holds the platform in place until launch.

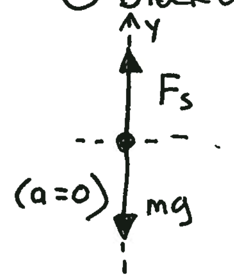
A block of mass m is gently placed on the platform, but the spring only compresses a distance $h/3$ before coming to equilibrium, and thus cannot lock into place (top right). To fully compress the spring and lock the mechanism, you must therefore push the block down with an external applied force of magnitude P. Assume that you start pushing the moment you place the block (bottom left) and continue with a constant force until the latch engages (bottom right).

What is the smallest value for P that will allow the lock to engage? Express your answer as a multiple of the block's true weight, mg.

Hint: you may assume the block ends at rest, at the moment the latch engages.



① Block at rest on spring: equilibrium: $\vec{a}=0$



$\langle +kh/3\rangle + \langle -mg\rangle = 0$

$\boxed{k = \frac{3mg}{h}}$ → use this to eliminate k from later steps

② Compressing spring: external force does work so $\boxed{\text{system energy is not conserved}}$

Use block itself as system, $\Delta E_{sys} = W_{ext}$ (Energy Principle) becomes $\Delta K = W_{TOT}$ (Work-KE theorem)

but $K_i = K_f = 0$, so $\boxed{\Delta K \equiv 0}$ → total work by all forces must sum to zero

① Work by gravity: $W_g = \vec{F}_g \cdot \vec{\Delta s} = (-mg)(\vec{\Delta y})$ → $\boxed{W_g = +mgh}$ $(\vec{\Delta y} = \langle -h\rangle)$

② Work by spring $W_s = -\frac{1}{2}k(\Delta s_f)^2 + \frac{1}{2}k(\Delta s_i)^2$
$= -\frac{1}{2}kh^2 + 0$ → $W_s = -\frac{1}{2}kh^2 = -\frac{1}{2}\left(\frac{3mg}{h}\right)h^2$ after subbing for k

$\boxed{W_s = -\frac{3}{2}mgh}$

③ Work by applied force $W_p = \vec{P} \cdot \vec{\Delta s} = \langle -P\rangle \cdot \langle -h\rangle$ → $\boxed{W_p = +Ph}$

So $W_{TOT} = 0 = P\cancel{h} + mg\cancel{h} - \frac{3}{2}mg\cancel{h}$

$\boxed{P = \frac{1}{2}mg}$

It is also possible to solve this using U_{spring} and U_{grav}

Form A

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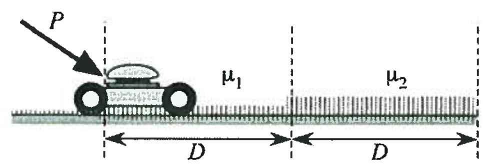
The following problem will be hand-graded. Show all supporting work for this problem.

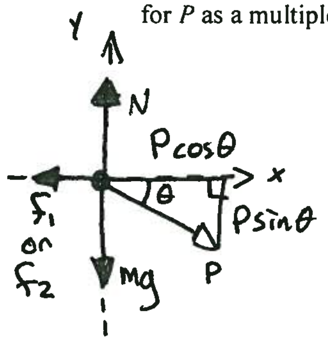
[II] *(20 points)* A lawn mower of mass m is pushed throught heavy grass, which causes a resistive force behaving just like kinetic friction, with a resistive coefficient μ. Starting from rest in short grass ($\mu_1 = 0.20$), the mower is pushed by a constant force P, directed at an angle 35° below the horizontal. The mower picks up speed as it travels a distance D, at which point it hits taller grass ($\mu_2 = 0.30$). Although the same pushing force is maintained, the extra resistance of this grass causes the mower to slow back down, bringing it to halt after an additional distance D.



Identify all forces acting on the mower, and determine expressions for the work done by each force during the time it is acting on the mower.

Yes, even the "zero work" forces

Use your expressions, and the Work-Energy Principle, to find the magnitude of the pushing force, P. Express your answer for P as a multiple of mg.



Forces: normal force : $\boxed{W_N = 0}$ because force $\perp$ displacement

grav force : $\boxed{W_g = 0}$ " " " "

Push P : $W_P = \vec{P}\cdot\vec{\Delta s} = P_x \Delta X = (P\cos\theta)(2D)$

$\rightarrow \boxed{W_P = 2PD\cos\theta}$

$\Rightarrow$ Two different friction forces, each acting only through displacements $\vec{\Delta X_i} = \langle +D \rangle$

$f_i = \mu_i N$ where N is found from $\Sigma \vec{F}_y = 0$

$\langle +N \rangle + \langle -mg \rangle + \langle -P\sin\theta \rangle = 0 \rightarrow \boxed{N = mg + P\sin\theta}$

(same for both halves of displacement)

Hence, $W_i = \vec{f_i}\cdot\vec{\Delta s} = (-\mu_i N)(+D)$

So: low grass $\boxed{W_1 = -\mu_1(mg + P\sin\theta)D}$

high grass $\boxed{W_2 = -\mu_2(mg + P\sin\theta)D}$

Mower begins and ends at rest, so $\Delta K = 0 \rightarrow \boxed{W_{TOT} = 0}$

$$W_N + W_g + W_P + W_1 + W_2 = 0$$

$$0 + 0 + 2P\cancel{D}\cos\theta + (-\mu_1[mg + P\sin\theta]\cancel{D}) + (-\mu_2[mg + P\sin\theta]\cancel{D}) = 0$$

distance D drops out!

$$2P\cos\theta - P(\mu_1 + \mu_2)\sin\theta = (\mu_1 + \mu_2)mg$$

$$\boxed{P = \frac{\mu_1 + \mu_2}{2\cos\theta - (\mu_1 + \mu_2)\sin\theta}\cdot mg}$$

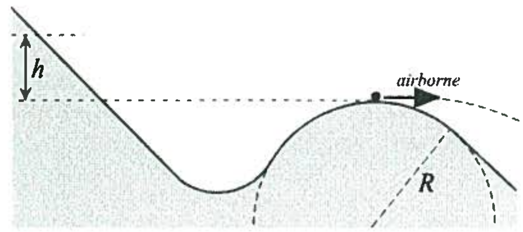
Plug in values for μ_1, μ_2, and θ:

$$\boxed{P = 0.37\,mg}$$

Please do not write above this line

The following problem will be hand-graded. Show all supporting work for this problem.

[III] *(20 points)* A skateboard park has a large hemispherical hill of radius R in a low-lying area. From a higher nearby slope, a boy sets off from rest and passes over the lower hill. From what maximum height can the boy start, without losing contact with the ground as he passes over the highest point of the hill? (That is, the goal is for the boy NOT to become airborne, as shown in the figure!)



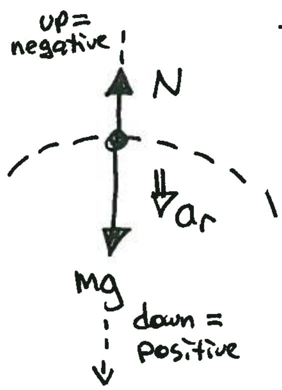
Treat the boy and skateboard as a single particle, and assume negligible friction between the skateboard and the ground.

Give your answer as a height h above the apex of the lower hill, expressed as a multiple of the hill's radius of curvature R.

① Passing over top of hill is a circular motion situation

$\rightarrow$ downward-directed radial acceleration, $a_r = V^2/R$

$\rightarrow$ "losing contact" would mean $N \rightarrow 0$, so we'll require some non-zero N (but close to zero!)



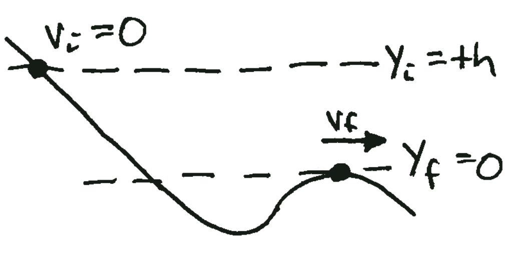
$$\Sigma \vec{F}_r = m\vec{a}_r$$

$$\langle -N \rangle + \langle +mg \rangle = m \langle +V^2/R \rangle \rightarrow \text{consider limit when } N \approx 0$$

$$\rightarrow \boxed{V^2 \approx gR}$$

[Note: since we actually want $N > 0$, we need $V^2 < gR$]

② Conservation of energy problem, since there is negligible friction/drag



$$K_i + U_i = K_f + U_f$$

$$0 + mgh = \tfrac{1}{2}mV^2 + 0 \quad \text{using } V^2 \text{ from above}$$

$$mgh = \tfrac{1}{2}m(gR)$$

$$\text{so } \boxed{h = \tfrac{1}{2}R}$$

[Since we want to remain in contact, we want $N \gtrsim 0$, and thus $V^2 \lesssim gR$. That requires $h \gtrsim \frac{1}{2}R$]

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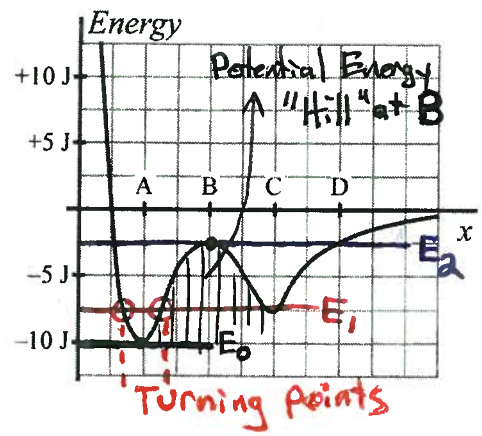
The next two questions involve the following situation:
The figure at right displays the potential energy curve for an object that moves along the x-axis. There are no other forces acting on the object, apart from the conservative force that gives rise to this PE curve.



$\rightarrow E_0 = K_0 + U_0 = -10J$

Question value 4 points

(1) If the object $\boxed{\text{begins at rest at position A.}}$ how much external work must be done on the system in order for the object to reach position C?

(a) No amount of work will allow the object to reach position C.

~~(b) $W_{ext} = -2.5$ J~~

(c) $W_{ext} = +2.5$ J

~~(d) $W_{ext} = -7.5$ J~~

$\boxed{\text{(e) } W_{ext} = +7.5 \text{ J}}$

doing negative work on an object at rest, at a PE minimum, is $\boxed{\text{IMPOSSIBLE}}$

① Answer must involve POSITIVE work

② Small positive work gives system total energy E_1
At that energy object has two turning points near A
$\Rightarrow$ not possible to reach C, due to "Hill" at B

③ Object needs enough added energy to get "over the hump" at Hill B, so $\boxed{W_{ext} = +7.5 J}$

Question value 4 points

(2) How many stable equilibrium postitions does this system have?

(a) Three.
(b) Four.
$\boxed{\text{(c) Two.}}$
(d) Zero.
(e) One.

"Stable equilibrium" is defined as being a $\boxed{\text{PE minimum}}$

We clearly see $\boxed{\text{two}}$ such minima in the PE curve
$\rightarrow$ one at A and one at C

[B = PE maximum = Unstable equilibrium]

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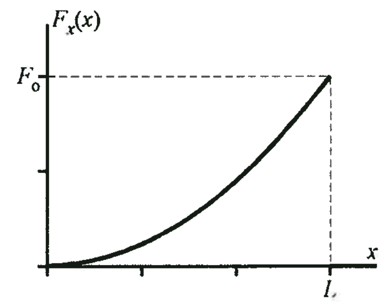
(3) A variable conservative force acts on an object that moves along the x-axis, given by the functional expression:

$$F_x(x) = F_0 \frac{x^2}{L^2} \text{ for } 0 \le x \le L,$$

where F_0 and L are constants. What is the change in the object's potential energy, as it moves from $x = 0$ to $x = L$?



(a) $\Delta U = +F_0 L$

(b) $\Delta U = +\frac{F_0 L}{3}$

$\boxed{\text{(c) } \Delta U = -\frac{F_0 L}{3}}$

(d) $\Delta U = 0$ becaue the force is conservative.

(e) $\Delta U = -F_0 L$ * right sign!

For a conservative force,

$$\Delta U = -W_{cons} = -\int F_x \, dx$$

• Since F_x = positive and displacement = positive, work will be positive

→ so correct answer must have $\boxed{\text{negative } \Delta U}$ *

$$\Delta U = -\int_{x=0}^{x=L} F_0 \frac{x^2}{L^2} dx = \frac{-F_0}{L^2} \int_0^L x^2 dx$$

$$= -\frac{F_0}{L^2}\left[\frac{L^3}{3}\right] \longrightarrow \boxed{\Delta U = -\frac{F_0 L}{3}}$$

Question value 8 points

(4) A construction worker drops a 2.0 kg hammer while standing on a girder that is 25 m above the ground. The hammer strikes the ground moving at 18 m/s. How much work was done on the hammer by aerodynamic drag?

(a) ~~+324 J~~ positive? ick!

(b) −490 J

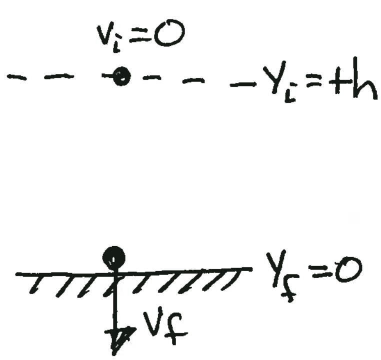
$\boxed{\text{(c) } -166 \text{ J}}$

(d) zero J, because energy was conserved

(e) ~~+490 J~~ positive? ick!

$$\Delta E_{sys} = W_{ext}$$

↳ Work by drag is external to system



$$\Delta K + \Delta U = W_{ext}$$

$$(K_f - K_i) + (U_f - U_i) = W_{ext}$$

$$\left(\tfrac{1}{2} m v_f^2 - 0\right) + (0 - mgh) = W_{ext}$$

$$(+324 \text{ J}) + (-490 \text{ J}) = W_{ext}$$

$$\boxed{-166 \text{ J} = W_{ext}}$$

Note that drag doing positive work is an absurd conclusion

Form A

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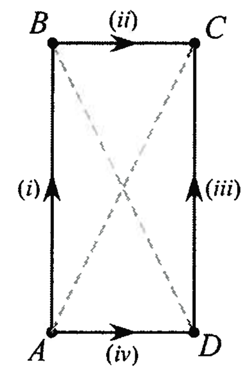
Question value 8 points

(5) A block moves in the *xy* plane, subject to a single conservative force. Points *A*–*D* are located at the corners of a rectangle of dimension $d \times 2d$. Paths (*i*) through (*iv*) connect these points as shown. The work done by the conservative force, along each path, are given by:

$$W_i = +5\text{ J} \qquad W_{ii} = -4\text{ J}$$
$$W_{iii} = +4\text{ J} \qquad W_{iv} = -3\text{ J}$$



If the obect moves directly acroos the diagonal, from *D* to *B*, how much work is done by the force?

(a) $W_{D\to B} = -2\text{ J}$

(b) $W_{D\to B} = -8\text{ J}$

(c) $W_{D\to B}$ cannot be computed from the information given.

(d) $\boxed{W_{D\to B} = +8\text{ J}}$

(e) $W_{D\to B} = +2\text{ J}$

Conservative force: Work is independent of path

So $W(A\to B) = W(A\to D\to B)$

$W_{(i)} = W_{(iv)} + W_{D\to B}$

$W_{D\to B} = W_{(i)} - W_{(iv)} = (+5\text{ J}) - (-3\text{ J})$

$\boxed{W_{D\to B} = +8\text{ J}}$

Question value 8 points

(6) You are standing around talking to your friends, when your bookbag slips off your shoulder at falls to the ground, where it comes to rest. Considering the system consisting of just your bookbag, starting from the moment it starts to slip, what work is done on the bag?

(a) Friction with your shoulder does negative work, gravity does positive work, and the normal force with the ground does negative work. ~~The total work done is negative.~~ Nope

(b) Friction with your shoulder does negative work, gravity does positive work, and the normal force with the ground does negative work. The total work done is zero.

(c) Friction with your shoulder does negative work and the normal force with the ground does negative work. The total ~~work done is negative.~~ Nope

(d) Friction with your shoulder does negative work, gravity does negative work, and the normal force with the ground does positive work. ~~The total work done is negative.~~ Nope

(e) Friction with your shoulder does positive work and the normal force with the ground does negative work. The total work done is zero.

- Start and end at rest: $\Delta K = 0$ so $\boxed{W_{TOT} \text{ must} = 0}$
- bag displaces downward
- Friction, normal force = up and gravity = down

$\boxed{W_f \text{ and } W_N = \text{neg} \quad W_g = \text{pos}}$