Physics 2211M
Summer 2018
Test 1

form A

First name (please write as legibly as possible within the boxes)

SOLUTION SET

Last name

nine-digit GTID

Instructions:

1) Please use **dark-colored ink** or **heavy pencil strokes**; this test will be scanned and graded electronically, and it is important for your work to be legible on the scanned page.

2) **DO NOT ERASE** any of your work and overwrite it with new work—this will interfere with the legibility of your scanned test. Please draw a line through invalid work that you wish us to disregard.

3) No not include any loose scratch work on a separate page along with your test—extra pages are not scanable. If you need additional workspace, please use the provided blank space on pages 2 or 9. Be sure to point out, on the main problem, when you have additional work on the scratch page(s).

4) For each free response question, show all work necessary to support your answer. Clearly indicate your final answer by underlining it, or boxing it in.

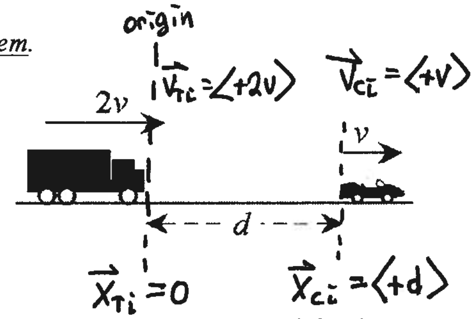
5) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

Please do not write above this line

The following problem will be hand-graded. *Show all supporting work for this problem.*

[I] *(20 points)* A moving van is driving at speed $2v$ when it emerges from a fog bank, and sees a sportscar a distance d directly in front of it, moving in the same direction at a speed v. The driver of the van immediately applies the breaks, giving the van a backward-directed acceleration of some magnitude a. At the same instant, the driver of the sportscar sees the van behind him and steps on the gas pedal, giving the car a forward-directed acceleration of magnitude $2a$.



What minimum value for a will prevent the two vehicles from colliding? Express your answer in terms of d and v.

Hint: to avoid a collision, the car and van must match velocities by the time the van catches up to the car.

Choose coord system with origin at truck, direction of travel = positive

- Truck must slow down, so $\vec{a}_T = \langle -a \rangle$

$$\vec{V}_{TF} = \vec{V}_{Ti} + \vec{a}_T \Delta t \rightarrow \boxed{\vec{V}_{TF} = \langle +2V \rangle + \langle -a \rangle \Delta t}$$

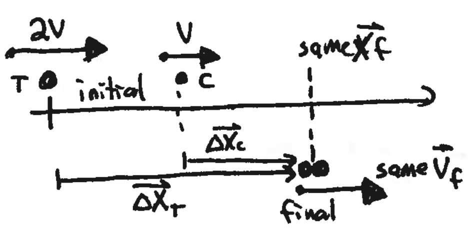
$$\vec{X}_{TF} = \underbrace{\vec{X}_{Ti}}_{0} + \vec{V}_{Ti}\Delta t + \tfrac{1}{2}\vec{a}_T \Delta t^2 \longrightarrow \boxed{\vec{X}_{TF} = \langle +2V \rangle \Delta t + \tfrac{1}{2}\langle -a \rangle \Delta t^2}$$

- Car must speed up, so $\vec{a}_c = \langle +2a \rangle$

$$\vec{V}_{cF} = \vec{V}_{ci} + \vec{a}_c \Delta t \longrightarrow \boxed{\vec{V}_{cF} = \langle +V \rangle + \langle +2a \rangle \Delta t}$$

$$\vec{X}_{cF} = \vec{X}_{ci} + \vec{V}_{ci}\Delta t + \tfrac{1}{2}\vec{a}_c \Delta t^2 \longrightarrow \boxed{\vec{X}_{cF} = \langle +d \rangle + \langle +V \rangle \Delta t + \tfrac{1}{2}\langle +2a \rangle \Delta t^2}$$

Now, per the hint, require velocities to match at moment positions match



① $\langle +2V \rangle + \langle -a \rangle \Delta t_m = \langle +V \rangle + \langle +2a \rangle \Delta t_m$

$(2V - V) = (2a + a)\Delta t_m$

→ time to match velocities is $\boxed{\Delta t_m = V/3a}$

(use this Δt_m in position equations)

② $\vec{X}_{TF} = \vec{X}_{cF} \longrightarrow \langle +2V \rangle \Delta t_m + \frac{1}{2}\langle -a \rangle \Delta t_m^2 = \langle +d \rangle + \langle +V \rangle \Delta t_m + \frac{1}{2}\langle +2a \rangle \Delta t_m^2$

$(2V - V)\Delta t_m + \frac{1}{2}(-a - 2a)\Delta t_m^2 = d$

$V\left(\frac{V}{3a}\right) + \frac{1}{2}(-3a)\left(\frac{V^2}{9a^2}\right) = d$

$\frac{V^2}{3a} - \frac{1}{2}\frac{V^2}{3a} = d \longrightarrow \frac{V^2}{6a} = d \Longrightarrow \boxed{a = \frac{V^2}{6d}}$

Form A

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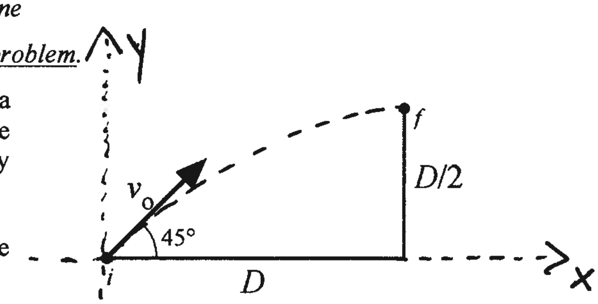
The following problem will be hand-graded. *Show all supporting work for this problem.*

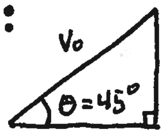


[II] *(20 points)* A compressed-air cannon fires a t-shirt into the crowd at a basketball game. The shirt is launched at an angle of 45° above the horizontal. It travels a horizontal distance D, at which point it is caught by a fan in the stands, who is at an elevation $h = D/2$ above the launch point.

Find an expression for the initial speed v_0 with which the t-shirt left the cannon. Express your answer in terms of g and D

Note: we don't really know for sure (yet) whether shirt is on the way up, at highest point, or on the way down

① Decompose $\vec{V}_0$:



$\vec{V}_{y0} = V_0 \sin 45° = \frac{1}{\sqrt{2}} V_0$

$\vec{V}_{x0} = V_0 \cos 45° = \frac{1}{\sqrt{2}} V_0$

② Horizontal motion is uniform: $\vec{\Delta x} = \vec{V}_x \Delta t$

let Δt_F = full time of flight until caught

$\langle +D \rangle + \langle +\tfrac{1}{\sqrt{2}} V_0 \rangle \Delta t_F \rightarrow \boxed{\Delta t_F = \frac{\sqrt{2} D}{V_0}}$

③ Vertical motion is free-fall: $\vec{\Delta y} = \vec{V}_{0y} \Delta t + \tfrac{1}{2} \langle -g \rangle \Delta t^2$

so, for full flight $\langle +\tfrac{D}{2} \rangle = \langle +\tfrac{1}{\sqrt{2}} V_0 \rangle \Delta t_F + \tfrac{1}{2} \langle -g \rangle \Delta t_F^2$

substitute

$$\frac{D}{2} = \frac{V_0}{\sqrt{2}} \left(\frac{\sqrt{2} D}{V_0} \right) - \frac{1}{2} g \left(\frac{2D^2}{V_0^2} \right)$$

$$\frac{D}{2} = D - \frac{gD^2}{V_0^2}$$

$$\frac{gD^2}{V_0^2} = \frac{D}{2} \rightarrow 2gD = V_0^2$$

$$\boxed{V_0 = \sqrt{2gD}}$$

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The following problem will be hand-graded. Show all supporting work for this problem.

[III] *(20 points)* You are in a sailboat, not far from shore. You feel the wind blowing past your boat in a direction $\theta_1 = 18°$ north of west, with a speed $v_1 = 8.8$ m/s. Instruments tell you that the boat is moving through the water at a speed $v_2 = 3.6$ m/s, with the bow (front) of the boat directed $v_2 = 12°$ south of west. Your nautical experience tells you that in these waters, there is a steady ocean current that flows due north with a speed $v_3 = 2.4$ m/s. What would a person on shore measure for the wind velocity? Express you answer as a speed and an angle relative to one of the cardinal directions (N, S, E, or W).

Hint 1: A valid pictorial representation of the problem adds clarity.

Hint 2 Work the problem symbolically for as long as possible, saving numerical substitutions for the last steps. Graders may deduct points for lack of clarity, if you make them wade through a slew of numbers on the page.

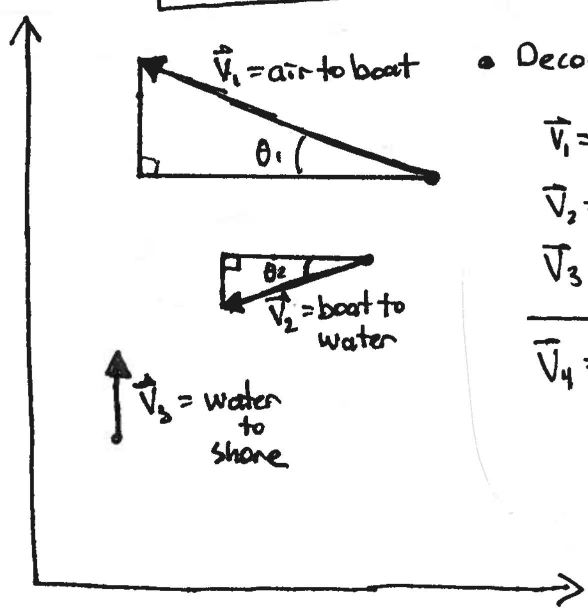
TWO relative velocity statements: A = Air, B = Boat, W = Water, S = Shore

① $\vec{V}_{AW} = \vec{V}_{AB} + \vec{V}_{BW}$

② $\vec{V}_{AS} = \vec{V}_{AW} + \vec{V}_{WS}$

Combine: $\vec{V}_{AS} = \vec{V}_{AB} + \vec{V}_{BW} + \vec{V}_{WS}$

Note that $\vec{V}_{AB}$, $\vec{V}_{BW}$, and $\vec{V}_{WS}$ are all given to us → just add, as vectors



• Decompose each vector, separately — then add components

$$\vec{V}_1 = \vec{V}_{AB} = \langle -V_1\cos\theta_1\rangle\hat{\imath} + \langle +V_1\sin\theta_1\rangle\hat{\jmath} = \langle -8.37\,\text{m/s}\rangle\hat{\imath} + \langle +2.72\,\text{m/s}\rangle\hat{\jmath}$$

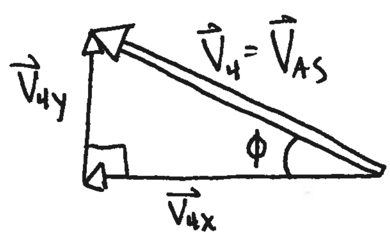
$$\vec{V}_2 = \vec{V}_{BW} = \langle -V_2\cos\theta_2\rangle\hat{\imath} + \langle -V_2\sin\theta_2\rangle\hat{\jmath} = \langle -3.52\,\text{m/s}\rangle\hat{\imath} + \langle -0.75\,\text{m/s}\rangle\hat{\jmath}$$

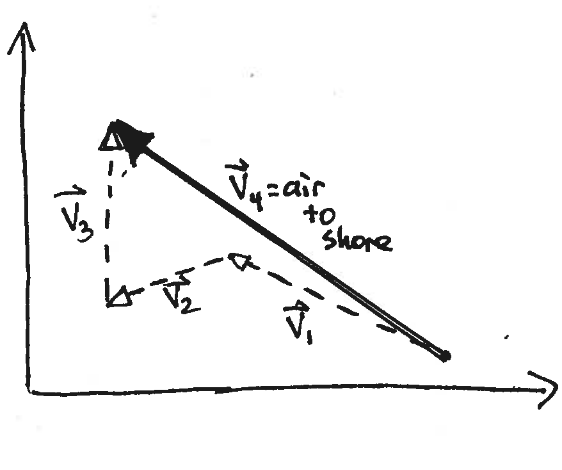
$$\vec{V}_3 = \vec{V}_{WS} = \langle 0\rangle\hat{\imath} + \langle +V_3\rangle\hat{\jmath} = 0\,\hat{\imath} + \langle +2.40\,\text{m/s}\rangle\hat{\jmath}$$

$$\vec{V}_4 = \vec{V}_{AB} + \vec{V}_{BW} + \vec{V}_{WS} = \underbrace{\vec{V}_{AS} = \langle -11.89\,\text{m/s}\rangle\hat{\imath} + \langle +4.37\,\text{m/s}\rangle\hat{\jmath}}$$

our desired vector, in cartesian form

→ "recompose" to get magnitude and direction of $\vec{V}_{AS}$





$V_4 = \sqrt{V_{4x}^2 + V_{4y}^2} = 12.67$ m/s rounds to $\boxed{13\text{ m/s}}$

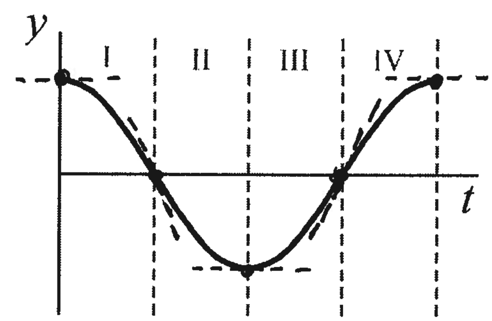
$\phi = \tan^{-1}\left(\frac{|V_{4y}|}{|V_{4x}|}\right) = 20.18°$ rounds to $\boxed{20°\text{ North of West}}$

(pos y, neg x = quadrant II, which is N of W)

Form A

Please do not write above this line

The next two questions involve the following situation:
A block hangs from a vertical spring. The block is lifted upward a bit and released, after which is bounces up and down. Based on the block's vertical motion, a graph of position versus time is constructed as shown below.



Question value 4 points

(1) During which interval(s) is the block *moving away from the origin*?

(a) During intervals II and III.

(b) During interval IV only.

(c) During intervals I and III.

(d) During intervals II and IV.

(e) During interval III only.

"moving away": either ① $\vec{y}$ is negative and $\vec{V}_y$ is negative [slope = neg]
or ② $\vec{y}$ is positive and $\vec{V}_y$ is positive [slope = pos]

→ I: y = pos, v_y = neg ✗
II: y = neg, V_y = neg ✓
III: y = neg, V_y = pos ✗
IV: y = pos, V_y = pos ✓

Question value 4 points

(2) During which interval(s) is the block *gaining speed*?

(a) During intervals I and IV.

(b) During interval II only.

(c) During interval III only.

(d) During intervals I and III.

(e) During intervals II and IV.

speed = magnitude of $\vec{V}_y$ = |slope of curve|

speed increasing means slope "becoming greater"
(neg slope getting steeper or pos slope getting steeper

I: slope starts zero, ends steeply negative ✓
II: slope starts steeply negative, ends zero ✗
III: slope starts zero, ends steeply positive ✓
IV: slope starts steeply positive, ends zero ✗

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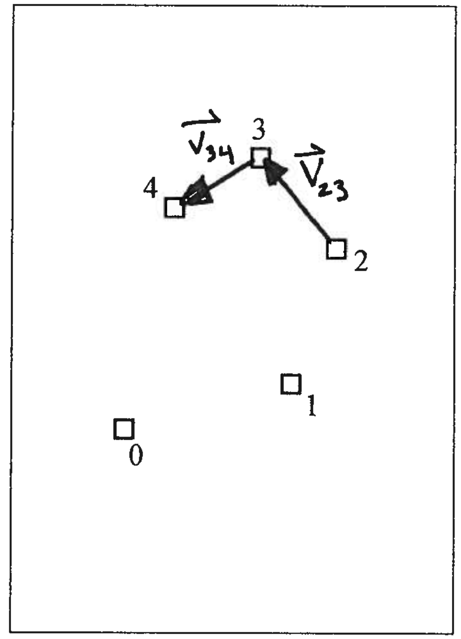
Question value 8 points

(3) The figure at right depicts the several frames of the motion diagram for an object moving in 2D. Which of the arrows below best depicts the acceleration vector for the object during frame 3?



(a) ↙

$\boxed{\text{(b) } \downarrow}$

(c) ↖

(d) $(\vec{a}_3 = 0)$

(e) ←

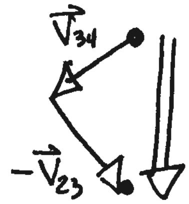
$\vec{a} = \frac{\Delta\vec{V}}{\Delta t} = \frac{\vec{V}_f - \vec{V}_i}{\Delta t}$

at frame 3:

$\vec{a}_3 \approx \frac{\vec{V}_{34} - \vec{V}_{23}}{\Delta t}$

$\boxed{\text{graphically:}}$

$\vec{V}_{34} - \vec{V}_{23} = \vec{V}_{34} + (-\vec{V}_{23})$



$\Delta\vec{V} \approx$ straight down,

so $\boxed{\vec{a} = \frac{\Delta\vec{V}}{\Delta t} \approx \text{"straight down"}}$

Question value 8 points

(4) An audiophile has a variable-speed turntable that he uses to play his old vinyl records. The turntable has two angular speed settings: $\omega_1 = 33$ rpm for albums, and $\omega_2 = 45$ rpm for singles. Suppose that the turntable is rotating at speed ω_1 when it is suddenly switched over to the higher speed setting ω_2. If it takes two full revolutions to achieve the higher speed, what angular acceleration α does the turntable experience? You may assume that α is constant.

(a) $\alpha = 0.59 \text{ rad/s}^2$

(b) $\alpha = 0.97 \text{ rad/s}^2$

$\boxed{\text{(c) } \alpha = 0.41 \text{ rad/s}^2}$

(d) $\alpha = 0.13 \text{ rad/s}^2$

(e) $\alpha = 0.62 \text{ rad/s}^2$

1 rpm = revolutions per minute $\Rightarrow \frac{\text{rev}}{\text{min}}$

$= \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) \times \left(\frac{1 \text{ min}}{60 \text{ sec}}\right) = \frac{\pi}{30} \text{ rad/sec}$

constant angular accel is "just like" constant linear accel

$V_f^2 = V_i^2 + 2a\Delta x$ (linear)

$\omega_f^2 = \omega_i^2 + 2(\alpha)(\Delta\theta)$ angular

$\rightarrow \boxed{\alpha = \frac{\omega_f^2 - \omega_i^2}{2\Delta\theta}}$

- convert ω_i/ω_f from rpm to rad/sec
- "two revolutions" means $\Delta\theta = 2(2\pi) = 4\pi$ rad

$\alpha = \frac{(45 \cdot \frac{\pi}{30} \text{ rad/s})^2 - (33 \cdot \frac{\pi}{30} \text{ rad/s})^2}{2(4\pi \text{ rad})} = \frac{[(1.5)^2 - (1.1)^2]\pi^2}{8\pi} \frac{\text{rad}}{s^2} = \frac{[(1.5)^2 - (1.1)^2]\pi}{8} \text{ rad/s}^2$

$= \boxed{0.408 \text{ rad/s}^2}$

Form A

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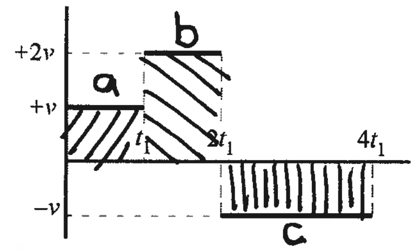
Question value 8 points

(5) The figure at right depicts the velocity vs. time graph for a particle moving along the x-axis. What is the average velocity for the particle during the interval shown?



$\boxed{\text{(a)} \quad v_{av} = v/4}$

(b) $v_{av} = v/2$

(c) $v_{av} = 5v/4$

(d) $v_{av} = 5v/3$

(e) $v_{av} = 3v/2$

$$\vec{V}_{av} = \frac{\Delta \vec{X}}{\Delta t} = \frac{\Delta \vec{X}_a + \Delta \vec{X}_b + \Delta \vec{X}_c}{\Delta t_a + \Delta t_b + \Delta t_c}$$

$\Delta t = t_f - t_i$

so $\Delta t_a = t_1 - 0 = t_1$

$\Delta t_b = 2t_1 - t_1 = t_1$

$\Delta t_c = 4t_1 - 2t_1 = 2t_1$

displacement = area under velocity curve = height × width

$\Delta \vec{X}_a = +(v)(t_1)$

$\Delta \vec{X}_b = +(2v)(t_1)$

$\Delta \vec{X}_c = -(v)(2t_1)$

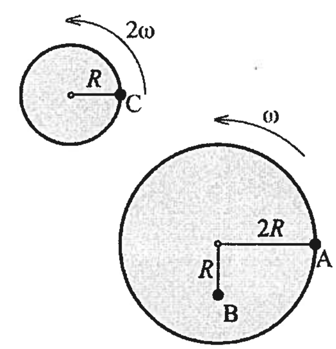
so:

$$\vec{V}_{av} = \frac{+vt_1 + 2vt_1 - 2vt_1}{t_1 + t_1 + 2t_1} = \frac{vt_1}{4t_1}$$

$$\boxed{\vec{V}_{av} = \langle +V/4 \rangle}$$

Question value 8 points

(6) Two wheels are spinning at constant angular speeds as shown in the figure at right. Rank, from greatest to least, the magnitudes of the acceleration vectors for the indicated points A, B, and C.



(a) $a_C > a_A = a_B$

(b) $a_A > a_C = a_B$

$\boxed{\text{(c)} \quad a_C > a_A > a_B}$

(d) $a_B = a_A > a_C$

(e) $a_C > a_B > a_A$

constant angular speed ω

implies: zero angular accel, α

$\Downarrow$

zero tangential accel, $\vec{a}_T$

$\Downarrow$

only component of vector $\vec{a}$ is radial, $\vec{a}_r$ (or $\vec{a}_{centripetal}$)

$\Rightarrow a_r = \frac{V_{Tan}^2}{R}$ but $V_{Tan} = \omega R$

so $a_r = \omega^2 R$

$a_A = \omega^2(2R) = 2\omega^2 R$

$a_B = \omega^2(R) = \omega^2 R$

$a_c = (2\omega)^2 R = 4\omega^2 R$

$$\boxed{a_c > a_A > a_B}$$